Mathematics

Grade 10

Government of Nepal

Ministry of Education

Curriculum Development Center

Sanothimi, Bhaktpur

Mathematics

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Preface

The curriculum and curricular materials have been developed and revised on a regular basis with the aim of making the education objective-oriented, practical, relevant and job oriented. It is necessary to instill the feelings of nationalism, national integrity and democratic spirit in students and equip them with morality, discipline and self-reliance, creativity and thoughtfulness. It is essential to develop in them the linguistic and mathematical skills, knowledge of science, information and communication technology, environment, health and population and life skills. It is also necessary to bring in them the feeling of preserving and promoting arts and aesthetics, humanistic norms, values and ideals. It has become the need of the present time to make the students aware of respect for ethnicity, gender, disabilities, languages, religion, cultures, regional diversity, human rights and social values so as to make them capable of playing the role of responsible citizens. This textbook has been developed in line with the Secondary Level Mathematics Curriculum, 2071 (2014 AD), grade ten by incorporating the recommendations of various education commissions and the feedback obtained from various schools, workshops and seminars, interaction programs attended by teachers, students and parents.

In bringing out the textbook in this form, the contribution of the Executive Director of CDC Mr. Krishna Prasad Kapri, Prof. Dr. Ram Man Shrestha, Laxmi Narayan Yadav, Baikunth Prasad Khanal, Krishna Prasad Pokhrel, Raj Kumar Mathema, Goma Shrestha, Anirudra Prasad Nyaupane and Durga Kandel are highly acknowledged. The contents and language of this book were edited by Harish Pant and Ramesh Prasad Ghimire. The layout and illustrations of the book were done by Jayram Kuinkel. CDC extends sincere thanks to all those who have contributed in developing this textbook.

This book contains a variety of learning materials and exercises which will help learners to achieve the competency and learning outcomes set in the curriculum. Each unit deals with all mathematical skills and the subject matters required to practice various learning activities. There is uniformity in the presentation of the activities which will certainly make it convenient for the students. The teachers, students and other stakeholders are expected to make constructive comments and suggestions to make it a more useful learning material.

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Contents

Unit: 1

Sets

1.0 Review:

Discuss on the following topics:

I. Definition and types of sets.

II. Union of two sets and their cardinal numbers.

III. Intersection of two sets and their cardinal numbers.

IV. Difference and symmetric difference of two sets.

V. Complement of a set and Venn-diagram.

After discussion, prepare a group report in newsprint or cardboard papers and then present to the class.

We have already discussed above topics in previous classes. Now we are going to discuss the problem of sets related to cardinality.

Moreover, we have to know about equal and equivalent sets.

Suppose there are three sets: A = {a, e, i, o, u}, B = {2, 4, 5, 7, 8} and C = {a, e, i, o, u}

Here, n(A) = n(B) = n(C) = 5. So A, B and C are equivalent sets.

Also, set A and set C have equal number and the same elements. They are called equal sets.

Remember all equal sets are equivalent sets but all equivalent sets may not be equal sets.

1.1 Problem Including Two Sets

Consider the following sets

U = {The set of natural numbers from 1 to 15}

A = {The set of even numbers from 1 to 15}

B = {The set of odd numbers from 1 to 12}

C = {The set of prime numbers from 1 to 15}

Now, draw Venn- diagrams of the following cases:

(i) Set A and set B.

(ii) Set B and set C.

The Venn-diagram of set A and B

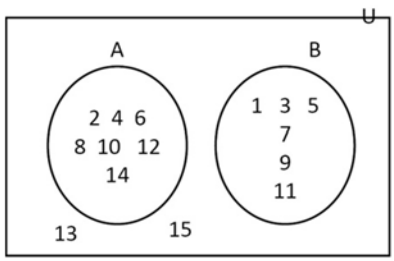


Fig. 1

The Venn-diagram of set B and C

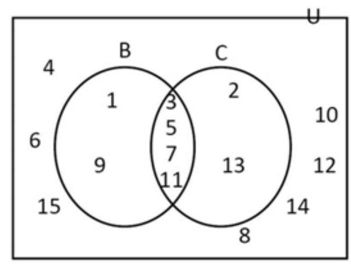


Fig. 2

List the elements of $A\cup B$, $A\cap B$, $B\cup C$ and $B\cap C$ by using above diagrams.

Here, in figure 1, $A\cup B$ ={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14}

Hence, $n(A\cup B) = 13$

And, $A\cap B = \phi$

So, $n(A\cap B) = 0$

Again, $B\cup C$ = {1, 2, 3, 5, 7, 9, 11,13}. This implies $n(B\cup C) = 8$

and, $B\cap C$ = {3, 5, 7, 11}. This implies that $n(B\cap C) =4$

Now, compare the sum of cardinal numbers of set A and set B with cardinal number of set $A\cup B$.

Similarly, compare the $n(B) + n(C)$ with $n(B\cup C)$ and find the conclusion.

Here, $n(A) = 7$, $n(B) = 6$

$n(A) + n(B) = 7 + 6 =13$

Also, $n(A\cup B) = 13 = n(A) + n(B)$

> Here, if A and B be two disjoint subsets of universal set U,
> then $n(A\cup B) = n(A) + n(B)$

Also, $n(B) = 6$ and $n(C) = 6$

$n(B\cup C) =8$

$n(B) + n(C) = 6+6 = 12$

So, $n(B\cup C) = 8 = 12-4$

$= 6 + 6 - 4$

$\therefore n(B\cup C) = n(B) + n(C) - n(B\cap C)$

> If A and B are two intersecting subsets of universal set U, then
> $n(A\cup B) = n(A) + n(B) – n(A\cap B)$

From the above Venn-diagrams, we can conclude that:

i. $n(A \cup B) = n(A) + n(B)$ (For disjoint sets only)

ii. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

iii. $n_o(A) = n(A) - n(A \cap B)$

iv. $n(A \cup B) = n_o(A) + n_o(B) + n(A \cap B)$

v. $n(U) = n(A \cup B) + n(\overline{A \cup B})$

Example 1:

Draw a Venn-diagram and find

i. $n(A \cup B)$ ii. $n(A \cap B)$ iii. $n_o(A)$ iv. $n(\overline{A \cup B})$

Where,

U = {set of natural number less than 21}

A = {set of factors of 12}

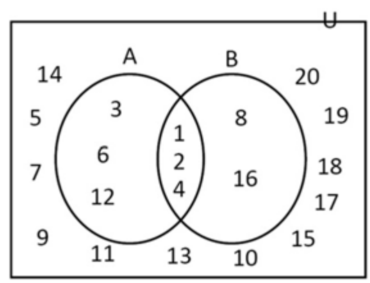
B= {set of factors of 16}

Solution: We have,

U = {1, 2, 3, 4, 5 ,...... , 20}

A = {set of factors of 12} = {1, 2, 3, 4, 6, 12}

B= {set of factors of 16} = {1, 2, 4, 8, 16}



Here, $n(U) = 20$

$n(A) = 6$

$n(B) = 5$

Now by using Venn-diagram

i. $A \cup B = \{1, 2, 3, 4, 6, 8, 12, 16\} \Rightarrow n(A \cup B) = 8$

ii. $(A \cap B) = \{1, 2, 4\} \Rightarrow n(A \cap B) = 3$

iii. $n_o(A) = n(A) - n(A \cap B) = 6 - 3 = 3$

iv. $\overline{A \cup B} = U - (A \cup B) = \{5, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19, 20\}$

Therefore, $n(\overline{A \cup B}) = 12$

Example 2:

A survey of students of Gyanjyoti Higher Secondary school shows that 45 students like mathematics and 41 students like science. If 12 students like both the subjects, how many students like either mathematics or science?

Solution:

Let, M = set of students who like mathematics

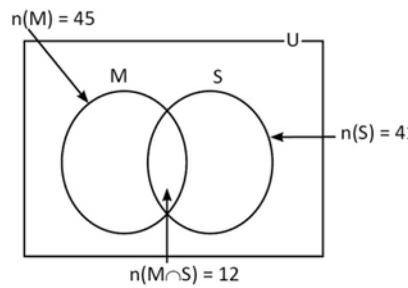
S = set of students who like science

Then, by question, n(M) = 45

n(S) = 41

Number of students who like both math & science = n(M∩S) = 12

The Venn-diagram shows the survey result of the number of students who like either mathematics or science [n(MUS)] = ?



Now, we have, n(M∪S) = n(M) + n(S) –n(M∩S)

= 45 + 41 – 12 = 74

Example 3:

A survey carried among 850 villagers shows that 400 of them like to make a water tank, 450 like to make an irrigation plant and 150 like to make both of them. Represent the information in a Venn–diagram and find:

i. the number of people who like to make a water tank only.

ii. the number of people who like to make either a water tank or an irrigation plant.

iii. the number of villagers who like neither of them.

Solution:

The Venn – diagram is as given.

Let,

T = set of villagers who like to make a water tank

I = set of villagers who like to make an irrigation plant

Then, by question,

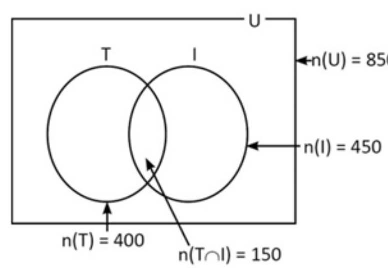
n(U) = 850, n(T) = 400, n(I) = 450

n(T∩I) = 150

(i). n_0(T) = ?

(ii) n(T∪I) = ?

(iii) n($\overline{T \cup I}$) = ?



Now,

(i) n_0 (T) = n(T) - n(T∩I) = 400 – 150 = 250

(ii) n(T∪I) = n(T) + n(I) - n(T∩I)

= 400 +450 – 150 = 700

(iii) n($\overline{T \cup I}$) = n(U) – n(T∪ I)

= 850 – 700 = 150

Example 4:

In a survey of some people, 73% like to drink tea, 85% like to drink coffee and 65% like to drink tea as well as coffee. If 210 people like neither tea nor coffee, then find the total number of people taken part in the survey. Also, by a Venn-diagram show how many of them like at least one of the given drink ?

Solution:

Suppose the total number of people = 100. n(U) = 100

Let T = set of people who like to drink tea.

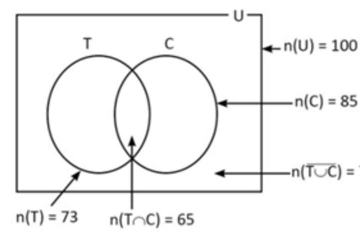
C = set of people who like to drink coffee

By the question n(T) = 73, n(C) = 85 and $n(T\cap C) = 65$

$n(T\cup C) = n(T) + n(C) - n(T\cap C)$

$= 73 + 85 - 65$

$= 93$



Also, the people who neither like tea nor coffee is

$n(\overline{T\cup C}) = n(\cup) - n(T\cup C)$

$= 100 - 93 = 7$

Hence, 7% people like neither of them.

The number of people who do not like both coffee and tea = 210

i.e. $n(\overline{T\cup C}) = 210$

Let the total number of people taken part in survey be x.

Then, 7% of x = 210

Or, $\frac{7x}{100} = 210$

Or, $x = 3000$

The total no of people taken part in survey is 3000.

Now, The number of people who like at least one drink is

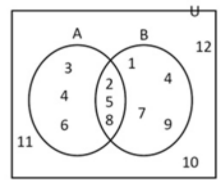
$n(T\cup C) = n(U) - n(\overline{T\cup C})$

$= 3000 - 210$

$= 2790$

Exercise 1.1

1. **Use the given Venn-diagram and find the followings:**

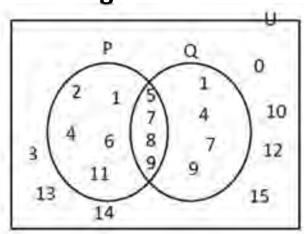


(a) $n(A)$ (b) $n(B)$ (c) $n(A \cap B)$

(d) $n(A \cup B)$ (e) $n_o(A)$ (f) $n_o(B)$

(g) $n(\overline{A \cup B})$

2. **Use the given Venn-diagram and verify the following identities.**



(a) $n_o(P) + n_o(Q) = n(P-Q) + n(Q-P)$

(b) $n(\overline{P \cup Q}) = n(U) - n(P \cup Q)$

(c) $n_o(P) + n_o(Q) + n(P \cap Q) = n(P \cup Q)$

3.(a) In a survey of a school, 300 students favour to play volleyball, 250 favour to play cricket and 110 favour both of the games. Draw a Venn-diagram and calculate

i. the number of students who play cricket only.

ii. the number of students who play either volleyball or cricket.

(b) In a survey, after their SEE, 190 students wanted to be an engineer, 160 wanted to be a doctor and 120 wanted to be both. If 300 students were interviewed, draw a Venn-diagram and find the number of students who wanted to be neither of them.

(c) In a survey of 120 adults, 88 drink cold drinks and 26 drink soft drink. If 17 of them drink neither of them, find out the number of adults who drink both cold and soft drink by using a Venn-diagram.

4.(a) In a survey of youths, it was found that 85% liked to do something in their village, 60% liked to go to foreign employment. If 5% of them did not like both of them, find:

I. The percent of youths who like to do something in their village only.

II. The percent of youths who like foreign employment only.

III. Draw a Venn–diagram to illustrate the above information.

(b) In a certain exam of grade ten, 75% students got high score in mathematics, 65% students got high score in English. If 6% of them did not get high score in both mathematics and English, then calculate:

i. the percent of students who got high score in both the subjects.

ii. the total number of students who got high score either in mathematics or in English if 300 students had attended the exam.

5.(a) In a class of 37 students, the number of students who like marshal arts only is double than the number of students who like athletics only. If 3 students like both and 4 like none of the games, find out how many students like:

i. Marshal arts ii. Athletics

(b) A survey was conducted in a group of 100 students of a school. The ratio of students who like mathematics and computer is 3:5. If 30 of them like both subjects and 10 of them like none of them, construct a Venn-diagram to find the number of students who like

i. Mathematics only ii. computer only iii. at most one subject

6. **(Project work)** Work in a group of 4 students or suitable group. Ask the students in a class to mark one of the following in your school.

i. Like playing

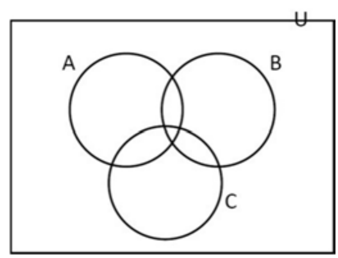
ii. Like dancing

iii. Like both playing and dancing.

iv. Like none of playing and dancing.

Draw a Venn-diagram of your data and present it to the class.

1.2 Problems Including Three Sets

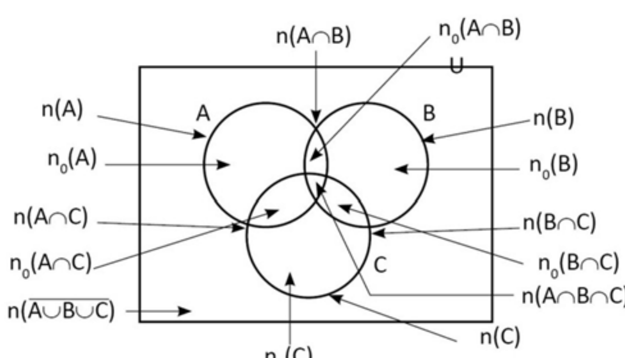


In the adjoining Venn – diagram, three intersecting subsets of Universal set are given. List different disjoint sets from the Venn-diagram.

The disjoint sets are A only, B only, C only, A and B only, B and A only, all A, B and C, and neither A, B or C.

Then their cardinality is denoted as follows $n_o(A)$, $n_o(B)$, $n_o(C)$, $n_o(A\cap B)$, $n_o(B\cap C)$, $n_o(A\cap C)$, $n_o(A\cap B\cap C)$ and $n_o(\overline{A \cup B \cup C})$.

Then, the Venn diagram is as follows



By Venn–diagram

(i) $n(A\cup B\cup C) = n_o(A) + n_o(B) + n_o(C) + n_o(A\cap B) + n_0(B\cap C) + n_0(C\cap A) + n(A\cap B\cap C)$

(ii) $n(A) = n_o(A) + n_o(A\cap B) + n_o(A\cap C) + n(A\cap B\cap C)$

Also,

$n(A\cup B\cup C) = n[A\cup(B\cup C)]$

$= n(A) + n(B\cup C) – n[(A\cap B) \cup (A\cap C)]$

$= n(A) + n(B) + n(C) - n(B\cap C) - [n(A\cap B) + n(A \cap C) – n\{(A\cap B) \cap (A\cap C)\}]$

$= n(A) + n(B) + n(C) - n(B\cap C) - n(A\cap B) - n(A \cap C) + n(A\cap B \cap C)$

$\therefore n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B \cap C)$

Based on above formula we can derive the following formulas for calculation.

(i) $n_o(A) = n(A\cup B\cup C) - n(B\cup C)$ = elements lie only in set A.

(ii) $n_o(B) = n(A\cup B\cup C) - n(A\cup C)$ = elements lie only in set B.

(iii) $n_o(C) = n(A\cup B\cup C) - n(A\cup B)$ = elements lie only in set C.

(iv) $n_o(A) = n(A) - n(A\cap B) - n(A\cap C) + n(A\cap B\cap C)$

(v) $n_o(B) = n(B) - n(A\cap B) - n(A\cap C) + n(A\cap B\cap C)$

(vi) $n_o(C) = n(C) - n(A\cap C) - n(B\cap C) + n(A\cap B\cap C)$

(vii) $n_o(A\cap B) = n(A\cap B) - n(A\cap B\cap C)$ = elements lie only in set $A\cap B$.

(viii) $n_o(B\cap C) = n(B\cap C) - n(A\cap B\cap C)$ = elements lie only in set $B\cap C$.

(ix) $n_o(A\cap C) = n(A\cap C) - n(A\cap B\cap C)$ = elements lie only in set $A\cap C$.

(x) $n(A\cup B\cup C) = n(U) – n(\overline{A \cup B \cup C})$

Example 1:

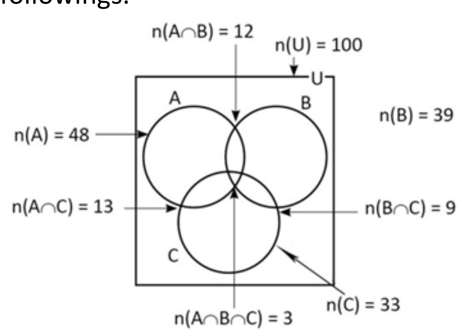
Use the adjoining Venn–diagram and calculate the followings.

(i) $n_o(A)$

(ii) $n_o(B\cap C)$

(iii) exactly two of them

(iv) $n(\overline{A\cup B\cup C})$



Solution:

(i) $n_o(A) = n(A) - n(A\cap B) - n(A\cap C) + n(A\cap B\cap C)$

$= 48 – 12 – 13 + 3 = 26$

(ii) $n_o(B\cap C) = n(B\cap C) - n(A\cap B\cap C) = 9-3 = 6$

(iii) $n_o(A\cap B) = n(A\cap B) - n(A\cap B\cap C) = 12-3 = 9$

$n_o(A\cap C) = n(A\cap C) - n(A\cap B\cap C) = 13-3 = 10$

$\therefore$ Exactly two of them $= n_o(A\cap B) + n_0(B\cap C) + n_0(C\cap A)$

$= 6 + 9 + 10$

$= 25$

(iv) now $n(A\cup B\cup C)$ $= n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C)$

$= 48 + 39 + 33 - 12 - 9 - 13 + 3$

$= 89$

$\therefore n\overline{A\cup B\cup C})$ $= n(U) - n(A\cup B\cup C)$

$= 100 - 89 = 11$

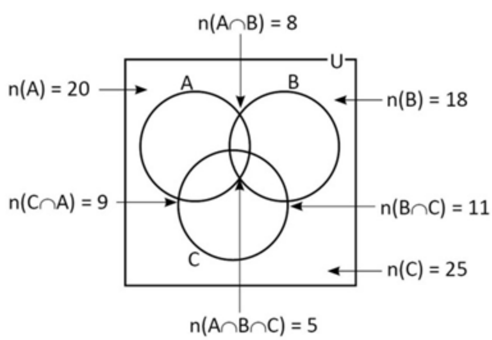
Example 2:

Given that $n(A) = 20$, $n(B) = 18$, $n(C) = 25$, $n(A\cap B) = 8$, $n(A\cap C) = 9$, $n(B\cap C) = 11$ and $n(A\cap B\cap C) = 5$. Represent this information in a Venn- diagram and find:

i) $n(A\cup B\cup C)$ ii) exactly one of them.

Solution:

The Venn- diagram representing the given information is as given below:



By using the Venn-diagram

i. $n(A\cup B\cup C)$ $= n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C)$

$= 20 + 18 + 25 - 8 - 11 - 9 + 5 = 40$

ii. $n_o(A) = n(A) - n(A\cap B) - n(A\cap C) + n(A\cap B\cap C)$

$= 20 - 8 - 9 + 5 = 8$

$n_o(B) = n(B) - n(A\cap B) - n(B\cap C) + n(A\cap B\cap C)$

$= 18 - 8 - 11 + 5 = 4$

$n_o(C) = n(C) - n(A\cap C) - n(B\cap C) + n(A\cap B\cap C)$

$= 25 - 11 - 9 + 5 = 10$

$\therefore$ exactly one of them $= n_o(A) + n_0(B) + n_0(C)$

$= 8 + 4 + 10 = 22$

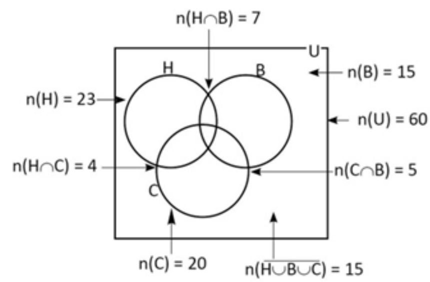
Example 3 :

In a survey of 60 students, 23 like to play hockey, 15 like to play basketball and 20 like to play cricket. 7 of them like to play both hockey and basketball, 5 like to play both cricket and basketball, 4 like to play both hockey and cricket and 15 students do not like to play any of these games. Draw a Venn-diagram and find:

(i) how many students like to play hockey, basketball and cricket.

(ii) how many students like to play hockey but not cricket.

(iii) how many students like to play hockey and cricket but not basketball.



Solution:

Let H, B and C denote the set of students who like to play the games hockey, basketball and cricket respectively.

By question,

$n(U) = 60$, $n(H) = 23$, $n(B) = 15$,

$n(C) = 20$, $n(H \cap B) = 7$, $n(C \cap B) = 5$, $n(H \cap C) = 4$

and $n(\overline{H \cup B \cup C}) = 15$

Now, $n(H \cup B \cup C) = n(U) - n(\overline{H \cup B \cup C})$

$= 60 - 15 = 45$

(i) We have to find $n(H \cap B \cap C) = ?$

$n(H \cup B \cup C) = n(H) + n(B) + n(C) - n(H \cap B) - n(B \cap C) - n(H \cap C) + n(H \cap B \cap C)$

Or, $45 = 23 + 15 + 20 - 7 - 5 - 4 + n(H \cap B \cap C)$

Or, $n(H \cap B \cap C) = 45 - 58 + 16 = 3$

(ii) Number of students playing hockey but not cricket = $n(H-C) = n(H) - n(H \cap C)$

$= 23 - 4 = 19$

(iii) Number of students playing hockey and cricket but not basketball

$= n_0(H \cap C) = n(H \cap C) - n(H \cap B \cap C) = 4 - 3 = 1$

Example 4:

Among the students in an examination, 42% offered mathematics, 35% offered English and 30% offered Computer. If 20% offered none of these subjects, 9% offered mathematics and Computer, 10% offered English and Computer and 11% offered mathematics and English. Draw a Venn diagram and find

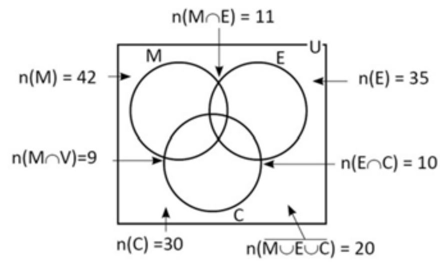
(i) the percent of students offering all three subjects

(ii) the percent of students offering mathematics only.

(iii) the number of students offering English and computer only if 200 students are attended the examination.

Solution:

In the Venn-diagram, let M, E and C denote the sets of students who offered the subjects mathematics, English and computer respectively.



Now, let n(U) = 100. then,

$n(M) = 42$, $n(E) = 35$, $n(C) = 30$

$n(M\cap E) = 11$, $n(E\cap C) = 10$, $(M\cap C) = 9$,

and, $n(\overline{M\cup E\cup C}) = 20$

(i) $n(M\cap E\cap C) = ?$

(ii) $n_0(M) = ?$

(iii) $n_0(E\cap C) = ?$

Now, $n(M\cup E\cup C) = n(U) - n(\overline{M\cup E\cup C})$

$= 100 - 20 = 80$

Percent of students who offered at least one subject = 80%

i. We have,

$n(M\cup E\cup C) = n(M) + n(E) + n(C) - n(M\cap E) - n(E\cap C) - n(C\cap M) + n(M\cap E\cap C)$

or, $80 = 42 + 35 + 30 - 11 - 10 - 9 + n(M\cap E\cap C)$

or, $n(M\cap E\cap C) = 80 - 107 + 30 = 3$

$\therefore$ 3% students offered all three subjects

ii. $n_o(M) = n(M) - n(M\cap E) - n(M\cap C) + n(M\cap E\cap C)$

$= 42 - 11 - 9 + 3 = 25$

$\therefore$ 25% students offered mathematics only.

iii. $n_o(E\cap C) = n(E\cap C) - n(M\cap E\cap C)$

$= 10 - 3 = 7$

$\therefore$ Percent of students who offered English and computer only is 7%.

The number of students who offered English and computer only is

7% of 200

$= \frac{7\times 200}{100} = 14$

$n_o(E\cap C) = 14$

Exercise 1.2

1. Let U = {x : x is a positive number less than 20}

   A = {1, 2, 3, 6, 7, 8, 9, 10},

   B = {2, 4, 5, 6, 10,12, 15} and

   C = {6, 8, 10, 12, 15, 16, 17, 18}.

   Then, draw a Venn diagram and calculate

   (a) n(A) (b) $n_0(B)$ (c) $n_o(A \cap C)$ (d) $n_o(A \cap B \cap C)$

   (e) $n(\overline{A \cup B \cup C})$ (f) only one of A or B or C

2. If U = 65 n(A) = 32 n(B) = 20, n(C) = 22, n (A ∩B) = 8,

   n (B ∩C) = 6, n(C ∩A) = 7 and n(A ∩B∩C) = 4, draw a Venn-diagram and calculate:

   (a) $n[(A \cup B) \cap C]$ (b) $n(A \cup B \cup C)$ (c) $n_o(B \cap C)$

   (d) exactly two of A or B or C (e) exactly one of three (f) neither of A or B or C.

3. If U = {The set of whole numbers less than 30}

   X = {The set of multiples of 2 less than 30}

   Y = {The set of multiples of 3 less than 30}

   Z = {The set of multiples of 5 less than 30}

   Represent the above sets in a Venn–diagram and verify

   (a) $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

   (b) $n(X \cup Y \cup Z) = n(X) + n(Y) + n(Z) - n(X \cap Y) - n(Y \cap Z) - n(X \cap Z) + n(X \cap Y \cap Z)$

   (c) $n(X \cup Y \cup Z) = n(X - Y) + n(Y - Z) + n(Z - X)$

4. (a) In a survey, 135 students appeared in an examination, 60 students got A+ grade in mathematics, 70 got A+ grade in science and 35 got A+ grade in social studies. 20 of them got A+ grade in mathematics and science, 15 got A+ grade in mathematics and social studies and 10 got A+ in science and social studies. If 5 did not get A+ grade in any of the three subjects, find how many of them got A+ grade in all three subjects.

   (b) In a survey of 100 students, 60 like to play football, 48 like to play volleyball and 40 like to play cricket. Similarly 32 of them like to play football and volleyball, 22 like to play football and cricket and 20 like to play both volleyball and cricket. If 5 students like to play all three games, represent the above information in a Venn-diagram and find the number of students who like

   (i) none of the games

   (ii) exactly two of the given games

(iii) only of the three games.

(c) Among the applicants in a certain vacant post, it is found that 70 are qualified in statistics, 60 are in computer and 50 in English. Also, 30 are qualified in statistics and computer, 20 in computer and English and 25 in English and statistics. If 20 are qualified in all three subjects and each are qualified in at least one subject, then;

(i) represent the information in a Venn-diagram

(ii) find the number of applicants who are qualified only in computer.

(iii) find total number of applicants.

5. (a) In a survey of a community, 40% favour Dashain, 45% favour Tihar and 55% favour Chhath. 10% of them favour both Dashain and Tihar, 20% favour Tihar and Chhath and 15% favour Chhath and Dashain. Then;

(i) represent the above information in a Venn – diagram.

(ii) calculate the percent of people who favour all three festivals.

(iii) if 80 community members have taken part in the survey, find the number of people who favour exactly one festival.

(b) In a survey of tourists who have arrived in Tribhuvan International Airport, 65% want to go to Pokhara, 55% like to go to Lumbini and 40% like to go to Ilam. Also, 30% like to go to Pokhara and Lumbini, 20% like to go to Lumbini and Ilam and 25% like to go to Ilam and Pokhara. If 10% like to go all the three places, then;

(i) represent the above data in a Venn-diagram.

(ii) what percent of tourists like to go to exactly two places?

(iii) what percent of tourists do not like to go to any of the places?

(c) For vacation, some students were asked whether they like to go picnic, hiking or tour. The result was 60% students like to go picnic, 45% hiking, 20% tour, 15% picnic and hiking, 12% hiking and tour, 10% tour and picnic and 7% none of them. If 15 students like all three programs;

(i) represent the above information in a Venn-diagram

(ii) how many students are taking part in the programs?

(iii) how many students like to go picnic only?

6. (a) In a survey of 100 people, 65 read daily newspapers, 45 read weekly newspapers, 40 read monthly newspapers, 25 read daily as well as weekly, 20 read daily as well as monthly and 15 read at least one type of newspaper. Find:

(i) how many people read all three types of newspaper.

(ii) the number of people who read exactly two newspapers.

(b) In a survey, 50% like cold drinks, 30% like hot drinks and 40% like juice. Likewise, 20% of them like cold and hot drinks, 18% like cold drinks and juice, 12% like hot drinks and juice and 5% like all three drinks. Represent the above information in a Venn–diagram and find percent of people who like:

(i) at least one of the three drinks. (ii) exactly two types of drinks.

(iii) exactly one type of drink. (iv) none of the drinks.

7. Work in the group of students. Each student of every group takes data from one of each class of the school with following opinionnaire:

(a) Like coffee (b) Like tea

(c) Like green tea (d) Like coffee and tea

(e) Like tea and green tea (f) Like green tea and coffee

(g) Like all three of the above (h) Like none of the above

Then represent the collected data in a Venn-diagram and present it to the classroom.

Unit: 2 Tax and Money Exchange

2.0 Review:

Before starting the discussion about VAT and money exchange we have to review the following concept.

Profit and loss:

If C.P. and S.P. are cost price and selling price of an article respectively, then

Profit (P) = S.P. – C.P. when [S.P. > C.P.]

Loss (L) = C.P. – S.P. when [C.P. > S.P.]

Also, Profit/Loss % = $\frac{\text{net profit/Loss}}{\text{cost price}} \times 100\ \%$

Tax and Income Tax

The compulsory contribution levied by the government to its people or the business forms or companies which is paid in terms of money is called tax. The government pays back this money in terms of service such as security and welfare, communication, education, health, etc. Income tax is such type of tax which a person has to pay certain amount for the excess of income at certain rate fixed by the government.

The income which is not taxable is called the allowance and income above the allowance is called a taxable amount. For example, according to fiscal year 2073/74, the following table gives the allowance and taxable amount.

	Assessed as individual	**Assessed as couple**	**Tax rate**
(First tax sales) Allowances	Rs 3,50,000	Rs. 4,00,000	1%
Next	Rs.1,00,000	Rs. 1,00,000	15%
Balance exceeding	Rs 4,50,000	Rs. 5,00,000	25%

Please find other rates and discuss with friends.

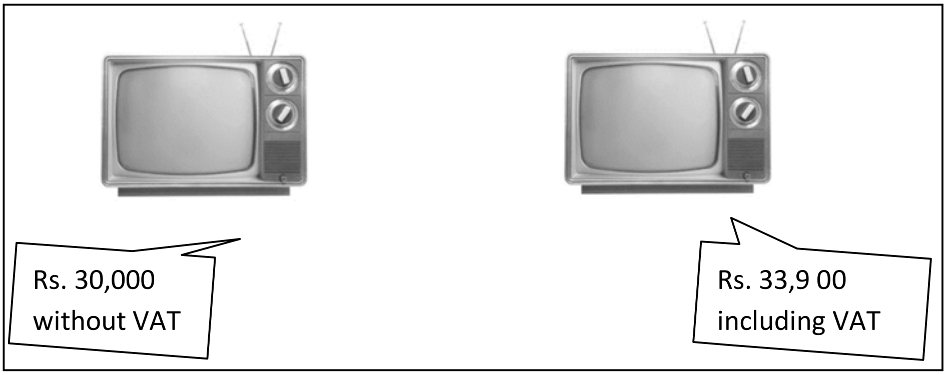
Discount:

If an allowance is given to an agent for the distribution of goods is called Trade Discount. The price deduction in the marked price of the goods is known as the discount. The discount is generally expressed as percentage and calculated on the marked price.

Discount% = $\frac{\text{True discount}}{\text{Marked Price}} \times 100\ \%$

2.1 Value added Tax (VAT)

Observe the following two cases and answer the given questions.



(a) What is the cost of two TV sets?

(b) What is the given condition in the cost of two TV sets?

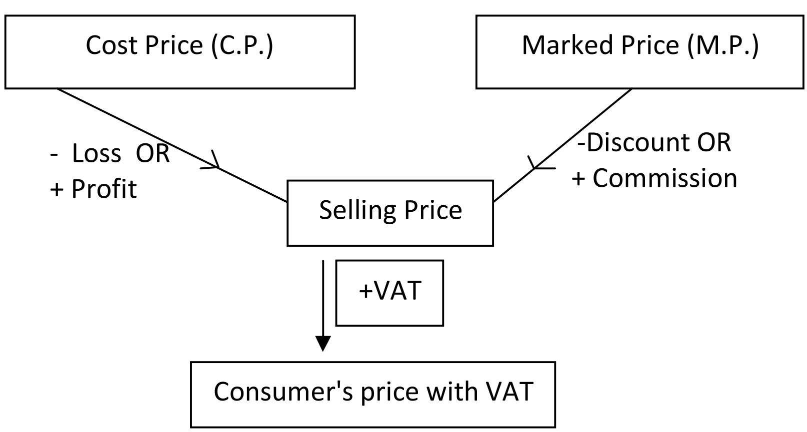
(c) Which TV set will be paid high?

(d) Which condition is economic to purchase? Explain.

From the above questions, we can conclude that the first TV set has no VAT added and second includes VAT. So, VAT is the amount which is added to the selling price of any good or service.

Therefore, the additional amount (consumption tax) that the consumer has to pay due to purchases of any goods or service is called value added tax (VAT). It is generally calculated in percentage. The VAT rate in our present situation is 13% (Fiscal year 2073/74).

The VAT is changeable to the supply of goods and services including transport, insurance, commission to the agent local taxes and profit. In case of discount the VAT is chargeable after subtracting the discount amount. i.e



The VAT percent = $\frac{\text{SP with VAT- SP}}{\text{SP}} \times 100\ \%$

The list of goods and services on which VAT is levied and the rate is according as the government provides in every fiscal year. Visit to the nearest Tax Office and collect such goods or services.

Example 1:

Calculate VAT amount of the following

(a) Selling price = Rs. 45,000 and VAT =13%

(b) Marked price = Rs. 14,000, discount = 10% and VAT = 13%

Solution:

(a) We have,

SP = Rs. 45,000

VAT = 13%

VAT amount = 13% of Rs. 45,000

$= \frac{13 \times 45000}{100}$

= Rs. 5850

(b) MP = Rs. 14,000

Discount = 10%

VAT = 13%

VAT amount = ?

We have, discount amount = 10% of Rs.14,0 00 = Rs. 1400

Selling price (SP) = MP – Discount

= 14000 – 1400

= Rs. 12,600

VAT amount = 13% of SP = 13% of Rs. 12,600

$= \frac{13 \times 12,600}{100}$

= Rs. 1638

Example 2:

The marked price of a computer set is Rs 28,000. If the seller provides 5% discount and then have to pay 13% VAT, what amount will be paid by consumer?

Solution:

Here, Marked price (MP) = Rs 28,000

Discount = 5%

VAT = 13%

Discount amount = 5% of Rs.28,000

$= \frac{5\times28000}{100}$

= Rs.1400

The selling price of computer set = Rs.28,000 – Rs.1400

= Rs. 26,600

VAT amount = 13% of Rs.26,600

$= \frac{13\times26600}{100}$ = Rs.3,458

The consumer has to pay Rs 26,600 + Rs.3,458 = Rs. 30,058

Example 3:

Enjal paid Rs 15,255 for a smart phone with 10% discount and 13% VAT. Find marked price of that smart phone.

Solution:

Here, Discount = 10%

VAT = 13%

Paid amount = Rs. 15,255

Marked price (MP) = ?

Let the marked price be x

By question, Discount = 10% of $x = \frac{x\times10}{100} = \frac{x}{10}$

The price after discount = MP – Discount

$= x - \frac{x}{10} = \frac{9x}{10}$

The VAT is levied for $\frac{9x}{10}$

VAT amount = 13% of $\frac{9x}{10} = \frac{9x\times13}{10\times100} = \frac{117x}{1000}$

Now, True price = Rs 15,255

Or, $\frac{9x}{10} + \frac{117x}{1000}$ = 15,255

Or, $\frac{900x+117x}{1000} = 15{,}255$

Or, $1017x$ = 1,52,55,000

Or, $1017x$ = 1,52,55,000

Or, $x = \frac{15255{,}000}{1017}$ = 15,000

∴ MP = Rs.15,000

Example 4

Timilsina Suppliers sold some construction materials of amount Rs. 4,40,000 to Adhikari suppliers with 10% profit and 13% VAT. Adhikari supplier added Rs. 5,000 as transportation charge, 10% profit and Rs. 250 local tax at their cost price and sold to consumer. Find VAT amount paid by the consumer if he/she has to pay 13% VAT.

Solution:

For Timilsina suppliers

Cost price (CP) = Rs, 4,40,000

Profit = 10% of Rs. 4,40,000 = Rs. 44,000

VAT = 13%

Total cost price = Rs. 4,40,000 + Rs. 44,00 = Rs. 4,84,000

The selling price with VAT = Rs. 4,84,000 + 13% of Rs. 4,84,000
= Rs. (4,84,000 + 62,920)
= Rs. 5,46,920

Again, for Adhikari Suppliers

Cost Price (CP) = Rs. 5,46,920

Profit = 10% of Rs. 5,46,920 = Rs. 54,692

Transportation exp. = Rs. 5,000

Local Tax = Rs. 250

Total price to be paid without VAT = Rs. (5,46,920 + 5,46,92 + 5,000 + 250)
= Rs. 6,06,862

VAT amount to be paid by a consumer = 13% of Rs. 6,06,862
= Rs. 78,892.06

Exercise 2.1

1. **Define the following terminologies:**

(a) Profit percent	(b) Loss percent	(c) Discount
(d) Bonus	(e) Income tax	(f) VAT

2. **Calculate the price with VAT of the following cases:**

(a) MP = Rs. 4,200	VAT = 13%		
(b) MP = Rs. 14,700	Discount = 10%	VAT = 13%	
(c) MP = Rs. 70,000	Discount = 5%	VAT = 13%	
(d) MP = Rs. 1,20,000	Discount = 12%	VAT = 13%	
(e) MP = Rs. 4,00,000	Profit = 20,000	Discount = 10%	VAT = 13%

3. **Calculate the Marked Price of each of the following.**

(a) Discount = 15%	VAT = 13%	paid amount = Rs.5,763	MP =?
(b) Discount = 25%	VAT = 13%	paid amount = Rs.3,390	MP =?
(c) Discount = 15%	VAT = 13%	paid amount = Rs. 7,49,190	MP = ?
(d) Bonus = 5%	VAT = 13%	paid amount = Rs.13,378	MP = ?
(e) Bonus = 4%	VAT = 13%	paid amount = Rs.1,99,784	MP = ?

4.(a) The marked price of a mobile set is Rs 6,000. What will be the price of that mobile set if 13% VAT is levied after allowing 5% discount? Find it.

(b) The marked price of an electric water heater is Rs. 5200. If the shopkeeper allows 5% discount and adds 13% VAT, than how much will the customer pay for that heater? Find it.

(c) The marked price of a motorcycle was Rs. 1,85,000. What would be the price of the motorcycle if 13% VAT was levied after allowing 10 % festival discount? Find it.

(d) The marked price of a camera is Rs 16,000. If the camera is sold with 15% discount and 13% VAT, then find the amount of VAT.

5.(a) Pemba bought a TV set with 13% VAT after 15% discount for Rs 7,203.75. What will be actual price of TV? Also find the VAT amount

(b) An electric equipment is sold at Rs 1,11,870 after allowing 10% discount and 13% VAT. Find VAT and discount amount .

(c) The price of a cycle after allowing 15% discount and 13% VAT is Rs.19,323. Find the amount of VAT levied and marked price.

(d) A photocopy machine was sold at 12% discount with 13% VAT. If the customer paid Rs. 3,57,984 then find VAT amount and discount amount.

6.(a) The price of a calculator is Rs 3000 excluding 13% VAT in the shop A, while its price is Rs 3,277 including VAT in the shop B. Which shop is cheaper and by how much?

(b) 13% VAT is levied on a handicraft after 10% discount. If the VAT amount is Rs 910, then find the marked price and selling price of it with VAT.

(c) A shopkeeper has to pay 7% bonus for selling some goods. If he paid Rs 17,500 bonus and then 13% VAT, what will be marked price and the price including VAT?

(d) Amrit bought a smart watch for Rs.23,391. If he gets 10% discount of amount Rs. 2,300, find the rate of VAT?

7.(a) Sonia sold a machine of price Rs. 1,50,000 adding 13% VAT to Binod. Binod sold it to Enjila by adding transportation cost Rs. 4,000, profit Rs. 7,000 and Rs. 1,500 local tax. If Enjila has to pay 13% VAT, find the VAT amount paid by Enjila.

(b) A dealer of electric oven sold an induction heater at Rs. 4,200 with 13% VAT to a retailer. The retailer added transportation cost of Rs. 250, profit 15% and local tax Rs. 150 and sold to a consumer. How much amount will be paid for that heater if he/she has to pay 13% VAT.

8. Collect the bills of electricity and drinking water of your home or neighbors or the office near to your home. Compare the VAT rate and VAT amount in different bills. Prepare a group report and present to the classroom.

2.2 Money Exchange:

Discuss on the following questions:

(a) Suppose you are going to educational tour to India or China for a week. Can you take Nepali Rupee there and can spend this directly?

(b) Basanti is a business person. She imports goods from other country. Can she pay the bills of her import directly using Nepalese rupees?

The system of money that a country uses is called currency. Every country has their own currency like as Rupee for Nepal, Yen in Japan, Mark in Germany, etc. Also the values of the currencies are different. The economic condition of the country adjusts the increase or decrease of the value of currency.

To visit one country to another country, we need the currency of that country. For example, if we are going to visit China, we need Chinese Yuan. So we have to change Nepali rupees to Chinese Yuan.

The exchange rate of currency of a country to another country will be determined by the government or the central bank of the country. That exchange rate is called the foreign currency exchange rate. In our country, the exchange rate is decleared by Nepal Rastra Bank. The rate of exchange of 2073 phalgun 9 (20 February 2017) is as follows.

Nepal Rastra Bank

Central Office

Foreign Currency Exchange Department

Money Exchange Rate Declared by Nepal Rastra Bank

Currency	Unit	Buying Rate	Selling rate
Indian Rupees ₹	100	Rs. 160.00	Rs. 160.15
Open market Exchange Rate (For the use of Rastra Bank)			
Currency	Unit	Buying Rate	Selling rate
US $	1	Rs.106.80	Rs. 107.40
Euro	1	Rs. 113.64	Rs.114.27
Pound Sterling £	1	Rs. 133.64	Rs.134.36
Swiss Frank	1	Rs. 106.65	Rs. 107.25
Australian dollar	1	Rs.82.37	Rs.82.83
Canadian Dollar	1	Rs.81.94	Rs.82.40
Singapore Dollar	1	Rs.75.33	Rs.75.75
Japanese Yen	10	Rs.9.40	Rs.9.45
Chinese Yuan	1	Rs. 15.58	Rs. 15.66
Saudi Arabia Riyal	1	Rs.28.48	Rs.28.64
Quatrain Riyal	1	Rs.29.33	Rs.29.49
Thai Bhatt	1	Rs.3.05	Rs.3.07
U.A.E Dirham	1	Rs.29.08	Rs.29.24
Malaysian Ringgit	1	Rs.23.97	Rs.24.11
South Korean wan	100	Rs.9.38	Rs.9.43
Swedish Corner	1	Rs.12.01	Rs.12.08
Desish Corner	1	Rs. 15.28	Rs.15.37
Honkong Dollar	1	Rs. 13.76	Rs. 13.84
Kuwaiti Dinar	1	349.75	351.71
Bahrain Dinar	1	283.32	284.91

Example 1:

Convert the following currencies into Nepalese rupees. (Use buying rate)

i. $750 ii. Singapore dollar 120 iii. 50 Japanese yen

Solution:

We have,

i. $750 = Rs. (750 x 106.80)
= Rs. 80,100

ii. Singapore dollar 120 = Rs. 120×75.33
= Rs. 9,039.60

iii. We have,
Japanese yen 10 = Rs. 9.40

Japanese yen 1= Rs. $\frac{9.40}{10}$

Japanese yen 50 = Rs. $\frac{9.40}{10} \times 50$ = Rs. 47

Example 2:

By using the above rate, convert the following currencies

i. 1 Canadian Dollar into Japanese Yan

ii. 250 Australian dollars into Swish Frank.

Solution:

i. We have 1 Canadian dollar = Rs. 81.94

Also, Rs.9.40 = 10 Japanese Yen

By chain rule , 1 Canadian dollar × Rs.9.40 = Rs. 81.94×10 Japanese Yen

Or, 9.40 Canadian dollar = 819.4 Japanese Yen

Or, 1 Canadian dollar = $\frac{819.4}{9.4}$ Japanese Yen

= 87.17 Japanese Yen

ii. 250 Australian dollar = Rs. (250× 82.37)= Rs. 20592.5

Also, Rs. 106.65 = 1 Swiss Frank

Therefore, 250 Australian Dollar × Rs. 106.65 = Rs. 20592.50 × 1 Swiss Frank

Or, 250 Australian Dollar = $\frac{20592.5}{106.65}$ Swiss Frank

= 193.08 Swiss Frank

Example 3:

Bijay needs $ 5,000 for America tour. If a broker takes 2% commission for exchange, how much Nepalese rupees will Bijay require? Find it.

Solution:

Required amount = $ 5,000

Commission rate = 2%

We know that $1 = Rs. 107.40 (selling rate)

$ 5,000 = Rs. (107.40 x 5,000) = Rs. 5,37,000

Again commission = 2% of Rs. 5,37,000 = Rs. 10,740

∴ Total Rupee required = Rs. 5,37,000 + Rs. 10,740 = Rs. 5,47,740

(Note: For buying foreign currencies the selling rate of bank is buying rate of us)

Example 4:

Shristi bought some Australian Dollar from Rs. 1,50,000. After 4 days the Nepali Rupee devaluated by 5%. How much profit or loss did she get if she exchanges that to Nepali rupee on that day.

Solution:

The amount with Shristi = Rs. 1,50,000

Rate of exchange = 1 Australian dollar = Rs. 82.83 (selling rate)

∴ From Rs. 82.83 we get 1 Australian dollar

From Rs. 1,50,000 we get $\frac{1}{82.83} \times 1{,}50{,}000$ = 1,810.94 Australian dollar

After 4 day the rate of devaluation = 5%

∴ The rate of exchange for 1 Aus$ = Rs. 82.83 + 5% of Rs. 82.83

= Rs. 82.83 + Rs. 4.14

= Rs. 86.97

∴ 1810.94 Australian dollar = 1810.94 x Rs. 86.97

= Rs. 157,497.45

Shristi gets profit and profit amount = Rs. 1,57,497.45 - 1,50,000

= Rs. 7,497.45

Exercise 2.2

Use the table of exchange rate given above to calculate the following:

1. Convert the following in to Nepalese currency (use buying rate)

(a) ₹ 1325	(b) Quatrain Riyal 5050
(c) South Korean Won 9,75,000	(d) Swiss Frank 650
(e) Australian Dollar 7560	(f) Singapore Dollar 9560
(g) Malaysian Ringgit 5350	(h)UAE Dirham 1200
(i) Canadian Dollar 25450	(j) Chinese Yuan 9600

2 (a) Bishnu earn US $ 9 per hour. If he works 42 hours per week, how many Nepali Rupees will Bishnu earn in a week? Find it.

(b) If the range of income of a teacher per month is $1000 – $3200, find it in Nepali rupees.

(c) What will be per week income of Parbati in NC if she earns 15 Australian Dollars per hour and works 8 hours per day for 5 days? Find it.

(d) Dorje goes Malaysia for 2800 Malaysia Ringgit per month. If he gets 120 Ringgit per month as bonus, find his income in Nepali currency.

3.(a) How much Singapore dollar should be exchanged to get Rs. 24,180.93? ($1 =Rs. 75.33)

(b) How much Euro should a student of a University of England pay for exam fee if he/she will pay Rs. 9,659.40 from Nepal? Find it.

(c) A person invested Rs 3,87,139 for study an Australia. How much Australian Dollar did that person invest? Find it. (1$ = 82.37 rupees)

(d) Kunti has deposited Rs 7,56,400 in an international bank in Nepal. How much amount will she obtain in the following countries? (Round of into the nearest hundred)

i. Japan	(10 Yen = Rs 9.40)
ii. Canada	(Canadian dollar 1 = Rs 81.94)
iii. America	($ 1 = Rs 106.80)
iv. India	(₹ 100 = Rs 160)
v. South Korea	(100 wan = Rs 9.38)
vi. Europe	(€ 1 = Rs 113.64)

4.(a) How much American Dollar will we obtain from 100 Canadian Dollar?

(b) How much Euro will we get from 5 Pound Sterling?

(c) How much Indian Rupee will we get from 1500 Japanese Yen?

(d) How much American dollar will we get from 24,000 Indian Rupee?

(e) If a person earns Australian dollar 3432 per year. How much American dollar will he earn?

(f) Sushila spent US \$ 4500 for medicine in America. What amount in Japanese currency did she spend? Find it.

5.(a) Surya needs \$ 45000 for tour. If the bank takes 2% commission for exchange, how much Nepali rupee does he require? Find it.

(b) The amount from remittance £9000 has to be exchanged in Napali rupees. How much will we obtain if the bank takes 1.5% commission?

(c) From Nepali rupee for \$ 600 what amount of Japanese yen we found after 2% commission.

(d) A project was contracted with total amount of Chinese Yuan 10,00,000 after 10% devaluation of Nepali rupee, how much should the contractor add to complete the contract?

6.(a) A laptop was bought at Canadian \$ 770. If the tax of 20% and 13% VAT should be paid, find the least selling price of it in Nepali rupee that prevents the shopkeeper from loss?

(b) By selling an object in ₹17,000 the shopkeeper gain 20% profit. Find the selling price of that object in NRS to get 25% profit.

(c) The ticket of Nepal airline from Kathmandu Bangkok is about Rs 25,000. Its value from Bangkok to Kathmandu is 8520 Bhatt in Thailand. What percent of ticket is expensive in Thailand in comparison to Nepal?

7. Make groups of 5 students in each group. Ask each group to make a package of tour of a foreign country except India. Estimate the total budget in different titles for the tour. Finally calculate the total cost in respective country's currency and then convert the currency into Nepali Rupee. Make report including all the process and costs. Present that report to the class.

Unit: 3 Compound Interest

3.0 Review

Observe the following cases and discuss in group.

Elina borrowed Rs. 4800 from Bipin at the rate of 10% per annum. Find the interest amount that

(i) Is calculated at the end of two years.

(ii) Is calculated every year for two years

(iii) interest to be paid for the interest of first year at the end of second year.

In the first case we have to find simple interest in which we calculate total interest at the end of the given time period. In this case we use the formula:

$$I = \frac{P \times T \times R}{100}$$

The interest in the case of (ii) and (iii) are differ from the simple interest. In this case, we have to calculate the interest of first year and then again calculate the interest amount after first year which is compound interest: Now we are going to discuss about compound interest.

3.1 Compound interest

If the principal Rs. P is deposited in a bank with the rate of R% for T years then the simple interest is given by,

Simple Interest (SI) = $\frac{P \times T \times R}{100}$ and Amount (A) = P+S.I.

In the above case, we calculate the interest of total at the end of the given time period. Now a days the banks and financial institutions calculate the interest in different ways. We can clarify by the following example:

Ashish borrows Rs 10,000 from a bank. If the bank charges 12% per annum interest then at the end of the first year he has to pay the interest as

$$I = \frac{10{,}000 \times 1 \times 12}{100} = \text{Rs. } 1200 .$$

As the end of the first year he has to pay Rs 11,200 in total. Due to some reasons if he is unable to pay the interest and principal at the end of the first year then Ashish has to pay the interest of Rs 11,200 thereafter. Thus the interest at the end of second year is Rs. $\frac{11{,}200 \times 1 \times 12}{100}$ =Rs 1344

The total interest is Rs. = (1,200 + 1,344) = Rs. 2,544

Also, Simple interest = Rs. $\frac{10{,}000 \times 2 \times 12}{100}$ = Rs. 2,400

The increase in interest is due to the fact that the principal for the second is more than the principal to the first year. The interest calculated in this condition is called compound interest.

When the interest at the end of each period of time is added to the principal and the amount at the end of each period of time is taken as the principal for the next period. This sum is called to be lent in compound interest.

Formula for compound interest

Suppose P = principal, T = time, R = rate of interest per year, (C.I) = compound interest at the end of T years.

Now, Principal	**Time (year)**	**Interest**
Rs. 100	1	Rs R
Rs. P	1	Rs $\frac{PR}{100}$. (I_1)

The amount at the end of first year= $\left(P + \frac{PR}{100}\right) = P\left(1 + \frac{R}{100}\right)$

The principal for second year is $P\left(1 + \frac{R}{100}\right)$

Then interest at the end of second year is;

$(I_2) = \frac{P.T.R}{100} = P\left(1 + \frac{R}{100}\right) \cdot \frac{R}{100}$

The amount is $\left[P\left(1 + \frac{R}{100}\right) + P\left(1 + \frac{R}{100}\right) \cdot \frac{R}{100}\right]$

$= P\left(1 + \frac{R}{100}\right)\left(1 + \frac{R}{100}\right) = P\left(1 + \frac{R}{100}\right)^2$

Similarly, the amount at the end of third year = $P\left(1 + \frac{R}{100}\right)^3$

Continuing in the same way,

The compound amount after T year is $P\left(1 + \frac{R}{100}\right)^T$

Compound amount (CA) $= P\left(1 + \frac{R}{100}\right)^T$

Now, compound interest (C.I) = C.A - P

$= P\left(1 + \frac{R}{100}\right)^T - P$

$= P\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$

Where T is always positive.

Note:

1. If compound interest in payable half yearly, then the interest rate obtained in this period will be $\frac{R}{2}\%$ and time period is 2 half year (ie 2T for T year)

   Then the compound amount (C.A) after T year is

$$\text{C.A} = P\left(1+\frac{R/2}{100}\right)^{2T} = P\left(1+\frac{R}{200}\right)^{2T}$$

$$\text{And C.I} = P\left\{\left(1+\frac{R}{200}\right)^{2T} - 1\right\}.$$

2. If the rate of interest for every time period is different i.e, R_1% for T_1, R_2% for T_2, then Amount A can be calculated as

$$\text{C.A} = P\left(1+\frac{R_1}{100}\right)^{T_1} . \left(1+\frac{R_2}{100}\right)^{T_2} \text{ and}$$

For 3 years.

$$\text{C.A} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right). \text{ and so on.}$$

Example 1:

Find simple interest of the following case

P = Rs. 45,000 T = 2years R =12.5%

Solution:

Principal (P) = Rs. 45,000, Rate (R) = 12.5%

Time (T) = 2 years

Simple interest (SI) = ?

We have

$$\text{SI} = \frac{PTR}{100}$$

$$= \frac{45{,}000 \times 2 \times 12.5}{100} = 11{,}250$$

Simple interest (i) = Rs. 11,250

Example 2:

Find the compound amount and then compound interest of Rs. 1,250 at the rate of 10% p.a. for 2 years. (i) Without using formula (ii) with using the formula of compound interest.

Solution: (i) Without using formula

For the first year

Here, Principal = Rs. 1,250

Time (T) = 1 year

Rate (R) = 10%

Interest at the end of first year (SI_1) = $\frac{PTR}{100} = \frac{1{,}250 \times 1 \times 10}{100}$

= Rs. 125

The amount (A) = Rs 1250 + Rs 125 = Rs 1,375

For the second year

P = Rs 1,375, T= 1 year, R = 10% and SI_2 = ?

Interest at the end of second year (SI_2) $= \frac{P \times T \times R}{100}$

$= \frac{1{,}375 \times 1 \times 10}{100}$

= Rs. 137.50

Total compound interest = $SI_1 + SI_2$ = Rs 125 + Rs 137.50

= Rs 262.50

Compound amount at the end of 2 years

= Rs. 1375 + Rs. 137.50

= Rs. 1,512.50

(ii) By using formula

P = Rs 1,250 R = 10% T = 2year C.I. = ? C.A. = ?

CI. $= P\left[\left(1+\frac{R}{100}\right)^{T}+1\right] = \text{Rs.}\,12.50\left[\left(1+\frac{1}{100}\right)^{2}-1\right]$ = Rs. 1250 [0.21]

= Rs. 262.50

and CA = P + CI = Rs.(1250 + 262.50) = Rs. 1512.50

Example 3:

Srijana deposited Rs 45,000 at the rate of 8% per annum for 3 years. Find compound amount and compound Interest.

Solution:

Here, Principal (P) = Rs 45,000

Time (T) = 3 years

Rate (R) = 8%

Compound Amount (A) = ?

Compound Interest (C.I) = ?

We have, C.A. $= P\left(1+\frac{R}{100}\right)^{T}$

$= 45{,}000\left(1+\frac{8}{100}\right)^{3}$

$= 45{,}000 \times (1.08)^3$

= 45,000 x 1.259712 = 56,687.04

Compound Amount = Rs 56687.04

Again Compound interest (C.I) = C.A – P

= Rs. (56,687.04 - 45,000)

= Rs. 11,687.04

Example 4:

What is the difference between compound interest and simple interest if Rs. 75,000 is borrowed at the rate of 10% p.a for 2 years ?

Solution: We have,

Principal (P) = Rs. 75,000

Rate (R) = 10% p.a

Time (T) = 2 years

We know that,

SI = $\frac{\text{PTR}}{100} = \frac{75{,}000 \times 2 \times 10}{100}$ = 15,000

SI = Rs 15,000

Again, Compound interest (C.I.) = $\text{P}\left\{\left(1+\frac{\text{R}}{100}\right)^{\text{T}}-1\right\}$

$= 75{,}000\left\{\left(1+\frac{10}{100}\right)^{2}-1\right\}$

$= 75{,}000\{(1.1)^2-1\}$

= 75,000 x 0.21 = 15,750

C.I = Rs. 15,750

Now, difference in CI and SI is

C.I – S.I = Rs. (15,750 - 15,000) = Rs. 750

Example 5:

Manju has taken Rs. 1,50,000 loan from a cooperative with 12% p.a. interest. If the cooperative compounded interest half early, find the total amount that Manju will return after 2 years

Solution:

Here, Principal (P) = Rs. 1,50,000

Rate (R) = 12% per half year

Time (T) = 2 years

C.A =?

We have C.A = $\text{P}\left(1+\frac{\text{R}}{200}\right)^{2\text{T}}$

$= \text{P}\left(1+\frac{12}{200}\right)^{2\times 2}$

= 1,50,000 $(1+0.06)^4$

= 1,50,000 x $(1.06)^4$

= 1,89,371.54

Manju will Return Rs. 1,89,371.54 after 2 years.

Example 6:

What will be the compound interest of Rs. 84,000 for 3 years if the rate of interest per annum for three years is 4%, 5% and 6% respectively.

Solution:

Here, P = Rs. 84,000

T = 3years

R_1 = 4% for 1st year

R_2 = 5% for 2nd year

R_3 = 6% for 3rd year

C.I =?

Now, Compound Amount (C.A) = $P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

= $84{,}000\left(1+\frac{4}{100}\right)\left(1+\frac{5}{100}\right)\left(1+\frac{6}{100}\right)$

= 84,000 (1.04 x 1.05 x 1.06)

= 84,000 x 1.15752 = 97231.68

Compound interest (C.I.) = C.A. – P

= 97,231.68 – 84,000

= Rs. 13,231.68

Example 7:

The difference between compound interest and simple interest of the certain amount for 3 years with 10% per annum is Rs 2,015, find the amount.

Solution:

Suppose Principal (P) = Rs. x

Time (T) = 3 years

Rate (R) =10%

Now, Simple Interest (SI) = $\frac{PTR}{100} = \frac{x\times3\times10}{100} = \frac{3x}{10} = 0.3x$

Again

C.I = $P\left\{\left(1+\frac{R}{100}\right)^{T}-1\right\}$

= $x\left\{\left(1+\frac{10}{100}\right)^{3}-1\right\}$

= $x\left\{\left(\frac{110}{100}\right)^{3}-1\right\}$

= $x(1.331-1)$

= $0.331x$

By question, C.I – S.I = 2,015

or, $0.331x - 0.3x = 2{,}015$

or, $0.031x = 2{,}015$

$\therefore\ x = \frac{2{,}015}{0.031} = 65{,}000$

Principal (P) = Rs 65,000

Example 8:

If the sum of money becomes Rs 7,260 in 2 years and Rs 7,986 in 3 years when compounded annually, find the sum and the rate of compound interest.

Solution:

Here, for 2 years

Let, Principal (P) = Rs. x

Time (T) = 2 years

Compound Amount (C.A) = Rs 7,260

We have, $\text{C.A} = \text{P}\left(1 + \frac{\text{R}}{100}\right)^{\text{T}}$

or, $7{,}260 = x\left(1 + \frac{\text{R}}{100}\right)^{2}$ .... (i)

Again, for 3 years

Principal (P) = Rs. x

Time (T) = 3 years

Compound amount (CA) = Rs 7,986

Also, $\text{C.A} = \text{P}\left(1 + \frac{\text{R}}{100}\right)^{\text{T}}$

or, $7{,}986 = x\left(1 + \frac{\text{R}}{100}\right)^{3}$

or, $7{,}986 = x\left(1 + \frac{\text{R}}{100}\right)^{2} . \left(1 + \frac{\text{R}}{100}\right)$ .... (ii)

From (i) and (ii), we get

$7{,}986 = 7{,}260\left(1 + \frac{\text{R}}{100}\right)$

Or, $\left(1 + \frac{\text{R}}{100}\right) = \frac{7986}{7260}$

Or, $\frac{\text{R}}{100} = \frac{7986}{7260} - 1$

Or, $\frac{\text{R}}{100} = 1.1 - 1$

Or, $\frac{\text{R}}{100} = 0.1$

$\therefore$ R = 0.1 x 100 = 10%

Alternatively from (i) and (iii)

$$7986 = \frac{7260}{\left(1+\frac{10}{100}\right)^2} . \left(1+\frac{R}{100}\right)^3$$

Or, $\left(1+\frac{R}{100}\right) = \frac{7986}{7260}$

Or, $\frac{R}{100} = 1.1 - 1$

Or, $\frac{R}{100} = \frac{7986}{7260} - 1 = 0.1$

$\therefore$ R = 0.1 x 100 = 10%

Again from (i), Principal (P) = $x = \frac{7260}{\left(1+\frac{10}{100}\right)^2}$

$$= \frac{7260}{(1.1)^2} = \frac{7260}{1.21} = \text{Rs. } 6,000$$

$\therefore$ Principal (P) =Rs. 6000 and R = 10%

Exercise 3

1. Define Simple interest and compound interest. Write the formula for calculating simple interest and compound interest.
2. Calculate the following

(a) P = Rs. 5600	R = 10.5%	T = 3year	SI = ?
(b) P = Rs. 8500	R = 10%	SI = Rs. 21250	T = ?
(c) R = 12%	T = 4years	SI = Rs. 8400	P =?
(d) P= Rs22,500	T = 2.5years	SI = Rs. 5906.25	R =?

3.(a) Without using the formula of compound interest, calculate the compound interest and compound amount of the following.

i. P = Rs40,000	T = 2 years	R = 7% compounded annually.
ii. P = Rs 86,00	T = 3 years	R = 5% compounded annually.
iii. P = Rs 10,15,000	T = 2 years	R for first year 5% and for second year 6%.
iv. P = Rs 10,000	T = 3 years	R_1 =4%m, R_2 =6% and R_3 = 7%

(b) By using formula, find the compound interest and compound amount of Q.No. 3(a).

4.(a) What will be the compound amount and compound interest if Rs 20,000 is deposited with 5% p.a for 3 years? Find it.

(b) How much amount will Enkita have with return to bank if she takes lone of Rs 50,000 with 10% p.a compounded annually for 3 years? Find it.

(c) Rashu has deposited Rs 1,50,000 in a bank. If the bank provides 6% p.a interest, find the compound amount and compound interest of Rashu after $2\frac{1}{2}$ years.

(d) A Co-operative in Armala has invested Rs 4,00,000 for an Agriculture farm. If it compounds the interest annually and the rate of compound interest is 12.5% per year, what amount will it get after 2 years? Find it.

5.(a) What will be the difference in simple interest and compound interest of Nisha if she deposits Rs 45,000 in a bank for 3years with 11% p.a interest ? Find it.

(b) Kaji detains a loan of Rs. 80,000. If the rate of interest is 12.5% per annum, find the difference between the compound interest and simple interest after 3 years.

(c) What sum of money will be different between compound interest and simple interest for 3 years if a bank provides 6% p.a interest of Rs 5,00,000? Find it.

(d) Ranju takes a loan of Rs 24,000 from a person with simple interest of 12.5% than she deposits that amount in a cooperative with same rate of compound interest. How much profit will she get after 3 years? Find it.

6. (a) Find the compound interest on Rs 4,000 for 2 years at the rate of 10% per annum compounded half yearly.

(b) What will be the compound amount and compound interest of Rs. 50,000 with rate of 8% p.a after 2 years if the interest is compounded half yearly? Find it.

(c) What will be the difference between annual compound interest and half yearly compound interest of amount Rs. 2,50,000 with rate of 12% p.a after 3 years? Find it.

(d) Pukar has taken a loan of Rs, 50, 000 with 10% per annum commanded half yearly. Rosani takes same amount with 12% per annum compoundable yearly. Find who has to pay more interest after 3 years?

7.(a) On what sum will the compound interest at 5 % p.a for 2 years compounded annually be Rs 164? Find it.

(b) Nanu borrowed a certain sum at the rate of 10 % p.a. If she paid compound interest Rs. 1,290 at the end of two years compounded annually, find the sum of money borrowed by her.

(c) What sum interested at the rate of 5 % per annum for 2 years will earn as the compound interest if the interest is payable yearly? Find it.

(d) Bipana lends sum of money to Yamuna at the rate of 8% per annum compound interest. If Bipana takes Rs 8,748 from Yamuna at the end of 2 years, what sum of money has Bipana lent to Yamuna? Find it.

(e) The annual compound interest of a sum with 10 % p.a is less than Rs 40 than the half yearly compound interest of same amount with same rate in one year. What will be the sum? Find it.

8.(a) Brinda borrowed a sum of money from Dipak. If she paid compound interest Rs. 18,205 at the end of 3 years with the rate of 10% p.a., find the principal.

(b) The compound amount of a sum in 2 years is Rs. 14,520 and in 3 years is Rs. 15,972. Find the principal and the rate of interest.

(c) What will be the sum and rate of interest if the compound amount of that sum in 2 yrs and 3 yrs are Rs. 10,580 and Rs 12,167 respectively? Find it.

(d) A sum of money invested at the compound interest payable yearly has interest in 2 years and 4 years are Rs. 4,200 and Rs. 9282 respectively. Find the rate of interest.

9.(a) Find compound interest of Rs. 70,000 with rate of 10% p.a for 3 years if it is i) compounded annually ii) compounded half yearly

(b) Divide Rs . 41,000 into two parts such that their amounts at 50% p.a compound interest compoundrd annually in 2 years and 3 years are equal.

(c) Nava borrows Rs. 10,000 from a bank at the rate of 12 % p.a simple interest and lend to Hari immediately at the same rate of compound interest. How much does Nava gain after 3 years? Find it.

(d) In what time Rs. 1,00,000 amount will be Rs. 1,21,000 at the rate of 10% p.a. compounded annually? Find it.

(e) In what time Rs. 25, 60,000 yields Rs. 8,58,801 compound interest at the rate of 15 % p.a compounded half yearly? Find it.

10. Divide the class into groups. Each group is requested to visit one business, occupational work and in budget. After that suppose they have to take loan from a financial –institution. Make different scheme and select which scheme is suitable and economic for their purpose. Prepare group report and present to the class. (Include service tax and income tax if possible)

Unit: 4

Population Growth and Depreciation

4.0 Review:

Discus about the following cases in group:

I. What will be the compound amount of Rs. 50,000 after 2 years if the rate of interest is 3% p.a?

II. What will be changed in above question if Rs. 50,000 is changed by the population of a town at certain time?

III. Suppose the number of virus in a patient is decreasing with the rate of 40% per hour. If the number of virus is 2×10^5 at starting of the medicine, find the number of virus after 3 hours?

In the above three cases; I and II are similar, II gives the growth per year and III gives depreciation per hour of the virus. Now we are going to discus about compound growth and compound depreciation in detail.

4.1 Population Growth:

The total number of inhabitants living in a place is called the population. The population is generally changing time to time. The relative increase in the population of the country represents the growth of population. The growth of population per year is called the annual growth rate .Generally, the population is increasing if it is not affected by extraneous conditions.

For example, in 2068 the population of a village council is P_0 and rate of growth is R%, what will be the population of that village council at the end of 2072?

Here, Initial population = P_o

Rate of growth = (R) = R%

Time = T year

Population after T year =P_T,

We can use the formula of compound amount as

$$P_T = P_o\left(1 + \frac{R}{100}\right)^T$$

Again, The increased population $= P_T - P_o = P_o\left(1 + \frac{R}{100}\right)^T - P_o$

$$= P_o\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$$

If, $R_1, R_2, R_3, \ldots\ldots R_T$ be the rates of growth of 1st, 2nd, 3rd, ...Tth different years

then, $P_T = P_o\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right)\ldots\ldots\left(1 + \frac{R_T}{100}\right)$

Example 1:

The population of a Village council at the end of the year 2068 was 45,500. If the rate of growth is 2.5%, find the population of that Village council at the end of 2071.

Solution:

Here, The population in 2068 = P_o =45,000

Rate of growth (R) = 2.5%

Time (T) = 3 years (2071 - 2068)

The population after 3 years (P_3) =?

We know, $P_T = P_o\left(1+\frac{R}{100}\right)^T$

or, $P_3 = P_o\left(1+\frac{R}{100}\right)^3$

$= 45,000 \times \left(1+\frac{2.5}{100}\right)^3$

= 45, 000 x 1.6769

= 48,460.07

= 48,460

Hence the population of the village council at the end by year 2071 was 48,460

Example 2:

The population of a town before 4 years was 765625 and growth rate is 5% .How many people will be added to town now?

Solution:

Here, Initial population (P_o) = 765625

Rate of growth (R) = 5%

Time (T) = 4 years

Increased population = ?

We know, the increased population $= P_o\left[\left(1+\frac{R}{100}\right)^T - 1\right]$

$= 7{,}65{,}625\left[\left(1+\frac{5}{100}\right)^4 - 1\right]$

= 7,65,625 x 0.2155

= 1,64,992.18

= 1,64,993 (about)

The increased population of that town is 1,64,993.

Example 3 :

The present population of a Municipality is 67,600 . If the rate of growth is 4%, find the population before 2 years.

Solution:

Here, Present population (P_T) = 67,600

Time (T) = 2 years

Growth rate (R) = 4%

Population before 2years (P_o) = ?

We know that $P_T = P_o\left(1+\frac{R}{100}\right)^T$

or, $67600 = P_o\left(1+\frac{4}{100}\right)^2$

or, $67600 = P_o\left(\frac{104}{100}\right)^2$

or, $67600 = P_o \times 1.0816$

or, $P_o = \frac{67600}{1.0816}$

$\therefore$ $P_o = 62{,}500$

Hence, the population of the Municipality before 2 years was 62,500.

Example 4:

The population of an urban area has increased from 27,000 to 64,000 in three years. Find the annual population growth rate.

Solution:

Here, Po = 27,000, P_T = 64,000, T = 3years and R = ?

We know that, $P_T = P_o\left(1+\frac{R}{100}\right)^T$

or, $64{,}000 = 27{,}000\left(1+\frac{R}{100}\right)^3$

or, $\frac{64000}{27000} = \left(1+\frac{R}{100}\right)^3$

or, $\left(\frac{4}{3}\right)^3 = \left(1+\frac{R}{100}\right)^3$

or, $\left(1+\frac{R}{100}\right) = \frac{4}{3}$

or, $\frac{R}{100} = \frac{4}{3} - 1$

or, R = 0.333 x 100

$\therefore$ R = 33.3%

Hence, the annual population growth rate is 33.3%.

Example 5:

The birth rate of the population of a town is 6% every year and death rate is 1% per year. If the population of the town is 3,38,000, find the population before 3 years.

Solution:

Here, birth Rate = 6%, death rate = 1 %

Rate of increase of population (R) = (6 -1)% = 5%

P_T = 3,38,000

P_o = ?

T= 3 years

We know, $P_T = P_o\left(1+\frac{R}{100}\right)^T$

or, $3{,}38{,}000 = P_o\left(1+\frac{5}{100}\right)^3$

or, $3{,}38{,}000 = P_o\left(\frac{105}{100}\right)^3$

or, $3{,}38{,}000 = P_0 \times 1.157625$

or, $P_o = \frac{338000}{1.157625}$

or, $P_o = 2{,}91{,}977.108 \approx 2{,}91{,}978$.

Hence, the population before 3 years was 2,91,978.

Exercise 4.1

1.(a) The population of a municipality was 1,85,220 before 3 years. If the rate of increase is 5%, find the present population of that municipality.

(b) At the end of 2011 AD, the population of Nepal was 2,64,94,504 and rate of growth was 1.35%.What was the population after 2 years?

(c) By the policy of prevention of eagles, it was found that the growth of eagle is 10% per annum. If the estimated population of eagles now is 5,000, find the population of eagles after 3 years.

(d) The population of householders in Kaski in 2068 was 1,25,673. If the rate of growth is 5% per year, what was the house holders' population in 2072?

2.(a) The population of a village council before 2 years was 28,500 . If the rate of growth is 2 % per year, estimate the increased population in 2 years.

(b) According to census 2068, the population of Pokhara was about 2,64,991 and growth rate was 10%. Estimate the growth of population of Pokhara at the end of 2072 B.S.

(c) The average rate of growth of a plant is 2% per month. It is measured that the plant is 4m tall in January 1st. Estimate the height of that plant at the end of April.

(d) The admission fee of a level was Rs. 6,500 before 4 years. If the policy is to increase fee by 10% every year, find the increased fee in 4 years.

3.(a) The present population of a city is 45,000 and rate of growth is 4%. Find the population of that city before 3 years.

(b) The rate of increase of bacteria of curd is 40% per hour. If the number of bacteria at 7 AM. is 10.12×10^{11}, find the number of bacteria before 5 hours.

(c) The price increase of a land is assumed as 10% per year. If the present value of that land is Rs 6, 00,000, find its value before 2 years.

4.(a) The population of a town at the end of 2013 was 40,000 and in 2 years it was increased by 4,100. Find the rate of growth.

(b) The population of a village council in 2012 A.D and 2014 A.D was 62500 and 67600 respectively. Find the rate of growth of population of that village council.

(c) The present number of students of a university is 21632 . Find how many years before the number of students of the university was 20,000 if the growth rate is 4%.

(d) In how many years the population of a town becomes 87880 from 40,000 if the rate of change is 30% per year?

5.(a) Three years ago the population of a city was 150,000. If the annual growth rates of population in the last three years were 2%, 4% and 5% respectively, find the population of the city after 3 yrs?

(b) The population of a village increases every year by 5%. At the end of two years the population of the village was 10,000. If 1025 people migrated to another place, find the population of the village before 2 years.

(c) Two years ago the population of a town was 54,000 with 5% growth rate. If 8,046 people migrated to town and 5089 migrated from that town to another places at present, find the present population of that town.

(d) Three year ago, the population of a town was 2,00000. The rate of growth in the first two years are 2% and 2.5% respectively and decrease in the third year by 1% due to natural disaster, find the increased population in 3 years?

6. Divide the class into suitable groups. Each group has to visit their respective ward office, village council officer other governmental offices. Collect the data (population) in different titles like use of internet, use of safe drinking water, educated, etc. Estimate the rate and population of your town village after two years.

4.2 Depreciation (Compound Depreciation):

Discuss in group of students

"Suppose you have to buy a computer with the fixed amount of money you have. You have to buy brand new or second hand computer. Which one is economic to you and why? Discuss.

Some kinds of machine, building, transportation equipment's, etc. are made for fixed term. After use of some time their value will decrease yearly at the certain rate. This is called deprecation .The decrease per unit time is called rate of deprecation.

The decrease in the value of the assets is obtained by charging a fixed percentage on the original cost. The simple deprecation is calculated as

D = $\frac{V_0 - V_T}{T}$ where V_o = original value, V_T = value after T years and T = number of years. If the deprecations is compounded, then according to fixed rate of depreciation R%, the value ofter T years (V_T) = $V_o\left(1 - \frac{R}{100}\right)^T$

The decreased value (V_D) = $V_0 - V_T = V_0 - V_0\left(1 - \frac{R}{100}\right)^T$

$= V_o\left[1 - \left(1 - \frac{R}{100}\right)^T\right]$

If the rate of depreciation is different for different years then,

$V_D = V_o\left(1 - \frac{R_1}{100}\right)\left(1 - \frac{R_2}{100}\right)\ldots\left(1 - \frac{R_T}{100}\right)$

Where R_1, R_2,….R_T are the rates of depreciation in 1^{st}, 2^{nd} ….. T^{th} year respectively.

Example 1:

The price of a machine is Rs 1,00,000 . If its price is decreased by the rate of 10% per year, what will be its value after 3 years? Find it.

Solution:

Here, The present price of a machine (V_o) = Rs 1,00,000

Depreciation rate (R) = 10% p.a

Time (T) = 3 years

Value after 3 years (v_3) = ?

We have, $V_T = V_o\left(1 - \frac{R}{100}\right)^T$

Or, $V_3 = V_o\left(1 - \frac{10}{100}\right)^3$

$= 1{,}00{,}000 \times \left(\frac{90}{100}\right)^3$

$= 1{,}00{,}000 \times 0.729$

$= 72900$

The value of machine after 3 years is Rs72,900.

Example 2:

The present price of a motorcycle is depreciated from Rs. 1,50,000 to Rs. 85,000 after use of 4 years, find the rate of depreciation.

Solution:

Here, Initial price of a motorcycle (V_o) = Rs. 1,50,000

The price of the motorcycle after 4 years (V_4) = Rs. 85,000

Time (T) = 4 years

The rate of depreciation (R) =?

We know that, $V_T = V_o \left(1 - \frac{R}{100}\right)^T$

or, $85000 = 1,50,000 \times \left(1 - \frac{R}{100}\right)^4$

or, $\left(1 - \frac{R}{100}\right)^4 = \frac{85000}{150,000}$

or, $\left(1 - \frac{R}{100}\right)^4 = \frac{17}{30}$

or, $1 - \frac{R}{100} = \left(\frac{17}{30}\right)^{1/4}$

or, $1 - \frac{R}{100} = 0.8676$

or, $\frac{R}{100} = 1 - 0.8676$

or, $R = 0.1324 \times 100$

$\therefore$ R = 13.24%

Example 3:

A person bought a taxi for Rs .6,25,000. He earned Rs. 2,50,000 in 2 years and sold it at the rate of 8% compound deprecation. Find his profit or loss.

Solution:

Here, buying price of a taxi (V_o)= Rs 6,25,000

Earned amount in 2 years = Rs. 2,50,000

Rate of depreciation (R) = 8%

Profit /Loss =?

We have, Selling price of the taxi after 2 years (V_T)= $V_o \left(1 - \frac{R}{100}\right)^T$

or, $V_2 = 6,25,000 \times \left(1 - \frac{8}{100}\right)^2$

or, V_2 $= 6,25,000 \times \left(\frac{92}{100}\right)^2$

$= 6,25,000 \times 0.8464$

$= 5,29,000$

$\therefore$ V_T = Rs.5,29,000

Now, the total value of taxi for him after 2 years

= Rs. 6,25,000 – Rs. 2,50,000 = Rs. 3,75,000

Bbut he sold the taxi at Rs. 5,29,000. So, he got profit.

His profit amount = Rs.(5,29,000 – 3,75,000) = Rs. 1,54,000

Example 4:

Due to political instability of a nation, a company's share price depreciated at the rate of 12% p.a for 3 years. If the present value of the shares is Rs. 85,184, how many shares of Rs 100 were sold 3 years ago? Find it.

Solution:

Present cost (V_T) = Rs. 85,184

Time (T) = 3years

Rate of depreciation (R) = 12%p.a.

Beginning cost = ?

Now, we have $V_T = V_o\left(1-\frac{R}{100}\right)^T$

Or, $85184 = V_o\left(1-\frac{12}{100}\right)^3$

Or, $85184 = V_o\left(\frac{88}{100}\right)^3$

Or, $85184 = V_o\left(\frac{22\times22\times22}{25\times25\times25}\right)$

Or, $V_o = \frac{85184 \times25 \times25 \times25}{22\times22\times22} = 8\times25\times25\times25$

$\therefore V_o$ = Rs. 1,25,000

∴ Total value of share before 3 yrs = Rs. 1,25,000

Price of each share is 100

i.e In Rs 100 → 1 share is sold

In Rs. 1,25,000 → $\frac{1}{100}\times125000$ = 1250 shares are sold.

∴ Total numbers of shares sold = 1250

Exercise 4.2

1.(a) The original value a electric heater is Rs. 3,000. If its value is depreciated by 10% p.a., find its value after 3 years.

(b) Enjal pays Rs. 16,00,000 for his car. If its value depreciates by 10% per year, find its value after 4 years.

(c) A company bought an extra power supply at Rs. 3,20,000 before 3 years. Due to regular supply of electricity it is sold at the rate of 15% compound depreciation per year, find its present Value .

(d) The number of virus count in a sample is decreasing with 10% per hour after use of medicine. If at 11.00 am the number of virus in sample is 2.3×20^7, what will be its number at 1:00 pm of that day? Find it.

2.(a) A farmer sold a tractor for Rs. 1,60,000 after using 2 years. If he purchased it for Rs. 2,50,000., what is its deprecation rate? Find it.

(b) Photocopy machine was bought for Rs. 64,000 and sold it for Rs. 27,000 after 3 years . Find the rate of compound depreciation.

(c) The present value of an apartment flat is Rs. 20,48,000. If the rate of compound deprecation is 15% per year, after how many years the value of this apartment flat will be Rs. 12,57,728? Find it.

(d) A rice mill was bought at Rs. 4,00,000. If the rate of deprecation is 30% per year, and sold at Rs.1,96,000, find how many years ago was it bought?

3.(a) A laptop depreciates each year by 5% of its beginning value. If its present value is Rs 95,000, find Its value before two years.

(b) Kopila purchased a vehicle for Rs. 2,40,000. By using this she earned Rs. 48,600 in 2 years and sold it at the rate of compound deprecation of 8% per year. Find her gain or loss.

(c) Amit bought a motorcycle for Rs. 72,900. If the rate of compound deprecation is 10% each year, find the price of that motorcycle .i) before 2 years ii) after 2 years.

(d) After the rate of deprecations 4% and 5% in two years, a machine was sold for Rs. 24,168. Find its original value before 2 years.

4.(a) Anju received Rs. 2430 by selling shares of a financial company with 10% per year deprecation after 2 years . How many shares of Rs 100 were sold before 2 years? Find it.

(b) A finance company's share price was depreciating at the rate of 10 % every year. If Rs. 710775 is the cost of the share at present, what was its cost 2 years ago? How many shares of each Rs 100 were there?

5. Work in group.

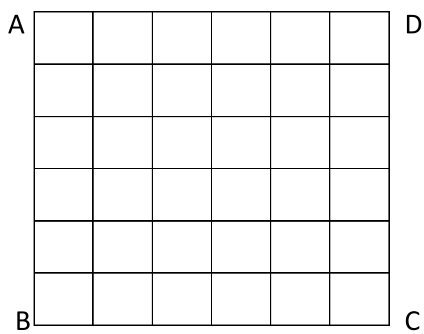
- Make groups of suitable number of students.
- Visit the nearest farm, machine, industry or other production plants.
- consult to manager or authorized person about
  - starting price
  - today's price
  - profit and loss
- Collect the data and find the rate of depreciation.
- Prepare a group report by each group and then present it to the class.

Unit: 5

Plane Surfaces

5.0 Review:

We have already discussed about the plane surfaces in the previous grades. Lets study the following figures and discuss on the following questions:



i) Which shape does of the above figure ABCD represent?

ii) What is the perimeter of the above figure?

iii) In how many ways the perimeter of the above figure can we find?

iv) What is the area of the above figure?

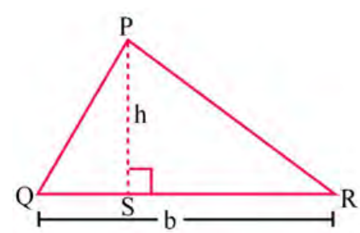
iv) After joining AC, the new figures are formed. What are the names of the figures?

vi) What are the area of that new figures?

5.1 Area of triangles.

a. When a base and height is given:

PQR be the triangle with base QR= b and altitude (height)

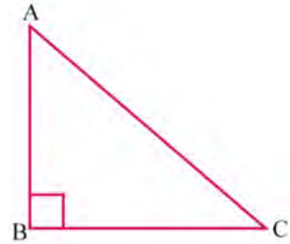


PS=h, then area of $\Delta PQR = \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} QR \times PS \qquad = \frac{1}{2} .b.h$

Therefore, area of $\Delta = \frac{1}{2}$ base $\times$ height

b. If the triangle is right angled triangle:



ABC be a right angled triangle with $\angle ABC = 90^o$, base BC = b and perpendicular AB = p.

We know that,

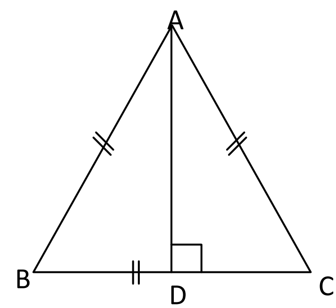
Area of any tangle is equal to half of the product of base and height.

Area of ΔABC $= \frac{1}{2}$ base $\times$ perpendicular

$= \frac{1}{2}$.BC. AB.

$\therefore$ Area of right angled triangle $= \frac{1}{2} \times b \times p$

c. Area of equilateral triangle:



The given triangle ABC is an equilateral triangle ABC in which AB = BC = AC.

A line AD is drawn by Joining the mid- point 'D' of the base BC and vertex 'A'.

So, AD is perpendicular to the base BC.

AD is also the height of ΔABC.

Let AB = BC = AC = a unit and AD = h unit. Then,

BD= DC $= \frac{a}{2}$ unit

Now, in the right angled triangle ADC,

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$

or, $(AC)^2 = (DC)^2 + (AD)^2$

or $a^2 = \left(\frac{a}{2}\right)^2 + h^2$

or, $a^2 = \frac{a^2}{4} + h^2$

or, $a^2 - \frac{a^2}{4} = h^2$

or, $\frac{3a^2}{4} = h^2$

$\therefore$ $h = \frac{\sqrt{3}}{2}a$

We know that, the area of the triangle $= \frac{1}{2}$ x base x height.

$\therefore$ Area of $\Delta ABC = \frac{1}{2}$.BC. AD.

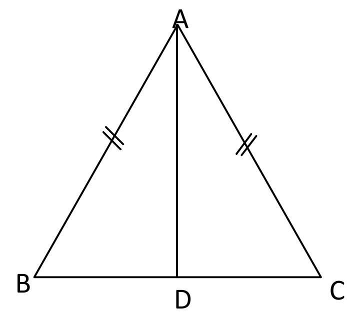
$= \frac{1}{2}$x a x h

$= \frac{1}{2} \text{x a x} \frac{\sqrt{3}}{2} \text{a}$

$= \frac{\sqrt{3}}{4} a^2$

$\therefore$ The area of an equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$.

d. Area of an isosceles triangle:



The given triangle ABC is an isosceles triangle in which AB = AC. A perpendicular AD is drawn to the base BC which bisects the base BC at D.

So, AD is the height of ΔABC.

Let AB = AC = a unit, AD = h units and BC = b unit.

Then, BD = DC = $\frac{1}{2}$b

Now, in the right angled triangle ADC,

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$

or, $h^2 = b^2 + p^2$

or, $(AC)^2 = (DC)^2 + (AD)^2$

or, $a^2 = \left(\frac{b}{2}\right)^2 + h^2$

or, $a^2 = \frac{b^2}{4} + h^2$

or, $\frac{4a^2-b^2}{4} = h^2$

$\therefore$ $h = \frac{\sqrt{4a^2-b^2}}{2} = \frac{1}{2}\sqrt{4 \times (\text{equal side})^2 - (\text{base})^2}$

We know that, area of the triangle = $\frac{1}{2}$ x base x height.

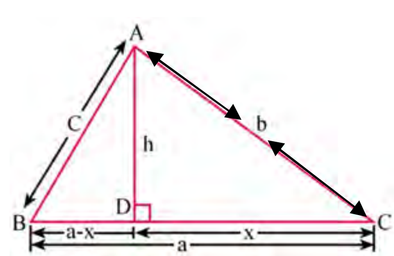
$\therefore$ area of ΔABC $= \frac{1}{2}$x BC x AD = = $\frac{1}{2}$ x b x h

$= \frac{1}{2} \times b \times \frac{\sqrt{4a^2-b^2}}{2}$

$= \frac{1}{4} b\sqrt{4a^2 - b^2}$

Hence, the area of an isosceles triangle $= \frac{1}{4} b\sqrt{4a^2 - b^2}$, where base side = b unit and two equal sides = a unit.

e. Area of a scalene Triangle



In the adjoining figure, ABC is a scalene triangle.
Where BC = a unit, CA = b unit and AB = c unit.
From vertex A, a perpendicular AD is drawn to BC.

Let AD = h and DC = x, then BD = $a- x$

The perimeter of ΔABC = a + b + c.

If the perimeter of ΔABC is denoted by 2s, then 2s = a + b + c.

$\therefore$ s = $\frac{a+b+c}{2}$, where S = semi-perimeter

Now, we try to find h and x in terms of a, b and c.

From the right angled ΔADC, $h^2=b^2-x^2$…………………… (i)

And from the right angled ΔADB, $h^2=c^2-(a-x)^2$……………… (ii)

From (i) and (ii), we get

$b^2 - x^2 = c^2-(a - x)^2$

or, $b^2-x^2 = c^2-a^2+2ax-x^2$

or, $2ax = b^2 - x^2 + a^2 - c^2 + x^2$

or, $2ax = a^2 + b^2 - c^2$

$$\therefore x = \frac{a^2+b^2-c^2}{2a}$$

Again, $h^2 = b^2-x^2$ [from equation (i)]

or, $h^2= b^2 - \left(\frac{a^2+b^2-c^2}{2a}\right)^2$

or, $h^2 = \left(b + \frac{a^2+b^2-c^2}{2a}\right)\left(b - \frac{a^2+b^2-c^2}{2a}\right)$

or, $h^2 = \left(\frac{2ab+a^2+ b^2-c^2}{2a}\right)\left(\frac{2ab-a^2+-b^2+c^2}{2a}\right)$

or, $h^2 = \left\{\frac{(a+b)^2 -c^2}{2a}\right\}\left\{\frac{c^2-(a-b)^2}{2a}\right\}$

or, $h^2 = \frac{(a+b+c)(a+b-c)(c+a-b)(c-a+b)}{4a^2}$

or, $h^2 = \frac{2s (2s-2c)(2s-2b)(2s-2a)}{4a^2}$ $\left[\because s = \frac{a+b+c}{2}\right]$

or, $h^2 = \frac{4s(s-a)(s-b)(s-c)}{a^2}$

$\therefore$ h $= \frac{2}{a}\sqrt{s(s-a)(s-b)(s-c)}$

Now, area of Δ ABC = $\frac{1}{2}$ x BC x AD = $\frac{1}{2}$.a.h [$\therefore$ Area of triangle = $\frac{1}{2}$ base × altitude]

$$= \frac{1}{2}.a.\frac{2}{a}\sqrt{s(s-a)(s-b)(s-c)}$$

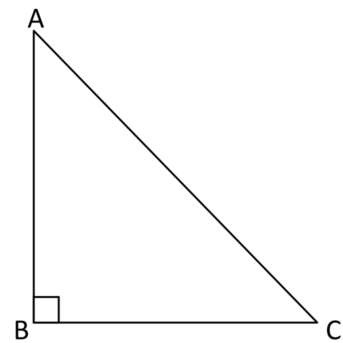
$\therefore\ \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

Hence, the area of a scalene triangle whose three sides are a, b and c is $\sqrt{s(s-a)(s-b)(s-c)}$ square units where s is semi perimeter of triangle.

This formula is known as Heron's formula for the calculation of area of a triangle.

Example 1:

In the given figure, ABC is a right angled triangle where $\angle$ ABC = 90°, AB = 8cm and BC = 6cm. Find the area of ΔABC.



Solution:

Here, in right angled triangle ABC,
base (BC) = 6cm and perpendicular AB = 8cm.

$\therefore$ Area of ΔABC $= \frac{1}{2}$ base $\times$ perpendicular

$= \frac{1}{2}$ BC.AB

$= \frac{1}{2} \times 6cm \times 8cm$

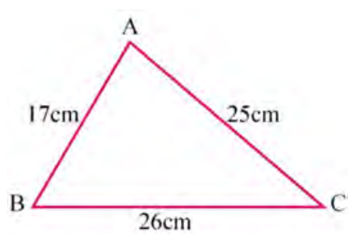
$= 24cm^2$

Thus, area of ΔABC is $24cm^2$.

Example 2:

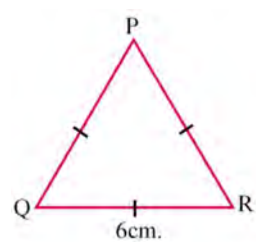
Find the area of the figures given below.

a)



(a)

b)



(b)

Solution:

a) In ΔABC, sides BC=a=26cm, AC=b=25cm, AB= c = 17cm.

$\therefore \quad s = \frac{a+b+c}{2} = \frac{26+25+17}{2} = 34cm.$

$\therefore$ Area of ΔABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{34(34-26)(34-25)(34-17)}$

$= \sqrt{34 \times 8 \times 9 \times 17}$

$= \sqrt{17 \times 17 \times 2 \times 2 \times 3 \times 3 \times 2 \times 2}$

= 17 x 2 x 3 x 2 = 204cm^2.

b) Here, ΔPQR is an equilateral triangle with side (a) = 6cm

$\therefore$ Area of ΔPQR $= \frac{\sqrt{3}}{4} a^2$

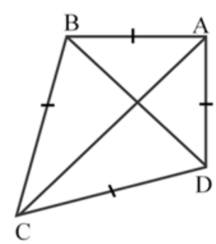
$= \frac{\sqrt{3}}{4} (6cm)^2 = 9\sqrt{3}\ cm^2$

$\therefore$ Area of ΔPQR $= 9\sqrt{3}\ cm^2$

Example3:

Find the area of the given adjoining figure where :

AC = 12cm and BD = 8cm



Solution:

ABCD is a kite where AB = AD and BC = CD. Here diagonal (AC) = d_1 = 12cm and diagonal BD = d_2 = 8cm.

We know, Area of a kite $= \frac{1}{2}(d_1 \times d_2) = \frac{1}{2} \times AC \times BD$

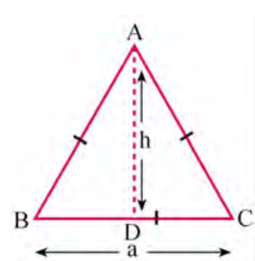
$= \frac{1}{2} \times 12cm \times 8cm = 48cm^2$

Thus, Area of the kite ABCD = 28cm^2.

Example 4:

Area of an equilateral triangle is $16\sqrt{3}$ cm^2. Find its side and height.

Solution:



Let, ABC is an equilateral triangle with side a and height AD = h.

Then area of $\Delta ABC = 16\sqrt{3}\ cm^2$

We have, Area of equilateral triangle $= \frac{\sqrt{3}}{4} a^2$

or, $16\sqrt{3}\ cm^2 = \frac{\sqrt{3}}{4} a^2$

or, $a^2 = 64cm^2$

$\therefore$ a = 8cm.

Again, area of $\Delta ABC = \frac{1}{2} \times a \times h$

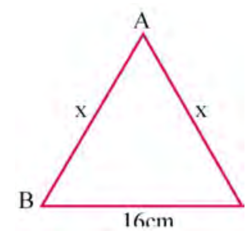
or, $16\sqrt{3}\ cm^2 = \frac{1}{2} \times 8\ cm \times h$

or, $h = 4\sqrt{3}$ cm.

Thus, side (a) = 12cm and height (h) = $4\sqrt{3}$ cm.

Example 5:

The area of an isosceles triangle is $120cm^2$ and its base is 16cm, find the length of its equal sides.



Solution:

Here, base (BC) = a =16cm

Let equal sides AB=AC=x

Then semi-perimeter

$S = \frac{a+b+c}{2} = \frac{16+x+x}{2} = 8+x$

Now area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

or, $120 = \sqrt{(8+x)(8+x-16)(8+x-x)\ (8+x-x)}$

or, $120 = \sqrt{(x+8)(x-8)(8\times8)}$

or, $120 = 8\sqrt{x^2-64}$

or, $15= \sqrt{x^2-64}$

or, $225= x^2-64$

or, $x^2= 289$

$\therefore x = 17$cm.

Hence, length of equal sides is 17cm.

Alternative way

Area of a isosceles ΔABC

$= \frac{1}{4} \times base\sqrt{4(equal\ side)^2 - (base)^2}$

or, $120 = \frac{1}{4} \times 16\sqrt{4x^2 - 16^2}$

or, 120 = 4 x $\sqrt{4x^2 - 16 \times 16}$

or, $30 = 2\sqrt{x^2 - 64}$

or, $15 = \sqrt{x^2 - 64}$

or, $225 = x^2 - 64$

or, $x^2 = 225 + 64$

or, $x^2 = 289$

$\therefore\ x = 17$cm

Example 6:

Perimeter of a triangle is 24cm. If its area is $24cm^2$ and one of its side is 8 cm, find the length of other two sides.

Solution:

One of the side (a) = 8cm

Let, other two sides be b and c

Since Perimeter (p) = 24cm

$\therefore$ a + b + c = 24cm

or, 8cm + b + c = 24cm

$\therefore$ b + c = 16cm ------------------ (i)

Semi-perimeter (s) = $\frac{p}{2} = \frac{24cm}{2}$ = 12cm

as area of the triangle is 24cm^2

$\sqrt{s(s-a)(s-b)(s-c)} = 24$

or, $\sqrt{12(12-8)(12-b)(12-c)} = 24$

or, $12\times4(144-12b-12c+bc) = 24 \times 24$

or, $144-12(b + c) + bc = 12$

or, $144-12\times16+bc = 12$ [$\therefore$ from equation (i) b + c = 16]

or, $144-192+bc = 12$ $\therefore$ bc = 60 .... (ii)

or, Substituting c = 16-b from (i) in (ii)

$b(16-b) = 60$

or, $16b - b^2 = 60$

or, $b^2-16b +60 = 0$

or, $b^2 -10b-6b+60 = 0$

or, $b(b-10)-6(b-10) = 0$

or, $(b-10)(b-6) = 0$

Either, b-10 = 0 i.e. b = 10

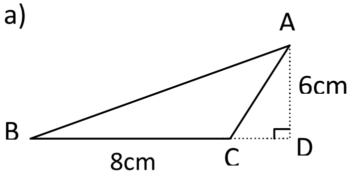
or b-6 = 0 i.e. b = 6

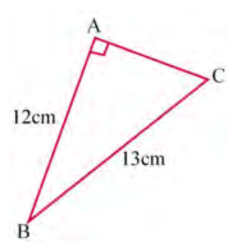
Here, if b = 6cm then c = 10cm and if b = 10cm then c = 6cm.

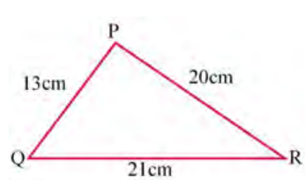
$\therefore$ The lengths of other two sides is 6cm and 10cm.

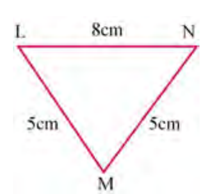
Exercise 5.1

1. Find the area of the following triangles.

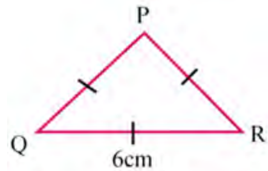
a)


b)


c)


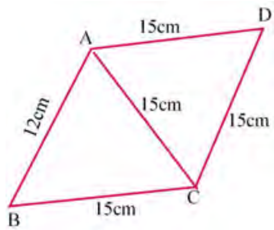
d)


e)

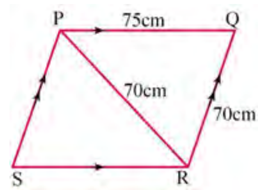


2. Find the area of the following quadrilaterals.

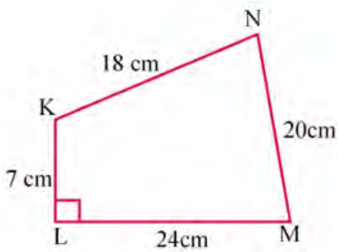
a)



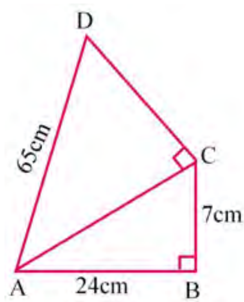
b)



c)

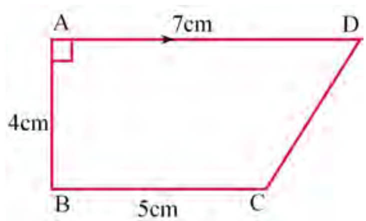


d)

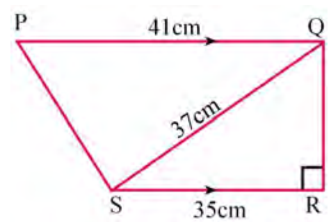


3. Find the area of the following figures.

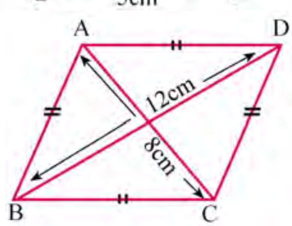
a)



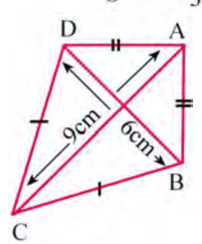
b)



c)



d)



4. **a)** Find the area of ΔABC in which the sides a = 5 cm, b = 6cm and c = 7cm.

b) Find the area of an equilateral triangle of side 12cm.

c) If the perimeter of an equilateral triangle is 30cm, find its area.

d) If the area of an equilateral triangle is $9\sqrt{3}$ cm^2, find its side.

5. a) Area of a triangle is $112cm^2$. If its base is 20cm, find its height.

b) If the area of an isosceles triangle is $240cm^2$ and its base is 20cm, find the length of its equal sides.

c) Find the perimeter and the area of an isosceles triangle with base 6cm and height 4cm.

d) Calculate the base and perimeter of an isosceles triangle having area $192cm^2$ in which the ratio of base and height is 3:2.

6. a) The sides of a triangle are in the ratio of 12:17:25. If the semi-perimeter of the triangle is 270cm, find its area.

b) Two sides of a triangle are in the ratio 15:14 and the third side is 26cm long. If the semi-perimeter of that triangle is 42cm, find its area.

c) Area and the perimeter of a right angled triangle are $6cm^2$ and 12cm respectively. Find the sides of the right angled triangle.

d) The hypotenuse of a right angled triangle is 50cm and the legs are in the ratio of 7:24. Find the area of the right angled triangle.

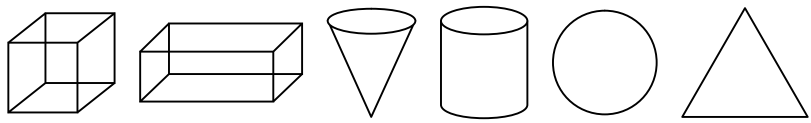
7. Measure the sides of any triangular land in feet. Then find the area of the land.

Unit: 6

Cylinder and Sphere

6.0 Review:

Observe the following figures and discuss on the following questions:



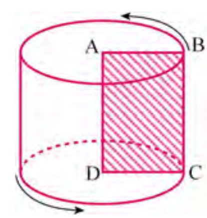
i) Write the name of the above different figures. Also, present the similarities and dissimilarities of the above figures in a table.

ii) What is the surface occupied by the above figures?

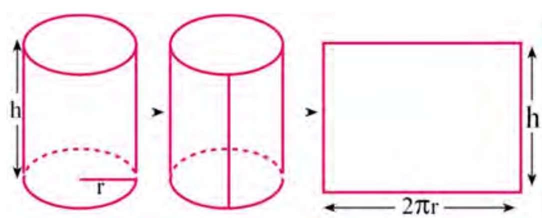
iii) What are the perimeter and area of the above figures?

iv) Are there any difference in unit of the perimeter and area of the above figures?

6.1 Cylinder



If we revolve a rectangle ABCD about its one side as its axis, it traces a solid figure called cylinder. Let a rectangle ABCD is revolved around taking the side AD as axis, it describes a cylinder of radius AB = CD and height AD = BC. The side AD is the axis of the cylinder, the point A and D are centres of circular base or cross-section. A cylinder is also called a circular based prism.



Let's take a hollow paper cylinder of vertical height h and radius of circular base r. Cutting vertically, Let's unfold it to form a rectangle with length = $2\pi r$ and breadth h. The area of the rectangle so formed is known as curved surface. Thus

Curved Surface Area (CSA) of the cylinder

= area of the rectangle

= length x breadth

= $2\pi r \times h = 2\pi rh$

= c x h, where c is circumference of base of the cylinder

$\therefore$ C.S.A. of the cylinder = $2\pi rh$ or C x h

Total surface area (TSA) of the cylinder = CSA + 2 (area of circular base)

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\,(r + h)$

$= c(r + h)$

Where c is the circumference of base.

As we know that cylinder is a circular based prism having circle on its base.

So, Volume of the cylinder = Area of base x height

$= \pi r^2 h$

i.e.

Volume of Cylinder = A x h or $= \pi r^2 h$

In terms of diameter (d)

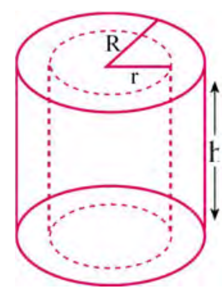
CSA = $2\pi rh = \pi dh$

TSA = $2\pi r\,(r + h) = \pi d\,(h + \frac{d}{2})$

Volume (v) = $\pi r^2 h = \frac{\pi}{4} d^2 h$.

Volume of material contained in a hollow cylindrical objects (pipes):

Let's consider a hollow cylinder of height h, external radius R and internal radius r, then external volume (V) = $\pi R^2 h$ and internal volume (v) = $\pi r^2 h$.



$\therefore$ Volume of the material contained in the hollow cylinder

$= V - v$

$= \pi R^2 h - \pi r^2 h$

$= \pi h(R^2 - r^2)$ (in terms of radii)

$= \frac{\pi}{4} h(D^2 - d^2)$ (in terms of diameters)

Example 1:

Calculate the curved surface area and total surface area of a cylinder whose radius and height are 14cm and 21 cm respectively.

Solution:

Here, radius of the cylinder (r) = 14cm

Height of the cylinder (h) = 21cm.

We have,

Curved surface area (C.S.A) $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 14^2 \times 21cm$

$= 1848cm^2$

And total surface area (T.S.A) $= 2\pi r(r + h)$

$= 2 \times \frac{22}{7} \times 14cm\ (14cm + 21cm)$

$= 3080cm^2$

Example 2:

The circumference of the base of a cylindrical drum is 88cm. If the sum of its radius and the height is 25cm, find its total surface area.

Solution: For the cylindrical drum,

Circumference (C) = 88cm,

Sum of radius and height = 25cm.

If radius and height be r and h respectively,

Then, C = 88cm.

or, $2\pi r = 88cm$

And r + h = 25cm.

∴ Total surface area (T.S.A) $= 2\pi r(r + h) = 88cm \times 25cm. = 2200cm^2$

Therefore, the total surface area of the drum is $2200cm^2$.

Example 3:

Find the volume of a cylinder whose radius of the base and height are 3.5 cm and 10cm respectively. ($\pi = \frac{22}{7}$)

Solution: Here,

Radius of the cylinder (r) = 3.5cm.

Height of the cylinder (h) = 10cm.

Volume of the cylinder (v) = ?

We know that,

Volume of the cylinder (V) = $\pi r^2 h$

$= \frac{22}{7} (3.5)^2 \text{x } 10$

$= \frac{22}{7} \text{x} 3.5 \text{ x } 3.5 \text{x } 10 = 385$cu.cm.

Example 4:

If the curved surface area of a solid cylinder is $1100cm^2$ and height 20cm, find its volume.

Solution:

Here, curved surface area (C.S.A) of the cylinder = $1100cm^2$

Height of the cylinder (h) = 20cm

Volume of the cylinder (v) = ?

We have, for a cylinder

C.S.A = 1100^2

or, $2\pi rh = 1100cm^2$

or, $2 \times \frac{22}{7} \times r \times 20cm = 1100cm^2$

or, $r = \frac{1100 \times 7}{880}$ cm

$\therefore$ r = 8.75cm.

Now, volume of the cylinder (v) $= \pi r^2 h$

$= \frac{22}{7} \times (8.75cm)^2 \times 20cm$

$= 4812.5cm^3$

Therefore, the volume of the cylinder is $4812.5cm^3$.

Example 5:

Total surface area of a solid cylindrical object is $110.88cm^2$. If the radius of the base and height of the object are in the ratio 4:5, find radius and the height.

Solution:

Here, total surface area of the cylindrical object,

T.S.A = $110.88cm^2$

If the base radius and height are r and h then r:h = 4:5

Let the radius be $4x$ and height $5x$, so that they will be in the ratio 4:5.

We have,

T.S.A = $110.88cm^2$

or, $2\pi r(r + h) = 110.88cm^2$

or, $2 \times \frac{22}{7} \times 4x(4x + 5x) = 110.88cm^2$

or, $9x^2 = \frac{110.88 \times 7}{44 \times 4} cm^2$

or, $x^2 = 0.49\text{cm}^2$

or, $x = 0.7\text{cm}$.

$\therefore$ $r = 4x = 4 \times 0.7\text{cm} = 2.8\text{cm}$

And $h = 5x = 5 \times 0.7\text{cm} = 3.5\text{cm}$

Therefore, radius is 2.8 cm and height 3.5cm.

Example 6:

Curved surface area and volume of a solid cylinder are 1355.2cm^2 and 4743.2cm^3 respectively. Find its total surface area.

Solution:

Here, for the solid cylinder

Curved surface area = 1355.2cm^2

or, $2\pi rh = 1355.2\text{cm}^2$------------- (i)

Volume = 4743.2cm^3

or, $\pi r^2 h = 4743.2\text{cm}^3$ ------------- (ii)

From (i) and (ii)

$$\frac{\pi r^2 h}{2\pi rh} = \frac{4743.2\text{cm}^3}{1355.2\text{cm}^2}$$

or, $\frac{r}{2} = 3.5\text{cm}$

$\therefore$ $r = 7\text{cm}$.

In equation (i)

$2\pi rh = 1355.2\text{cm}^2$

or, $2 \times \frac{22}{7} \times 7\text{cm} \times h = 1355.2\text{cm}^2$

or, $h = 30.8\text{cm}$.

Now, total surface area $= 2\pi r(r + h)$

$= 2 \times \frac{22}{7} \times 7\text{cm}(7\text{cm} + 30.8\text{cm})$

$= 1663.2\text{cm}^2$

Therefore, total surface area of the cylinder is 1663.2cm^2.

Example 7:

If the sum of the radius of the base and height of a cylinder is 21cm. and the curved surface area of the cylinder is 616sq. cm, find the total surfaces area of the cylinder.

Solution: Here,

The sum of radius (r) and height(h) of the cylinder = r + h = 21cm.

Curved surface area of the cylinder (C.S.A) = 616 sq.cm

Total surface area of the cylinder (T.S.A.) = ?

we have,

C.S.A. = 616sq.cm.

or, $2\pi rh = 616$

or, $2 \times \frac{22}{7} \times r \times h = 616$

or, $r \times h = \frac{616 \times 7}{44}$

or, $r \times h = 98$ .... (i)

According to questions,

$r + h = 21$

∴ $r = 21 - h$ .... (ii)

From equations (i) and (ii),

$(21-h) \times h = 98$

or, $21h - h^2 = 98$

or, $h^2 - 21h + 98 = 0$

or, $h^2 - 14h - 7h + 98 = 0$

or, $h(h-14) - 7(h-14) = 0$

or, $(h-14)(h-7) = 0$

Either, OR,

$h - 14 = 0$ $h - 7 = 0$

∴ $h = 14$ ∴ $h = 7$

If h = 14cm, then

r = 21 - 14 = 7cm.

If h = 7cm, then

r = 21 - 7 = 14cm.

Now, if r = 7cm and h = 14cm.

total surface area of the cylinder = $2\pi rh\ (r+h)$

$= 2 \times \frac{22}{7} \times 7\ (7 + 14)$

$= 44 \times 21 = 924\text{cm}^2$.

Again, If r = 14cm. and h = 7cm,

total surface area of the cylinder = $2\pi rh(r+h)$

$= 2 \times \frac{22}{7} \times 14(14 + 7)$

$= 88 \times 21 = 1848\text{cm}^2$.

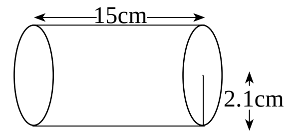
Exercise 6.1

1. **a)** Calculate the curved surface area and total surface area of a cylinder whose radius and height are 5cm and 14cm respectively.

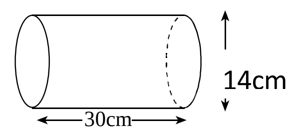
   b) Find the curve surface area and total surface area of a solid cylindrical object having diameter 7cm and height 8cm.

   c) Find the curve surface area and total surface area of a solid cylindrical object having radius 4.2cm and height 11cm.

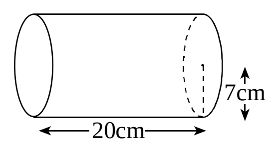
2. **a)** Find the curved surface area and total surface area of the given cylinder.



   b) Calculate the total surface area of the solid cylindrical object shown in the figure.



   c) Calculate the volume of the solid cylinder object shown in the figure.



3. If the sum of the radius of the base and height of a cylinder is 17.5cm.and the curved surface area of the cylinder is $308cm^2$, find the total surface area of the cylinder.
4. Find the volume of the cylinder whose radius of the base is 7cm. and height is 13cm.
5. Height and total surface area of a solid cylinder are 12.5cm and $3300cm^2$ respectively. Find the radius of the cylinder.
6. If total surface area of a cylinder is $4620cm^2$ and sum of its radius and height is 35cm, find its curved surface area.
7. If the curved surface area of a cylinder is $2464cm^2$ and height 28cm, find its volume.
8. Total surface area of a solid cylinder is 880sq.cm and the radius of the circular base is 4cm. Find its volume.
9. If the radius and height of a right cylinder are in the ratio 1:3 and total surface area is $1232cm^2$, find its volume.
10. The curved surface area of a cylinder is $880cm^2$ and its volume is $770cm^3$.Find its diameter and height.

11. The volume of a cylinder is $770cm^3$ and circumference of the circular base of the cylinder is 44cm. Find its height.

12. If the volume and curved surface area of a cylindrical can are $616cm^3$ and $176cm^2$respectively, find its diameter and height.

13. The internal and external diameter of a steel pipe of length 140cm are 8cm and 10cm respectively. Find the thickness of the pipe and volume of steel in it.

14. A metal pipe is 70cm in length. Its internal diameter is 24mm and thickness 1mm. If $1mm^3$ of the metal weighs 0.02gm, find the weight of the pipe.

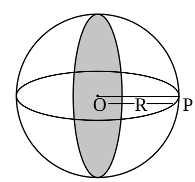
15. From a cubical tank of side 12cm, which is full of water, the content is poured into a cylindrical vessel of radius 3.5cm. What will be the height of the water level in the vessel?

16. Construct two cylinders by taking length and breadth as the base of the cylinder respectively from a piece of rectangular paper of length and breadth are 21cm and 14cm respectively. Find the volume of the both cylinders. Also, write your conclusion.

6.2 Surface Area and Volume of a Sphere

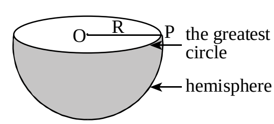
A sphere is a solid object, each point of its surface is equidistant from a fixed point inside it. The fixed point is called the centre and the constant distance is called the radius of the sphere. Objects such as globe, football, tennis ball, etc. are examples of spheres.

In the figure given alongside, O is the centre, P is a point on the surface. So OP is the radius of the sphere. The circular plane through the centre, dividing the sphere into two equal halves is called the greatest circle and the equal half spheres are called hemispheres.



For a sphere, if radius = R and diameter = d,

- Surface area(A) = $4\pi R^2 = \pi d^2$ sq. units
- Volume (V) = $\frac{4}{3}\pi R^3 = \frac{\pi}{6} d^3$ cu. units



For a hemisphere,

- Curved surface area (C.S.A) $= 2\pi R^2 = \frac{\pi}{2} d^2$ sq. units
- Total surface area (T.S.A) = C.S.A + area of circular face

$$= 2\pi R^2 + \pi R^2 = 3\pi R^2 = \frac{3}{4}\pi d^2 \text{ sq. units}$$

- Volume (V) = $\frac{2}{3}\pi R^3 = \frac{\pi}{12} d^3$ cubic units

Volume of a sphere:

Let the surface of the sphere is divided into an infinite number of small polygons each of which is practically a plane surface. Consider pyramids are formed on these polygons having height equal to the radius r of the sphere and vertex at the centre of the sphere. The sum of the bases of the pyramids is the whole surface of sphere and the sum of volume of all these pyramids is the volume of the sphere.

Volume of a small pyramid $= \frac{1}{3} \times$ area of base $\times$ height

$\therefore$ Volume of the sphere $= \frac{1}{3} \times$ sum of area of bases of the pyramids $\times$ height

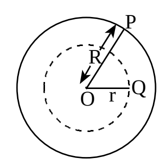
$= \frac{1}{3} \times$ surface area of sphere $\times$ radius

$= \frac{1}{3} \times 4\pi r^2 \times r$

$= \frac{4}{3}\pi r^3$

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

Volume of material contained in a hollow spherical shell:



Let external radius (radius of whole sphere) be R and inner radius (radius of inner hollow part) be r, then

External volume (volume of whole sphere) $= \frac{4}{3}\pi R^3$

Internal volume (volume of hollow part) $= \frac{4}{3}\pi r^3$

$\therefore$ Volume of the material contained in the shell $= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (R^3 - r^3)$ cu. units.

Surface Area and volume of a Hemisphere:

curved surface area of a hemisphere = half the surface of a sphere

$= \frac{1}{2} \times 4\pi r^2 = 2\pi r^2$

Total surface area of a hemisphere = curved surface area of the hemisphere + area of the base

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$ sq. unit.

Volume of the hemisphere = half of the volume of the sphere

$= \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$ cubic unit.

Example 1:

Find the surface area and volume of a sphere with radius 7cm.

Solution: Here,

radius of the sphere (r) = 7cm

surface area of the sphere (A) =?

Volume of the sphere (V) = ?

Now, Surface area of the sphere (A) = $4\pi r^2$

$$= 4\times \frac{22}{7}(7cm)^2$$

$$= 4\times \frac{22}{7} 49cm^2 \quad = 616cm^2$$

Volume of the sphere(v) $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7}(7cm)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 343cm^3$$

$$= 1437.33cm^3$$

Example 2:

Calculate the radius of a sphere having surface area154cm^2.

Solution: Here, radius of the sphere be 'r'.

Surface area (S.A) = $154cm^2$

We have,

Surface are of sphere (S.A) = $4\pi r^2$

or, $154cm^2 = 4\pi r^2$

or, $154cm^2 = 4 \times \frac{22}{7} r^2$

or, $r^2 = \frac{154 \times 7}{4 \times 22}$

$\therefore$ r = 3.5cm

Therefore, the radius of the sphere is 3.5cm.

Example 3:

If the total surface area of a sphere is 154cm^2, find its volume.

Solution: Here,

Surface area of the sphere (A) = $154cm^2$.

Volume of the sphere (V) = ?

Given that,

Surface area of the sphere = $154cm^2$.

or, $4\pi r^2 = 154cm^2$

or, $4 \times \frac{22}{7} r^2 = 154cm^2$

or, $r^2 = \frac{154\times7}{4\times22} cm^2$.

or, $r^2 = \frac{7\times7}{2\times2} cm^2$.

or, $r^2 = (\frac{7}{2} cm)^2$.

$\therefore r = \frac{7}{2}$cm.

Now, Volume of the sphere(v) $= \frac{4}{3}\pi r^3 .= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{2} cm\right)^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{7\times7\times7}{2\times2\times2} cm^3$

$= 179.67cm^3$

Example 4:

Find the total surface area of a hemispherical solid having radius 14cm.

Solution:

Here, radius of the hemisphere (r) = 14cm

Surface area (S.A) = ?

We have,

Total surface area of the hemisphere (T.S.A) $= 3\pi r^2$

$= 3 \times \frac{22}{7} (14cm)^2$

$= 1848cm^2$

Therefore, the surface area of the hemisphere is $1848cm^2$.

Example 5:

Find the curved surface area and total surface area of a hemisphere of diameter 28cm.

Solution:

Here, diameter of the hemisphere (d) = 28cm

Curved surface area (C.S.A) = ?

Total surface area (T.S.A) =?

We have,

Curved surface area of the hemisphere $= \frac{\pi}{2} d^2$

or, C.S.A $= \frac{1}{2} . \frac{22}{7} .(28cm)^2$

$= 1232cm^2$

and total surface area (T.S.A) $= \frac{3}{4}\pi d^2$

$= \frac{3}{4} \times \frac{22}{7} (28cm)^2 = 1848cm^2$

Therefore, C.S.A and T.S.A of the hemisphere are $1232cm^2$ and $1848cm^2$ respectively.

Example 6:

Volume of a hemispherical object is 2425.5 cubic cm. Find the total surface area of the hemisphere.

Solution:

Here, Volume of the hemisphere (v) = 2425.5 cm^3

Radius of the hemisphere (r) = ?

$\frac{2}{3}\pi r^3 = V$

or, $\frac{2}{3} \times \frac{22}{7} \times r^3 = 2425.5cm^3$

or, $r^3 = \frac{2425.5 \times 21}{44} cm^3$

$\therefore$ r = 10.5cm

Now, total surface area of the hemisphere $= 3\pi r^2$

$= 3 \times \frac{22}{7}(10.5cm)^2$

$= 1039.5cm^2$

Example 7:

What will be the volume and surface area when the radius of a sphere is doubled?

Solution:

Let radius of the sphere before doubling be r, then

Surface area (A) = $4\pi r^2$ ………….(i)

And Volume (V) = $\frac{4}{3}\pi r^3$ ………….(ii)

When radius is doubled,

New radius R = 2r,

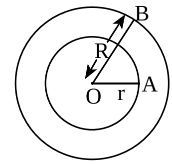
$\therefore$ New surface area $(A_1) = 4\pi R^2 = 4\pi(2r)^2 = 4(4\pi r^2) = 4A$

And new volume $(V_1) = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(2r)^3 = 8\left(\frac{4}{3}\pi r^3\right) = 8V$

Therefore, on doubling the radius of the sphere its surface area becomes 4 times and volume becomes 8 times more.

Example 8:

Find the volume of the material contained in a hollow sphere having external radius 6.3 cm and inner radius 5.6cm.



Solution:

Here, outer radius of the sphere (R) = 6.3cm and inner radius (r) = 5.6cm.

Now,

We have, Volume of material in the hollow sphere

$$V = \frac{4}{3}\pi(R^3 - r^3)$$

$$= \frac{4}{3} \times \frac{22}{7}\{(6.3\text{cm})^3 - (5.6\text{cm})^3\}$$

$$= \frac{88}{21} \times (250.047 - 175.616)\text{cm}^3$$

$$= \frac{88}{21} \times 74.431\text{cm}^3 = 311.9\text{cm}^3$$

Therefore, volume of the material in the hollow sphere is 311.9cm^3.

Example 9:

Three solid metallic spheres of radii 10cm, 8cm and 6cm respectively are melted to form a single solid sphere. Find the radius of the resultant sphere.

Solution:

Radii of the given metallic spheres are

r_1 = 10cm, r_2 = 8cm and r_3 = 6cm

Let radius of the resultant sphere be R, then

Volume of the resultant sphere = sum of the volumes of the given spheres.

$$\text{or, } \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$\text{or, } R^3 = (10\text{cm})^3 + (8\text{cm})^3 + (6\text{cm})^3$$

$$\text{or, } R^3 = 1728\text{cm}^3$$

$$\therefore R = 12\text{cm}.$$

Therefore, the radius of the resultant solid sphere is 12cm.

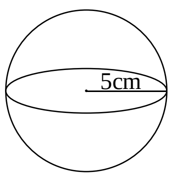
Exercise 6.2

1. Find the surface area of the sphere having
   - a) radius = 3.5cm b) radius = 7cm
   - c) diameter = 21cm d) diameter = 42cm

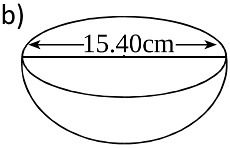
2. Find the volume of the sphere having
   - a) radius = 2.1cm b) radius = 14cm
   - c) diameter = 7.2cm d) diameter = 8.4cm

3. Calculate the surface area and volume of the following solids.

   a)



   b)



4. a) If the surface area of a spherical object is 616cm^2, find its radius.

   b) If the surface area of a ball is 38.5cm^2, calculate its diameter.

5. a) If the surface area of a sphere is 616cm^2, find its volume.

   b) If the surface area of a sphere is 2464cm^2, find its volume.

6. a) If the volume of a sphere is 4851cm^3, find its radius.

   b) A spherical ball has volume of $\frac{9\pi}{2}$ cm^3. Calculate its diameter.

7. Find the curved surface area and total surface area of the hemisphere having
   - a) radius = 4.2 cm b) diameter = 7.2 cm

8. a) If the volume of a solid in the form of a hemisphere is $\frac{2000\pi}{3}$ cm^3, find area.

   b) A big bowl in the form of a hemisphere has capacity of 65.4885 liters. Find its total surface area.

9. a) By how much will the surface area and volume of a sphere increase if radius is doubled?

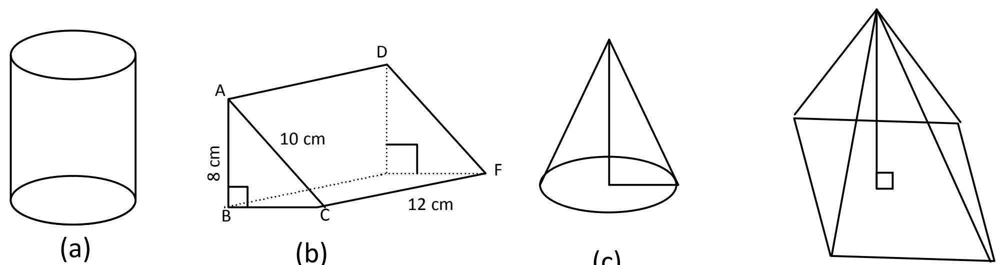
   b) Surface area of a sphere is πm^2. If its radius is doubled, find the difference in area.

10. Find the volume of the material contained in a hollow sphere having external radius 8.3 cm and internal radius 6.6 cm.
11. Three solid metallic spheres of radii 9cm, 12cm and 15cm respectively are melted to from a single solid sphere. Find the radius of the resultant sphere.
12. A solid metallic sphere of diameter 24cm is melted and cast to 3 equal small spheres. Find the radius of the small sphere so formed.
13. A solid metallic sphere of diameter 6cm is melted and cast to a solid cylinder of radius 3cm. Find the height of the cylinder.
14. A solid sphere of aluminum of diameter 6cm is melted and drawn into a cylindrical wire 2mm thick. Find the length of the wire.
15. A solid metal sphere of diameter 42cm is immersed into a cylindrical drum partly filled with water. If the diameter of the drum is 140cm, by how much will the surface of the water in the drum be raised?
16. Collect the different size of balls in your school. Then find their surface area and volume.
17. Collect a T.T. ball and a cricket ball. Then divide them into halves. Find the total surface area and volume of the halves of both balls. Present your answer to the class.

Prism and Pyramid

7.0 Review:

Study the following figures and discuss on the questions given below:



(a) (b) (c) (d)

i) What is the name of the above solid figures.

ii) Distinguish the prism and pyramid from the above figures.

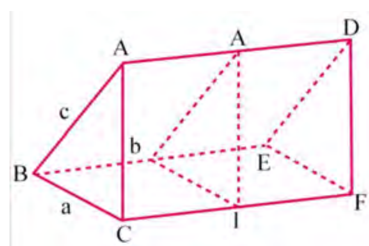
iii) How many surfaces are there in the above each figures?

iv) Write the name of the surface in the above figures.

v) Write the name of solid figures which have the same surface.

7.1 Surface Area and Volume of Triangular Prisms

A triangular prism consists triangular base or cross-section and three rectangular lateral faces. The given figure is a triangular prism with base ΔABC and ΔDEF which are congruent and lateral rectangular faces ABED, BCFE and ACFD. Let sides of base be a, b and c, area of cross-section A and length of the prism be l, then



Lateral surface area = Area of rectangle ABED + area of rectangle BCFE + area of rectangle ACFD.

$= c.l + a.l + b.l$

$= (a + b + c)l$

$\therefore$ L.S.A $=$ perimeter of the base $\times\ l = p.l$

Total surface area = 2 x Area of triangular base + L.S.A

$\therefore$ T.S.A $= 2A + p.l$

Volume of the prism = Area of triangular base × length

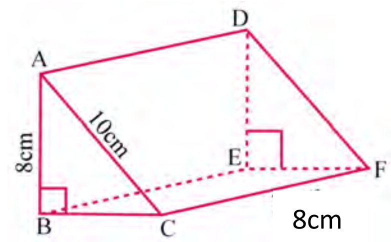
$\therefore V = A.l$

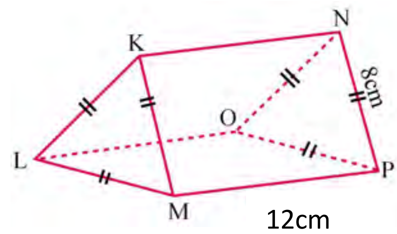
Note: Formula for area of triangular base depends upon the given triangle.

Example 1:

Find the lateral surface area and total surface area of the following triangular prisms.

a)





Solution:

a) The prism is a triangular prism. The base is a right angled triangle where AB =8cm, AC= 10cm and $\angle B = 90°$

$\therefore BC = \sqrt{AC^2 - AB^2} = \sqrt{(10cm)^2 - (8cm)^2} = \sqrt{100cm^2 - 64cm^2} = \sqrt{36cm^2} = 6cm.$

$\therefore$ Area of base (A) $= \frac{1}{2} \times AB \times BC = \frac{1}{2} \times 8cm \times 6cm = 24cm^2$

Height of the prism (h) = 18 cm

Now,

Lateral surface area (L.S.A) $=$ perimeter of $\Delta ABC \times$ height

$= (8+10+6)cm \times 18cm$

$= 24cm \times 18cm$

$= 432cm^2$

Total surface area $= 2A +$ L.S.A

$= 2 \times 24cm^2 + 432cm^2$

$= 48cm^2 + 432cm^2 = 480cm^2$

b) Base of the prism is an equilateral triangle with side a = 8cm.

$\therefore$ Area of triangular base (A) $= \frac{\sqrt{3}}{4} a^2$

$= \frac{\sqrt{3}}{4} \times (8cm)^2$

$= 16\sqrt{3}\ cm^2 = 27.7cm^2$

length of the prism(l) $= 12cm$

$\therefore$ lateral surface area (L.S.A) $= p \times l$

$= (3a) \times 12cm$

$= 3 \times 8cm \times 12cm$

$= 288cm^2$

Now,

Total surface area (T.S.A) $= 2A + L.S.A$

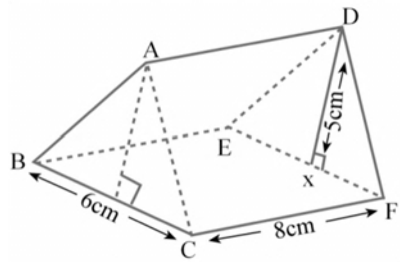
$= 2 \times 27.7cm^2 + 288cm^2$

$= 55.4cm^2 + 288cm^2$

$= 343.4cm^2$

Example 2:

Find the volume of the given triangular prism.



Solution: Here,

For the triangular base ABC

base (b) = BC = 6cm

height(h) = DX = 5cm

$\therefore$ Area of cross-section (A) $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 6cm \times 5cm = 15cm^2$

Length of the prism (l) = 8cm.

Now,

Volume of the prism (V) = base area × length

$= A \times l$

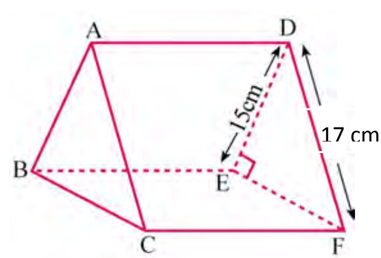
$= 15cm^2 \times 8cm$

$= 120cm^3$

Example 3:

If the volume of the given triangular prism is $840cm^3$, find the length of the prism.

Solution:



Here, the cross-section is a right angled triangle with AB = 15cm, and AC = 17cm.

$\therefore$ Using Pythagoras theorem

$BC = \sqrt{AC^2 - AB^2} = \sqrt{17^2 - 15^2} = 8cm$

Let l be the length of the prism, then

Volume = Area of base × length

or, $840cm^3 = \frac{1}{2} \times BC \times AB \times l$

or, $840cm^3 = \frac{1}{2} \times 8cm \times 15cm \times l$

$\therefore l = \frac{840cm^3}{60cm^2} = 14cm$

Therefore, length of the prism is 14cm.

Example 4:

Lateral surface area of a prism having equilateral triangular base is $108\sqrt{3}$ cm^2. If its length is $6\sqrt{3}$ cm, find its volume.

Solution: Here,

Lateral surface area (L.S.A) = $108\sqrt{3}$ cm^2

Length of the prism (l) = $6\sqrt{3}$ cm

Let length of the side of equilateral triangular base be 'a'.

Then,

L.S.A = P × l

or, $108\sqrt{3}$ cm^2 = 3a × l

or, $108\sqrt{3}$ cm^2 = 3 × a × $6\sqrt{3}$ cm

or, a = 6cm

$\therefore$ Area of triangular base (A) $= \frac{\sqrt{3}}{4} a^2$

$= \frac{\sqrt{3}}{4} (6\text{cm})^2$

$= 9\sqrt{3}$ cm^2

Now,

Volume of the prism (V) = A × l

$= 9\sqrt{3}$ cm^2 × $6\sqrt{3}$ cm

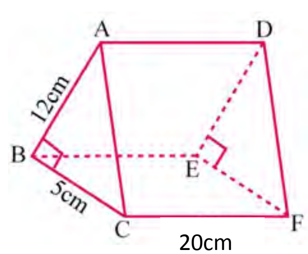
= 162cm^3

Therefore, the volume of the prism is 162cm^3.

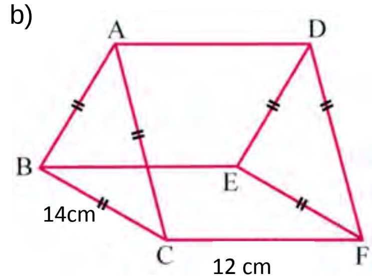
Exercise 7.1

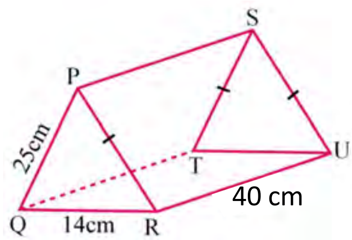
1. **Find the base area, lateral surface area and total surface area of the following triangular prism.**

a)



b)



c) 

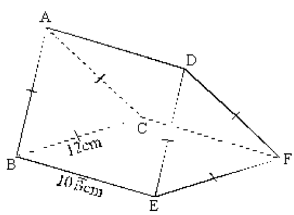
2. **a)** If the perimeter of the base of a triangular prism is 18cm and its height is 15cm, find the lateral surface area of the prism.

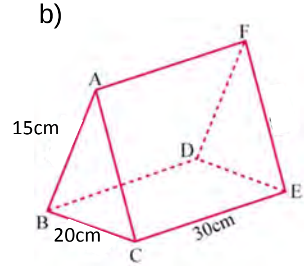
b) If the area of the base of a triangular prism is $18.4cm^2$ and its length 35cm, find the volume of the prism.

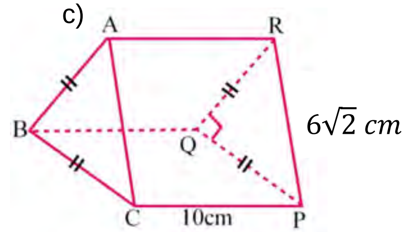
c) The area of the base of a triangular prism is $28.5cm^2$. If its lateral surface area is $300cm^2$, find the total surface area of the prism.

d) If the perimeter of the base of a triangular prism is 15.5cm and its lateral surface area is $248cm^2$, find the length of the prism.

3. Find the volume of the following triangular prisms:

a) 

b) 

c) 

4. **a)** The volume of a prism with equilateral triangular cross-section is $270cm^3$.If the length of the prism is $10\sqrt{3}$ cm, what is the length of the side of the equilateral triangular cross-section?

b) If height of the right-angled isosceles triangular based prism with volume 2000 cubic cm is 40cm, find the length of its sides.

5. **a)** Find the volume of a prism having triangular base of sides 10cm, 17cm and 21cm and length 25cm.

b) If the sides of the base of a triangular prism are 3cm, 4cm, and 5cm and total surface area is 156 square cm, find its height and volume.

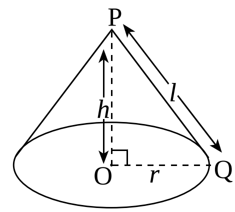
6. The cross-section of a triangular prism is a right-angled isosceles triangle with one of the equal side 6cm.If the length of the prism is $8\sqrt{2}$ cm, calculate the total surface area and volume of the prism.

7. Measure all the parts of a triangular prism which is available from your science practical room. Find the area of cross-section, lateral surface area and total surface area of the prism. Also find its volume.

8. Make different groups of the students of your class. Make two/two prisms of different size from the paper, wood, bamboo by each group of the students. Measure the all parts of each prism and then find cross-section, lateral surface and total surface area of it after that present the solution to the class.

7.2 Surface Area and Volume of Cones

If a right angled triangle POQ is revolved about one of the sides containing right angle, the solid thus formed is called a right circular cone.



Here, in the cone given in the figure, P is called vertex, O the centre and OQ radius (r) of the circular base and PQ is called the generator whose length is called slant height (l). OP is called vertical height (h).

A cone is also called circular based pyramid.

Relation between the vertical height (h), radius (r) and slant height (l),

$$l^2 = h^2 + r^2$$

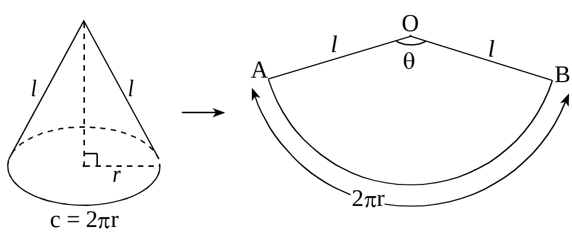
or, $$l = \sqrt{h^2 + r^2}$$

$$h = \sqrt{l^2 - r^2}$$

and $$r = \sqrt{l^2 - h^2}$$

Curved surface area of a cone

Let's consider a hollow cone of paper with open base of radius (r) and slant height (l). So, the circumference of the circular base c = 2πr.



Let the cone be unfolded to form a sector in which radius R = l and arc AB = 2πr. Let ∠AOB = θ be the angle of the sector (i.e. central angle).

We have, central angle

$$\theta = \frac{s}{r} = \frac{\text{length of arc}}{\text{radius}} \text{ in radians}$$

or, $$\theta = \frac{2\pi r}{l}$$

$$\therefore \theta l = 2\pi r \text{ .... (i)}$$

$$\left[\begin{array}{l}\text{If central angle is } 360^o\text{, area} = \pi R^2 \\ \text{If central angle is } \theta\text{, area} = \dfrac{\theta \,.\, \pi R^2}{360^o}\end{array}\right]$$

Now, area of sector $$= \frac{\theta.\pi r^2}{360^0}$$

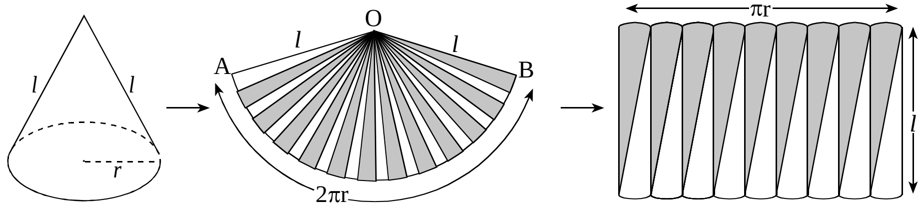
$$= \frac{\theta.\pi l^2}{2\pi} = \frac{\theta l^2}{2} = \frac{\theta l.l}{2} = \frac{\cancel{2}\pi r.l}{\cancel{2}} \quad \text{[from (i) } \theta l = 2\pi r]$$

∴ Area of sector = πrl, which is the curved surface area (C.S.A) of the cone.

∴ Curved surface area of the cone (C.S.A) = πrl.

Simply, C.S.A. of a cone = $\pi r l$ can be verified as ………

Let's consider a cone of paper with open base of radius r and slant height l.



(i) (ii) (iii)

$\therefore$ Circumference of the base c = $2\pi r$.

Now, the cone is unfolded to a sector whose radius is R = l and length of arc S = $2\pi r$

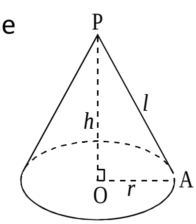
Then, the sector is divided into equal sectors as in the figure. Now the small sectors are arranged in opposite direction alternately as in the figure in 3rd step to form approximately a rectangle of length πr and breadth l.

$\therefore$ Area of the rectangle = l × b= $\pi r \times l = \pi r l$

Which is the curved surface area of the cone.

$\therefore$ Curved surface area of the cone (C.S.A.) = $\pi r l$

Total surface area (T.S.A) of a cone:



Total surface area (T.S.A) = curved surface area + area of plane circular base

$= \pi r l + \pi r^2$

$\therefore$ T.S.A $= \pi r(l + r)$

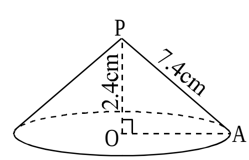
Volume of cone:

$V = \frac{1}{3}$ base area × vertical height $= \frac{1}{3}\pi r^2 h$.

Example 1:

Find the curved surface area and total surface area of the given cone.

Solution: Here,



Slant height of the cone (l) = 7.4cm

Vertical height (h) = 2.4cm.

We have,

Radius of the base (r) = $\sqrt{l^2 - h^2} = \sqrt{(7.4)^2 - (2.4)^2}$ = 7cm

Now, curved surface area (C.S.A.) = $\pi r l = \frac{22}{7} \times 7\text{cm} \times 7.4\text{cm} = 162.80\text{cm}^2$

Total surface area (T.S.A) = $\pi r(l + r) = \frac{22}{7} \times 7\text{cm}\ (7.4 + 7)\text{cm} = 316.8\text{cm}^2$

Example 2:

Find the volume of a right circular cone having base radius 14cm and slant height 14.8cm.

Solution: Here,

Radius of the base (r) = 14cm

Slant height (l) = 14.8cm

We have,

Vertical height (h) = $\sqrt{l^2 - r^2}$

$= \sqrt{(14.8\text{cm})^2 - (14\text{cm})^2}$

$= \sqrt{23.04\text{cm}^2} = 4.8\text{cm}$

Now, Volume of the cone (v) $= \frac{1}{3}\pi r^2 h.$

$= \frac{1}{3} \times \frac{22}{7} \times 14^2 \times 4.8\text{cm}^3 = 985.6\text{cm}^3$

Example 3:

If the volume of a right circular cone is 1796.256 cm^2 and the radius of the circular base 18.9cm, find the curved surface area.

Solution: Here,

Volume of the cone (V) = 1796.256cm^3

Radius of the base (r) = 18.9cm

Let, the vertical height be h and slant height l.

We have, $V = \frac{1}{3}\pi r^2 h$

or, $1796.256\text{cm}^3 = \frac{1}{3} \times \frac{22}{7} \times (18.9\text{cm})^2 \times h$

$\therefore h = 4.8\text{cm}$

$\therefore l = \sqrt{h^2 + r^2}$

$= \sqrt{(4.8)^2 + (18.9)^2}\text{cm}$

= 19.5cm

Now, curved surface area (C.S.A.):

$= \pi r l$

$= \frac{22}{7} \times 18.9\text{cm} \times 19.5\text{cm}$

= 1158.3cm^2

Example 4:

The sum of the radius of the base and slant height of a right cone is 6.4cm and total surface area of the cone is 28.16cm^2. Find the curved surface area of the cone.

Solution: Here,

T.S.A = 28.16cm^2

r + l = 6.4cm

We have,

T.S.A = πr(r + l)

or, $28.16cm^2 = \frac{22}{7} \times r \times 6.4cm$

or, $r = \frac{28.16 \times 7}{22 \times 6.4} cm$

$\therefore$ r = 1.4cm

Now, curved surface area = T.S.A – area of base

$= 28.16cm - \pi r^2$

$= 28.16cm - \frac{22}{7} \times (1.4cm)^2 = 22cm^2$

Example 5:

Find the volume of a right circular cone with slant height 19.5cm and curved surface area 1158.3cm^2.

Solution: Here,

Slant height (l) = 19.5cm

Radius of the base be r

Curved surface area (C.S.A) = 1158.3cm^2

We have,

C.S.A. = $\pi r l$

or, $1158.3cm^2 = \frac{22}{7} \times r \times 19.5cm$

or, r = 18.9cm

$\therefore$ Vertical height (h) $= \sqrt{l^2 - r^2}$

$= \sqrt{(19.5)^2 - (18.9)^2}$ cm

= 4.8cm

Now, Volume of the cone (V) $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} (18.9\text{cm})^2 \times 4.8\text{cm}$$

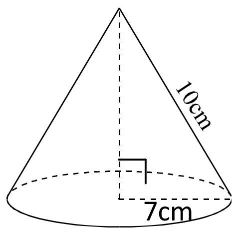
$$= 1796.256 \text{ cm}^2$$

Therefore, the volume of the cone is 1796.256cm^2.

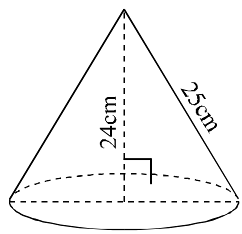
Exercise 7.2

1. Find the curved surface area and total surface area of the following right cones.

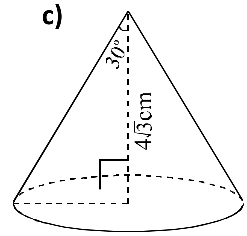
a)



b)

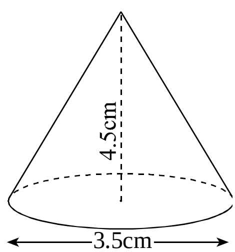


c)

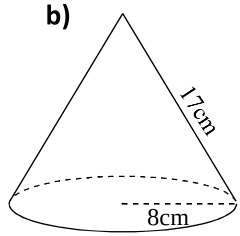


2. Find the volume of the following right cones.

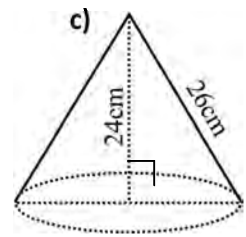
a)



b)



c)



3. **a)** Find the volume of the cone having vertical height 27cm and radius 7cm.

b) A cone has diameter of its base 14cm and slant height 25cm. Find its volume.

4. **a)** Find the total surface area of a cone whose radius of base 13cm and height 84cm.

b) The diameter of the circular base of a cone is 12cm and its vertical height is 8cm. Find the total surface area of the cone.

c) Find the curved surface area of a cone whose slant height and vertical height are 35cm and 28cm respectively.

5. **a)** Find the vertical height and volume of a right circular cone with slant height 25cm and curved surface area 550cm^2.

b) If the radius of a right circular cone is 4cm and curved surface area 62.8cm^2, find its volume. [$\pi = 3.14$]

6. **a)** Find the height of the cone whose volume is 1232cm^3 and diameter of its circular base is 14cm.

b) A right circular cone of height 21cm has a volume of $1078cm^3$. Calculate the diameter of the base of the cone.

c) Volume of a cone is $513.33cm^3$ and its vertical height is 10cm. Calculate the circumference of the base.

7. a) Total surface area of a cone is $594cm^2$ and its slant height 20cm. Find the diameter of its base.

b) Curved surface area of a cone is $2200cm^2$. If its slant height is 50cm, find the height of the cone.

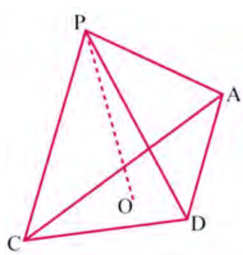
c) Curved surface area of a right circular cone is $8800cm^2$ and diameter of its base is 56cm. Find its height.

8 a) Total surface area and curved surface area of a cone are $770cm^2$ and $550cm^2$ respectively. Find its radius.

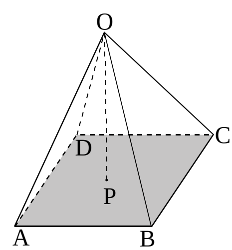
b) Total surface area and curved surface area of a cone are 1320 cm^2 and $704cm^2$, find its radius.

7.3 Pyramid

A pyramid is a solid having polygonal base and plane triangular faces meeting at a common vertex.



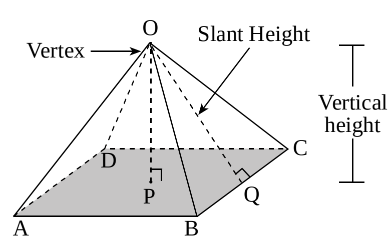
Triangular based pyramid



Square based pyramid

The base of a pyramid may be any polygon, triangle, square, rectangle, hexagon and so on. The length of the perpendicular drawn from the vertex to the base is called vertical height. If the vertical height falls at the centre of the base, the pyramid is known as right pyramid.

As shown in the figure, ABCD is the base of the pyramid. OAB, OBC, OCD, OAD are triangular faces, O is the vertex and OP which is perpendicular to the base is the vertical height. Height of a triangular face over its base is known as slant height. In this figure, OQ is the slant height of the face OBC.



Naming of a pyramid is made according to the shape of the base as triangular based pyramid, square based pyramid, rectangular based pyramid, etc.

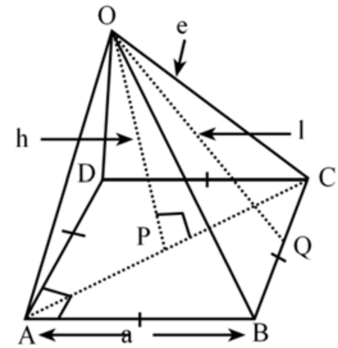
Square based pyramid.

Let's consider a square based pyramid with vertex O, vertical height OP = h

Slant height OQ = l

edge (OC) = e

diagonal (AC) = d



- Area of square base = a^2
- Area of square base = $\frac{1}{2}d^2$ where d = length of diagonal of square base.
- Area of a triangular face = $\frac{1}{2}$ a.l

→ Relation of the vertical height, side of the base and the slant height

Slant height (l) = $\sqrt{h^2 + \frac{a^2}{4}}$

Vertical height (h) = $\sqrt{l^2 - \frac{a^2}{4}}$

Length of side (a) = $2\sqrt{l^2 - h^2}$

→ Relation of the slant height, side of the base and edge of the triangular faces.

Slant height (l) = $\sqrt{e^2 - \frac{d^2}{4}}$

Side of base (a) = $2\sqrt{e^2 - l^2}$

→ Relation of the vertical height ,length of the diagonal and edge of the triangular faces.

Vertical height (h) = $\sqrt{e^2 - \frac{d^2}{4}}$

Length of diagonal (d) = $2\sqrt{e^2 - h^2}$

Length of side of base (a) = $\frac{d}{\sqrt{2}} = \sqrt{2(e^2 - h^2)}$

Volume of square based pyramid (v) $= \frac{1}{3}$ A x h[$\because$ A = base area]

$= \frac{1}{3}a^2 \times h$

Lateral Surface area(Triangular faces area) of square based pyramid (LSA) = $4\left(\frac{1}{2}al\right)$

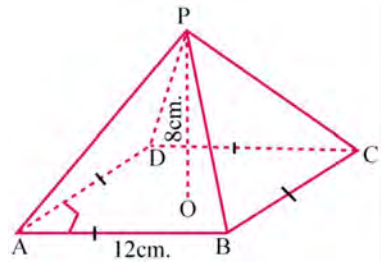
= 2al

Total surface area of square based pyramid (TSA) = $4\left(\frac{1}{2}al\right) + a^2$

$= 2al + a^2$

Example 1:

Find the triangular surface area, total surface area and volume of the square-based pyramid given in the figure alongside.



Solution: Here, side of the base (a) = 12cm

$\therefore$ Base area (A) $= a^2$

$= (12\text{ cm})^2$

$= 144\text{ cm}^2$

Vertical height (h) = 8cm

$\therefore$ Slant height (l) $= \sqrt{h^2 + \left(\frac{a}{2}\right)^2}$

$= \sqrt{(8\text{cm})^2 + \left(\frac{12\text{cm}}{2}\right)^2}$

$= \sqrt{(64 + 36)\text{cm}^2} = 10\text{cm}.$

$\therefore$ Area of the triangular faces $= 4.\left(\frac{1}{2}.a.\ l.\right)$

$= 2 \times 12\text{cm} \times 10\text{cm}$

$= 240\text{cm}^2$

$\therefore$ Total surface area = base area + area of triangular faces

$= 144\text{cm}^2 + 240\text{cm}^2$

$= 384\text{cm}^2$

And Volume (V) $= \frac{1}{3}$ base area $\times$ vertical height

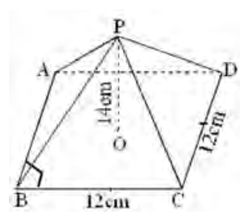
$= \frac{1}{3} \times 144\text{cm}^2 \times 8\text{cm}$

$= 384\text{cm}^3$

Therefore, total surface area of the pyramid is 384cm^2 and its volume is 384cm^3.

Example 2:

The adjoining figure is a square-based pyramid where the length of the side of the base is 12cm and vertical height is 8cm. Find the total surface area of the pyramid.



Solution: Here,

length of a side of the base (a) = 12cm.

height of the pyramid (h) = 8cm.

slant height of the pyramid (l) = ?

Total surface area of the pyramid (T.S.A.) = ?

In the figure,

OM = $\frac{1}{2}$ of BC = $\frac{1}{2} \times 12cm$= 6cm

OP = 8cm.

By, Pythagoras theorem in the right angled triangle POM,

$PM^2 = OP^2 + OM^2$

or, $l^2 = (8)^2 + (6)^2$

or, $l^2 = 64 + 36$

or $l^2 = 100$

$\therefore$ l = 10cm

Now, Total surface area (T.S.A.) $= 2al + a^2$

$= 2 \times 12cm + (12cm)^2$

$= 240cm^2 + 144cm^2$

$= 384cm^2$

Example 3:

The total surface area of a square-based pyramid is 360 sq. cm. and its slant height is 13cm. Find the length of the side of the base of the pyramid.

Solution:

Here,

The total surface area of the pyramid = 360 sq.cm.

Slant height (l) = 13cm

Length of the side of the base (a)= ?

We know that,

Total surface area of the pyramid = $a^2 + 2al$

or, $360 = a^2 + 2a \times 13$

or, $a^2 + 26a – 360 = 0$

or, $a^2 + 36a – 10a – 36a = 0$

or, $a(a + 36) – 10(a + 36) – a$

or, $(a + 16) (a – 10) = 0$

Either, or,

a + 36 = 0 a – 10 = 0

∴ a = -36 ∴ a = 10

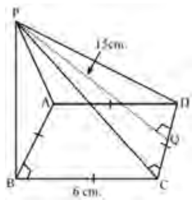
the length of the side is not negative.

∴ The length of the side is 10cm.

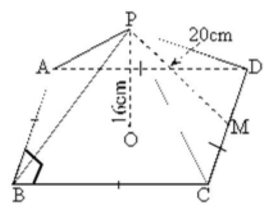
Exercise 7.3

1. Find the triangular faces area and total surface area (T.S.A) of the given square based pyramids.

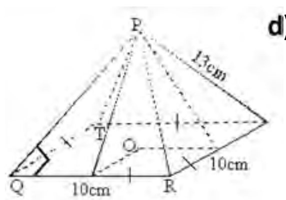
a)



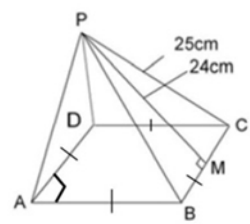
b)



c)

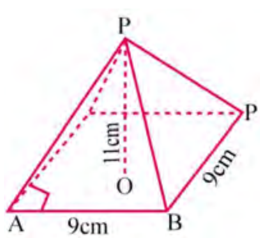


d)

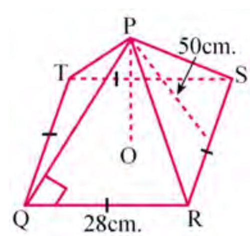


2. Find the volume of the following square based pyramids.

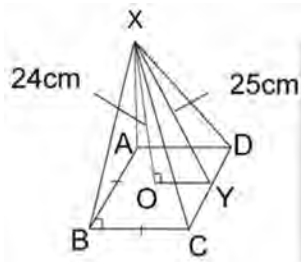
a)



b)

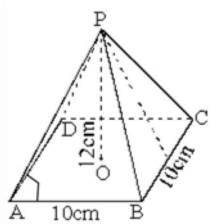


c)

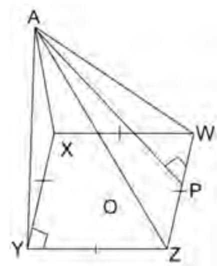


3. Find the total surface area and the volume of the given right pyramids with square bases.

a)



b)



AW = 13cm and AP = 12cm

4. **a)** The volume of a square based pyramid is $120cm^3$. If its height is 10cm, find the length of its base.

b) If the slant height of a triangular face of a square based pyramid of side 16cm is 17cm, find the volume of the pyramid.

c) Find the total surface area of a square based pyramid having vertical height 8cm and height of the triangular faces 10cm.

5. **a)** Find the lateral surface area and the volume of a square based pyramid where length of the side of the base is 12cm and the vertical height is 8cm.

b) Volume of a square based pyramid is $128cm^3$. If the vertical height and the length of the side of square base are in the ratio of 3:4, find the total surface area of the pyramid

6. **a)** The total surface area of a square based pyramid is $340cm^2$ and the length of slant height is 12cm, find the length of the side of base

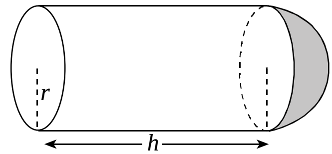
b) The volume of a square based pyramid in $2400cm^2$ and its height is 8cm. Find the total surface area of the pyramid.

c) The total surface area of a square based pyramid is $1920cm^2$ and the length of side base is 30cm. Find the vertical height of the pyramid.

7.4 Combined Solids

We often need to calculate surface area and volumes of combined solids.

1. Cylinder and hemisphere:



A radius of the solid (cylinder + hemisphere) be r, height of the cylindrical part be h then,

C.S.A. = ?

T.S.A. = Area of circular base + C.S.A. of cylindrical part + C.S.A. of hemispherical part.

T.S.A $= \pi r^2 + 2\pi rh + 2\pi r^2$

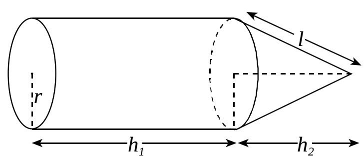
$= 3\pi r^2 + 2\pi rh$

$= \pi r(3r + 2h)$

And, volume= Volume of cylindrical part + Volume of hemispherical part

$= \pi r^2 h + \frac{2}{3}\pi r^3$

2. Cylinder and Cone:



C.S.A. = ?

Total surface area

T.S.A.= Area of circular base + C.S.A. of cylindrical part + C.S.A. of conical part

$= \pi r^2 + 2\pi rh_1 + \pi rl$

$= \pi r (r + 2h_1 + l)$

Where, r = radius

h_1 = height of cylindrical part

h_2 = height of conical part

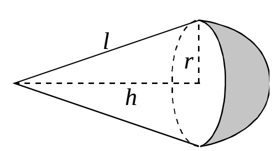
l = slant height of conical part

Total volume = Volume of cylindrical part + volume of conical part

$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$

$= \pi r^2 (h_1 + \frac{1}{3} h_2)$

3. Cone and Hemisphere:



If radius of the circular base be r, height and slant heights of conical part be h and l respectively, then

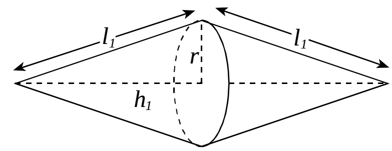
Total surface area= C.S.A of conical part + C.S.A of hemispherical part

$= \pi r l + 2\pi r^2$

Total Volume = Volume of conical part + Volume of hemispherical part

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

4. Cone and Cone:



If r be the radius of circular base, h_1 and h_2 be the vertical heights and l_1 and l_2 their slant heights, then,

Total surface area = C.S.A of cone 1 + C.S.A. of cone 2

$= \pi r l_1 + \pi r l_2$

$= \pi r (l_1 + l_2)$

Total volume = Volume of cone 1 + Volume of cone 2

$= \frac{1}{3}\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$

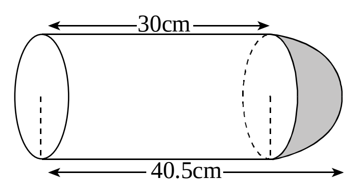
$= \frac{1}{3}\pi r^2 (h_1 + h_2)$

$= \frac{1}{3}\pi r^2 h$

(Where h is the total height of combined solid)

Example 1:

Find the surface area and volume of the solid shown in the figure which is the combination of a cylinder and a hemisphere.



Solution: Here,

Total height of the solid = 40.5cm

Height of the cylindrical part (h) = 30cm

$\therefore$ Radius of the circular base = 40.5cm – 30cm

or, r = 10.5cm

Now,

T.S.A. of the solid = Area of the circular base + C.S.A. of the cylindrical part + C.S.A. of the hemispherical part

$= \pi r^2 + 2\pi rh + 2\pi r^2$

$= 3\pi r^2 + 2\pi rh$

$$= 3\times \frac{22}{7} \times (10.5cm)^2 + 2 \times \frac{22}{7} \times 10.5cm \times 30cm = 3019.5cm^2$$

And volume of the solid = volume of the cylindrical part + volume of the hemispherical part

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \pi r^2\left(h + \frac{2}{3}r\right)$$

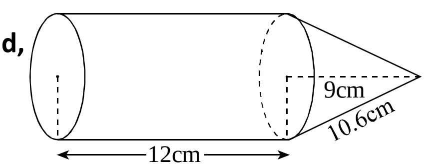
$$= \frac{22}{7} \times (10.5cm)^2\left(30 + \frac{2}{3} \times 10.5\right) cm$$

$$= 12820.5cm^3.$$

Therefore, the surface area is $3019.5cm^2$ and volume is $12820.5cm^3$.

Example 2:

Find the total surface area and volume of the given solid, which is the combination of a cylinder and a cone.



Solution: Here,

Vertical height of cone (h_1)= 9cm

Slant height (l_1)= 10.6cm

Radius of the base of cylinder (r)=$\sqrt{l_1^2 - h_1^2} = \sqrt{(10.6)^2 - 9^2} = 5.6cm$

Height of cylindrical part (h) = 12cm

Height of conical part (h_2) = 9 cm

Curved surface area of cone (C.S.A.) $= \pi rl$

$$= \frac{22}{7} \times 5.6cm \times 10.6cm = 186.56cm^2$$

Surface area of cylindrical part = Area of circular base + C.S.A.

$$= \pi r^2 + 2\pi rh$$

$$= \frac{22}{7} \times (5.6cm)2 + 2 \times \frac{22}{7} \times 5.6cm \times 12cm$$

$$= \frac{22}{7} \times 5.6\,(5.6 + 2 \times 12)cm^2 = 520.96cm^2$$

Total surface area of the solid (T.S.A.) = S.A. of cylindrical part + C.S.A. of cone

$$= 520.96\ cm^2 + 168.56cm^2 = 689.52cm^2$$

And volume of conical part (V_1) $= \frac{1}{3}\pi r^2 h_1$

$$= \frac{1}{3} \times \frac{22}{7} \times (5.6cm)^2 \times 9cm = 295.68cm^3$$

Volume of cylindrical part (V_2) $= \pi r^2 h_2$

$= \frac{22}{7} \times (5.6cm)^2 \times 12cm = 1182.72cm^3$.

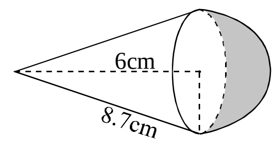
Therefore, the volume of the solid = volume of cylindrical part + volume of cone

$= 1182.72cm^3 + 295.68cm^3$

$= 1478.4cm^3$

Example 3:

Find the surface area and volume of the combined solid formed by cone and hemisphere given in the figure.



Solution: Here,

Vertical height of the conical part (h) = 6cm

Slant height (l) = 8.7cm

$\therefore$ Radius of the circular base (r) $= \sqrt{l^2 - h^2}$

$= \sqrt{(8.7cm)^2 - (6cm)^2}$

$= \sqrt{75.69 - 36}$ cm

$= 6.3cm$

Now, the surface area of the solid = C.S.A of cone + C.S.A of hemisphere

$= \pi r l + 2\pi r^2$

$= \pi r (l + 2r)$

$= \frac{22}{7} \times 6.3cm\ (8.7cm + 2\times6.3cm)$

$= 19.8cm \times 21.3cm$

$= 421.74cm^2$

And volume of the solid = volume of conical part + volume of hemispherical part

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 (h + 2r)$

$= \frac{1}{3} \times \frac{22}{7} \times (6.3cm)^2\ (6cm + 2\times6.3cm)$

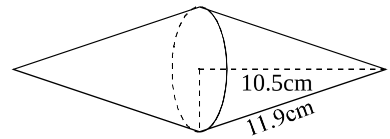
$= 6.6cm^2 \times 18.6cm$

$= 122.76cm^3$

Therefore, the surface area of the cone is $421.74cm^2$ and the volume is $122.76cm^3$.

Example 4:

A solid consist double cone each of vertical height 10.5 cm and slant height 11.9cm. Find the surface area and volume of the solid.



Solution: Here,

Slant height (l) = 11.9cm

Vertical height (h) = 10.5cm

$\therefore$ Radius of the circular base (r) $= \sqrt{l^2 - h^2}$

$= \sqrt{(11.9)^2 - (10.5)^2}$ cm

$= 5.6$cm

Now, curved surface area (C.S.A.) = $2(\pi rl)$

$= 2 \times \frac{22}{7} \times 5.6\text{cm} \times 11.9\text{cm}$

$= 418.88\text{cm}^2$

And volume = $2\left(\frac{1}{3}\pi r^2 h\right)$

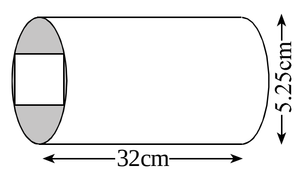
$= 2 \times \frac{1}{3} \times \frac{22}{7} \times (6.6\text{cm})^2 \times 10.5\text{cm}$

$= 689.92\text{cm}^3$

Therefore, the surface area is 4188.88cm^2 and volume is 689.92cm^3.

Example 5:

Find the volume of the material of the given cylindrical pipe if the base area of the hollow space as shown in the figure is maximum.



Solution: Here,

Height of the pipe (h) = 32cm

Diameter if the circular base (d) = 5.25cm

$\therefore$ External volume of the pipe (V) = $\frac{p}{4} d^2 h$

$= \frac{22}{7 \times 4} \times (5.25\text{cm})^2 \times 32\text{cm}$

$= 693\text{cm}^3$

For the rectangular base of the hollow space to be maximum, it must be square, whose diagonal is equal to the diameter of the cylinder.

$\therefore$ Volume of the hollow space (v) = $\frac{1}{2} d_1 \times d_2 \times h$

or, $\quad v = \frac{1}{2} d^2 \times h$

$= \frac{1}{2} \times (5.25\text{cm})^2 \times 32\text{cm}$

$= 441\text{cm}^3$

$\therefore$ Volume of the material of the pipe $= V - v$

$= 693\text{cm}^3 - 441\text{cm}^3$

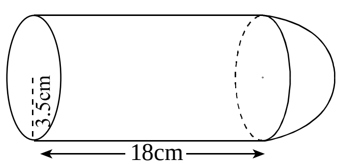
$= 252\text{cm}^3$

Therefore, the volume of the material of the pipe is 252cm^3.

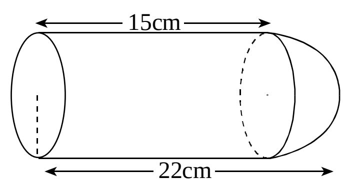
Exercise 7.4

1. **Find the curved surface area, total surface area and volume of the following solids which are the combination of cylinder and hemisphere:**

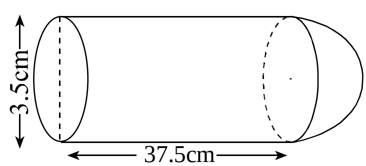
a)



b)

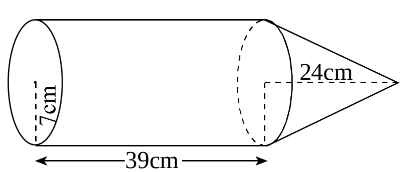


c)

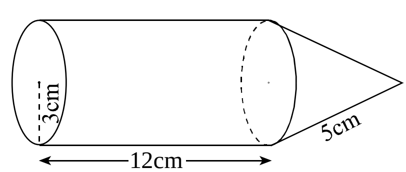


2. **Find the curved surface area, total surface area and volume of the following solids, which are the combination of cylinder and cone:**

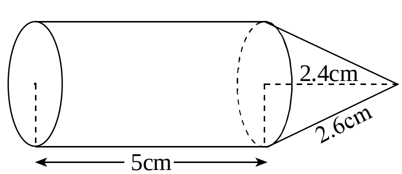
a)



b)

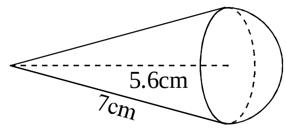


c)

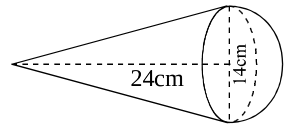


3. **Find the surface area and volume of the following solids which are the combination of a cone and a hemisphere:**

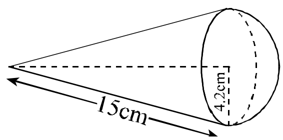
a)



b)

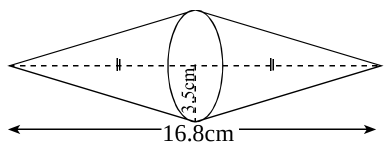


c)

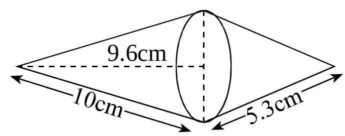


4. **Calculate the surface area and volume of the following solids which are the combination of two cones:**

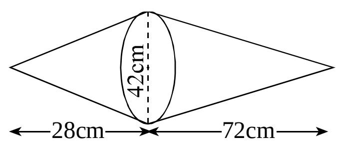
a)



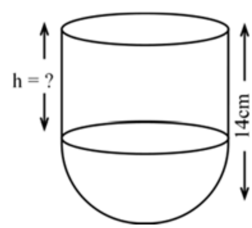
b)



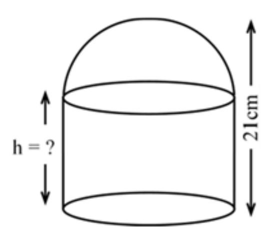
c)



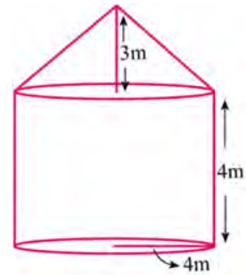
5. **a)** The total surface area of a given solid is $770 cm^2$ and total height is 14cm. Find the height of the cylinder.



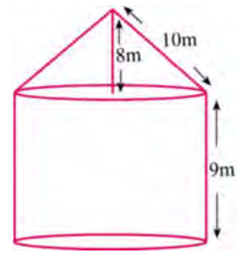
b) The total surface area of a given solid is $1078 cm^2$ and total height is 21cm. Find the height of the cylinder.



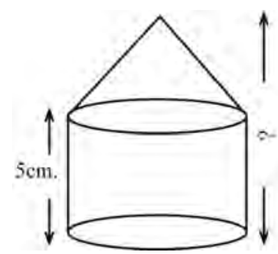
6. **a)** A tent is in the shape of a right circular cylinder of height 4m. with a cone of height 3m. over it. Find the total surface area of the tent.



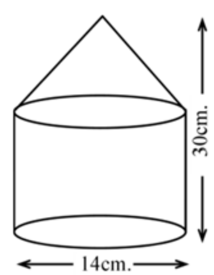
b) A tent is in the shape of a right circular cylinder of height 9m. and a cone of height 8m. and slant height 10m. over it, find the total surface area of the tent.



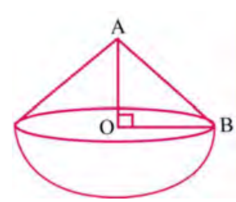
7. **a)** The volume and base area of a given figure are $600cm^3$ and $100cm^2$ respectively. If the height of the cylinder is 5cm,find the total height of the solid.



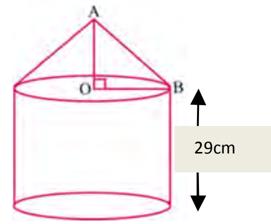
b) The total height of the solid object given alone side is 30cm. If the height of the cone and cylinder is in the ratio of 2:3, find the total volume of the solid object.



8. **a)** The ratio of the slant height and the radius of a conical part of the given combined solid is 5:3. The total volume of the given solid is $240\pi cm^3$. Calculate the total surface area of the solid.



b) The ratio of slant height and vertical height of the conical part of the given combined solid object is 13:12. The total surface area of the solid is $840\pi cm^2$. Calculate the volume of the given solid object.

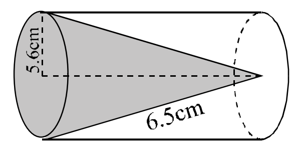


9. **a)** From a solid cylinder of height 12cm and base radius 5cm, a conical cavity of the same height and base is hollowed out. Find the surface area of the remaining solid.

b) A circus tent is cylindrical in shape up to the height of 12.5m and conical above it. If the diameter of the tent is 16m and height of the conical part is 15m, find the surface area of the tent.

10. **a)** A solid object is made of hemisphere and a cone. If the total height of the object is 14.6cm and height of the conical part is 9cm, find the surface area of the object.

b) A conical hole of base and height equal to a cylindrical wooden log is drilled out. Find the height of the log and volume of the remaining part.



7.5 Geometrical Bodies:

A Man discusses with his son who is studying in grade 10 about the cost estimation for bricks to construct the compound walls in the house. He asked his son to estimate the cost for bricks. Then immediately his son started to calculate the cost for the bricks at the rate of Rs. 14000 per 2000 bricks.

Total length of the compound (d) = 70m.

height of the compound wall (h) = 2.5m.

thickness of the compound wall (b) = 10cm.

∴the volume of the compound wall (V) $= l \times b \times h$

= 70 x 100cmx 10cm x 2.5 x 100cm

= $17500000cm^3$.

The size of a brick is 10cm x 5cm x 5cm

∴ The volume of a brick (v) = 10cm x 5cm x 5cm

= $250cm^3$.

To find the number of bricks, he divides the volume of the wall by volume of one brick.

So, number of bricks (N) $= \frac{V}{v}$

$= \frac{17500000cm^3}{250cm^3}$ = 70000

Now,

cost of 2000 bricks = Rs. 14000

cost of 1 brick = Rs. $\frac{14000}{2000}$

cost of 70000 bricks = Rs. 7 x 70000 = Rs. 4,90,000

Therefore, the boy estimated Rs. 4,90,000 the cost of bricks to construct the compound wall in his house.

Study the above activity and answer the following questions.

a) How many centimeter (cm) is equal to 1 meter (m)?

b) How many cubic cm. is equal to 1 cubic m?

c) Calculate the estimated labor charge for his activities.

d) Prepare the work plan to construct the compound wall in your house or school.

Similarly, can you help your family for the following activities?

- cost estimation of carpeting your room in your house.
- cost estimation of coloring the walls of the rooms in your house.
- cost estimation of constructing underground water tank in your house.
- cost estimation of constructing compound wall in your house.
- cost estimation of paving the tiles on the brands of your house.

Example 1:

The area of a square base of a water tank is $4m^2$ and the height of the tank is 2.75m. Find the capacity of the tank in litre.

Solution: Here,

Area of the tank (A) = $4m^2$

height of the tank (h) = 2.75m

∴volume of the tank (v) = A x h

= $4m^2$ x 2.75m

=$11m^3$.

Capacity of the tank = volume of the tank

$= 11m^3$

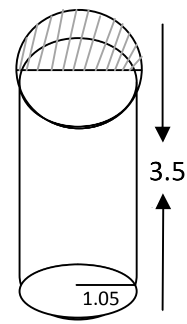
$= 11 \times 100 \times 100 \times 100cm^3$

$= \frac{11000000}{1000}$ liters

= 11000 litres

Example 2:

A person bought a cylinderical water tank of height 3.5m. and radius of base 1.05m with the upper part as a hemisphere. How much money did he pay for a tank of water at the rate of Rs. 1.50 per litre?



Solution: Here,

height of the tank = 3.5m

radius of the base (r) = 1.05m.

height of cylindrical part(h_1) = 3.5m - 1.05m.

=2.45m.

Volume of the tank (v) = volume of cylindrical part + volume of hemispheric part

$= \pi r^2 h_1 + \frac{2}{3} \pi r^2$

$= \pi r^2 (h_1 + \frac{2}{3} r)$

$= \frac{22}{7} (1.05m)^2 (2.45m + \frac{2}{3} \times 1.05m)$

$= \frac{22}{7} \times 1.1025 \times 3.15 m^3$

$= 10.91475\ m^3$.

= 1091475 x 1000 litres $[\because 1m^3 = 1000 litres]$

= 10914.75 litres

Rate of water (c) = Rs. 1.50 per liter.

$\therefore$ Total cost of the water = v x c

= 10914.75 liter x Rs. 1.50 liter

= Rs. 16372.125

Hence, the man paid Rs. 16372.12 for a tank of water.

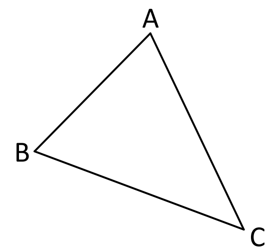
Example 3:

The adjoining figure is a triangular play ground where AB = 13 feet, BC = 14 feet and AC = 15feet. If the ground is paved by the stone at the rate of Rs. 25 per sq. feet, find the total cost for paving the ground.

Solution: Here,

According to question,

AB = 13 feet = c, BC = 14 feet = a and AC = 15 feet =b.



Semi perimeter of the triangular ground (s) = $\frac{a+b+c}{2}$

$= \left(\frac{14+15+13}{2}\right)$feet

= 21 feet.

Area of the ground (A) $= \sqrt{s\,(s-a)(s-b)(s-c)}$

$= \sqrt{21\,(21-14)(21-15)(21-13)}$

$= \sqrt{21 \times 7 \times 6 \times 8}$

= 84sq.feet.

Rate (c) = Rs. 25per sq.feet

Total cost (T) = c x A

= Rs. 25per sq. feet x 84sq. feet

= Rs. 2100

Exercise 7.5

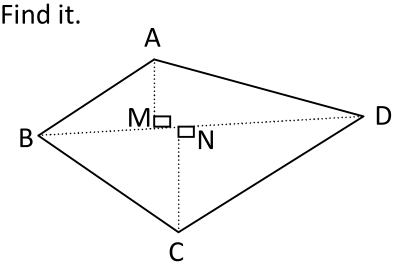
1. a) The area of square base of a water tank is 9m^2 and height of the tank is 2m. Find the capacity of the tank in litres.

   b) The internal dimension of a water tank is 2.5 long, 2.5 broad and 2 m. high. Find the total cost of the water if the tank is filled at the rate of Rs. 2 per liter.

2. a) A man bought a cylindrical water tank of height 2.5m. and radius of base 0.5m. with the upper part as a hemisphere. How much money did the man pay for the tank at the rate of Rs. 2 per liter? Find it.

   b) The internal diameter of a cylindrical water tank is 2m. and it is 3.5m. high. How many liters of water does it hold when it is full? What is the cost of water if the price of 1 liter water is Rs. 3? Find it.

3. The adjoining figure is a piece of land where BD = 24 feet, AM = 10 feet and CN= 14 feet. Find the total cost for paving the bricks at the rate of Rs. 7 per sq. feet.



4. A cylindrical water tank of height is 1.5m. and the diameter of the base 1.4m is surmounted by a cone of height 0.36m. Find the capacity of the tank in litres.

5. A ring if diameter 3.5 feet and height 1 feet is made from the cement and concrete. A well is made from such 32 rings. Find the cost for constructing the well at the rate of Rs. 1200 per ring. If the water level in the well is up to 18 rings, find the volume of the water in the well.
6. There are two pillars of the same shape and size in the gate of a house. The pillar is cuboid form of dimension 1 feet x 1 feet x 6 feet. and a pyramid of height 1 feet is kept on the above of each pillar. Find the total cost for the tile to put in the pillars at the rate of Rs. 52 per square feet.

Unit: 8

Highest Common Factor and Lowest Common Multiple

8.0 Review

In this unit, we are finding the H.C.F. and L.C.M of algebraic expressions by using factorization method. So, the process to find the factors of algebraic expressions are must important for the factors of the following algebraic expressions in the group.

i) $3x+6$ ii) $2x^2 - 6x - ax + 3a$ iii) $3x^2 - 27y^2$ iv) $x^3 + 8y^3$

v) $x^4 + 64$ vi) $x^2 + 1 + \frac{1}{x^2}$ (vii) $2x^2 - 5x - 3$

viii) $2x^3 + 6x^2 + 4x$ ix) $a^4 + a^2 + 1$ x) $m^3 - \frac{1}{27m^3}$

Can you find the factors of $a^2 - 3a$ and $a^2 - 9$?

Also, show the factors of these expressions in Venn-diagrams.

8.1 Highest Common Factor (H.C.F.)

Let us consider two numbers 12 and 42. Find the factors of 12 and 42.

Factors of 12 are 1, 2, 3, 4, 6, 12

Factors of 42 are 1, 2, 3, 6, 7, 14, 21, 42

Here, the common factors of 12 and 42 are 1, 2, 3, and 6. Among these factors, 6 is the highest factor of 12 and 42.

$\therefore$ 6 is the highest common factor of 12 and 42.

Similarly, let's take two algebraic expressions $5x^2y$ and $15xy^2$. Then

$5x^2y = 5 \times x \times x \times y$

$15xy^2 = 3 \times 5 \times x \times y \times y$

Here, the common factors of $5x^2y$ and $15xy^2$ are 5, x and y.

The product of 5, x and y is $5xy$.

$\therefore$ H.C.F. = $5xy$

The highest common factor of the given two or more algebraic expressions is an expression of the highest degree which is common to all given expressions that divides each given expressions without any remainder. It is written in the short form as H.C.F.

Note: If there is no common factor among the given expressions then the H.C.F of the given expressions is 1.

Example 1:

Find the H.C.F of $6x^3y^2$, $9x^2y^4$ and $12x^5y^3$

1st expression = $6x^3y^2 = 2 \times 3 \times x \times x \times x \times y \times y$

2nd expression = $9x^2y^4 = 3 \times 3 \times x \times x \times y \times y \times y \times y$

3rd expression = $12x^5y^3 = 2 \times 2 \times 3 \times x \times x \times x \times x \times x \times y \times y \times y$

The common factors in all three expressions = $3 \times x \times x \times y \times y = 3x^2y^2$

H.C.F of the given expressions = $3x^2y^2$

Example 2:

Find the H.C.F. of $x^3y - xy^3$ and $x^2+2xy+ y^2$

Solution:

Here,

1st expression $= x^3y - xy^3$

$= xy\,(x^2 - y^2)$

$= xy\,(x+y)\,(x-y)$

2nd expression $= x^2 + 2xy + y^2$

$= (x + y)^2$

$= (x+y)\,(x+y)$

$\therefore$ H.C.F $= x+y$

Example 3

Find the H.C.F of x^3+y^3 and $x^4 + x^2y^2 + y^4$

Solution:

Here, 1st expression $= x^3+y^3$

$= (x+ y)\,(x^2 - xy + y^2)$

2nd expression $= x^4 + x^2y^2 + y^4$

$= (x^2)^2 + 2x^2y^2 + (y^2)^2 - x^2y^2$

$= (x^2+y^2)^2 - (xy)^2$

$= (x^2 + y^2 + xy)\,(x^2 + y^2 - xy)$

$= (x^2 + xy + y^2)\,(x^2 - xy + y^2)$

H.C.F $= x^2 - xy + y^2$

Example 4:

Find the H.C.F of $3x^2 - 12$, $x^2 + 3x + 2$ and $x^4 + 8x$

Solution:

Here, 1st expression $= 3x^2 - 12$

$= 3(x^2 - 4)$

$= 3\{(x)^2 - (2)^2\}$

$= 3(x+2)(x-2)$

2nd expression $= x^2 + 3x + 2$

$= x^2 + 2x + x + 2$

$= x(x+2) + 1(x+2)$

$= (x+2)(x+1)$

3rd expression $= x^4 + 8x$

$= x(x^3 + 8)$

$= x\{(x)^3 + (2)^3\}$

$= x(x+2)(x^2 - 2x + 4)$

$\therefore$ H.C. F $= x + 2$

Example 5:

Find the H.C.F of $m^3 + n^3$, $m^6 - n^6$ and $m^4 + m^2n^2 + n^4$

Solution:

Here, 1st expression $= m^3 + n^3$

$= (m + n)(m^2 - mn + n^2)$

2nd expression $= m^6 - n^6$

$= (m^3)^2 - (n^3)^2$

$= (m^3 + n^3)(m^3 - n^3)$

$= (m + n)(m^2 - mn + n^2)(m - n)(m^2 + mn + n^2)$

3rd expression $= m^4 + m^2n^2 + n^4$

$= (m^2)^2 + 2m^2n^2 + (n^2)^2 - m^2n^2$

$= (m^2 + n^2)^2 - (mn)^2$

$= (m^2 + n^2 + mn)(m^2 + n^2 - mn)$

$= (m^2 + mn + n^2)(m^2 - mn + n^2)$

H.C.F $= m^2 - mn + n^2$

Exercise 8.1

1. **Find the H.C.F. of:**

(a) $4x^2y$ and $12xy^3$ (b) $12x^3y^2$ and $18x^2y^5$

(c) a^2bc^3 and a^3b^2c (d) $15x^2y^3$, $40x^3y^4$ and $55x^3y^5$

2. **Find the H.C.F. of:**

(a) $a^3b - ab^3$ and $a^2 + 2ab + b^2$ (b) $x^2 - 9$ and $x^2 - x - 6$

(c) $4x^3 - x$ and $4x^2 + 4x + 1$ (d) $x^2y - y^3$ and $x^3 + y^3$

3. **Find the H.C.F. of:**

(a) $x^3 - y^3$ and $x^4 + x^2y^2 + y^4$ (b) $a^3 + 1$ and $a^4 + a^2 + 1$

(c) $8m^3 + n^3$ and $16m^4 + 4m^2n^2 + n^4$ (d) $x^4 + 1 + \frac{1}{x^4}$ and $x^3 - \frac{1}{x^3}$

4. **Find the H.C.F. of:**

(a) $2a^2 - 8$, $a^2 - a - 2$ and $a^4 - 8a$ (b) $3x^2 - 8x + 4$, $x^4 - 8x$ and $x^2 - 4$

(c) $2m^3 + 16$, $m^2 + 4m + 4$ and $m^2 + 3m + 2$ (d) $4y^3 - y$, $2y^3 - y^2 - y$ and $8y^4 + y$

5. **Find the H.C.F. of:**

(a) $x^3 - y^3$, $x^6 - y^6$ and $x^4 + x^2y^2 + y^4$ (b) $x^3 - 1$, $x^4 + x^2 + 1$ and $x^6 - 1$

(c) $a^3 + b^3$, $a^6 - b^6$ and $a^4 + a^2b^2 + b^4$ (d) $x^3 + 1$, $x^4 + x^2 + 1$ and $x^3 - 1 - 2x^2 + 2x$

6. **Find the H.C.F. of:**

(a) $a^2 + 2ab + b^2 - c^2$, $b^2 + 2bc + c^2 - a^2$ and $c^2 + 2ac + a^2 - b^2$

(b) $x^6 - 1$, $x^4 + x^3 + x^2$ and $x^3 + 2x^2 + 2x + 1$

(c) $2x^2 - 3x - 2$, $8x^3 + 1$ and $4x^2 - 1$

(d) $2y^3 - 16$, $y^2 + 3y + 2$ and $2y^2 - 8$

(e) $4x^3 - 6x^2y + 9xy^2$, $16x^4 + 36x^2y^2 + 81y^4$ and $8x^3 + 27y^3$

8.2 Lowest Common Multiple (L.C.M)

Let's consider two numbers 8 and 12.

The multiples of 8 (M_8) = 8, 16, 24, 32, 40, 48, 56, 64, 72 …

The multiples of 12(M_{12}) = 12, 24, 36, 48, 60, 72, 84, ...

What are common multiples of 8 and 12?

The common multiples of 8 and 12 are 24, 48, 72 ….

But the least common multiple of 8 and 12 is 24?

Therefore, the lowest common multiple of 8 and 12 is 24.

We can also find the L.C.M by prime factorization method as.

$8 = 2 \times 2 \times 2$ $\qquad$ $12 = 2 \times 2 \times 3$

Here, first we find the H.C.F of 8 and 12. Then we multiply the H.C.F by the remaining factors of 8 and 12.

H.C.F of 8 and 12 = $2 \times 2 = 4$

The remaining factors of 8 and 12 are 2 and 3 respectively

L.C.M = H.C.F x remaining factor of 8 x remaining factor of 12 .

$= 4 \times 2 \times 3 = 24$

Thus, the least number exactly divisible by 8 and 12 is 24. We can use the same concept to find the L.C.M of the given algebraic expressions.

Let's consider two algebraic expressions $4x^2y$ and $6xy^2$. Then,

$4x^2y = 2 \times 2 \times x \times x \times y$

$6xy^2 = 2 \times 3 \times x \times y \times y$

H.C.F of $4x^2y$ and $6xy^2 = 2 \times x \times y = 2xy$

Remaining factors of $4x^2y = 2x$

Remaining factors of $6xy^2 = 3y$

L.C.M of $4x^2y$ and $6x^2$ = H.C.F. X remaining factors of $4x^2y$ X remaining factors $6xy^2$

$= 2xy \times 2x \times 3y$

$= 12x^2y^2$

Therefore, the lowest common multiple of the given two algebraic expressions is the product of the H.C.F. and the remaining factors.

Lowest common multiple of two or more than two algebraic expressions is an expression of the least degree which is exactly divisible by the given algebraic expressions. It is written as L.C.M in the short from.

Example 1

Find the L.C.M of $3a^2b$, $12ab^2$ and $15a^3bx$.

Solution:

Here, 1st expression = $3a^2b = 3 \times a \times a \times b$

2nd expression = $12ab^2 = 2 \times 2 \times 3 \times a \times b \times b$

3rd expression = $15a^3bx = 3 \times 5 \times a \times a \times a \times b \times x$

H.C.F = $3 \times a \times b = 3ab$

L.C.M = H.C.F $\times$ remaining factors

$= 3ab \times 2 \times 2 \times 5 \times a \times b \times a \times x$

$= 60a^3b^2x$

Example 2

Find the L.C.M of $2a^2 - 8$ and $a^2 - a - 6$

Solution:

Here, 1st expression = $2a^2 - 8$

$= 2 (a^2 - 4)$

$= 2\{(a^2) - (2)^2\}$

$= 2(a + 2) (a - 2)$

2nd expression $= a^2 - a - 6$

$= a^2 - 3a + 2a - 6$

$= a(a - 3) + 2 (a - 3)$

$= (a - 3) (a - 2)$

H.C.F = $a + 2$

L.C.M = H.C.F × remaining factors

$= (a+2) \times 2 \times (a - 2) \times (a - 3)$

$= 2(a + 2) (a - 2) (a - 3)$

Example 3:

Find the L.C.M of $y^3 - z^3$ and $y^4 + y^2z^2 + z^4$

Solution:

Here, 1st expression $= y^3 - z^3$

$= (y - z) (y^2 + yz + z^2)$

2nd expression $= y^4 + y^2z^2 + z^4$

$= (y^2)^2 + 2y^2z^2 + (z^2)^2 - y^2z^2$

$= (y^2 + z^2)^2 - (yz)^2$

$= (y^2 + z^2 + yz) (y^2 + z^2 - yz)$

L.C.M $= (y^2 + yz + z^2) (y - z) (y^2 + z^2 - yz)$

$= (y - z)(y^2 + yz + z^2) (y^2 - yz + z^2)$

Example 4:

Find the L.C.M of $x^3 - 9x$, $x^4 - 2x^3 - 3x^2$ and $2x^3 - 54$.

Solution: Here,

1st expression $= x^3 - 9x$

$= x(x^2 - 9)$

$= x(x + 3) (x - 3)$

2nd expression $= x^4 - 2x^3 - 3x^2$

$= x^2 (x^2 - 2x - 3)$

$= x^2 (x^2 – 3x + x - 3)$

$= x^2 \{x (x - 3) + 1(x-3)\}$

$= x^2 (x + 1) (x - 3)$

3rd expression $= 2x^3 – 54$

$= 2(x^3 -27)$

$= 2\{(x)^3- (3)^3\}$

$= 2(x- 3) (x^2 + 3x + 9)$

LCM $= (x - 3) \times 2(x + 3) \times (x + 1) \times x^2$ x $(x^2 + 3x + 9)$

$= 2x^2 (x^2 -9) (x + 1) (x^2 + 3x + 9)$

Example 5:

Find the L.C.M of $x^3 – y^3$, $x^6 – y^6$ and $x^4 + x^2y^2 + y^4$

Solution:

Here, 1st expression $= x^3 – y^3$

$= (x - y) (x^2 + xy + y^2)$

2nd expression $= x^6 – y^6$

$= (x^3)^2 – (y^3)^2$

$= (x^3 + y^3) (x^3 – y^3)$

$= (x + y) (x^2 – xy+ y^2) (x - y) (x^2 + xy + y^2)$

$= (x + y) (x - y) (x^2 – xy + y^2) (x^2 + xy + y^2)$

3rd expression $= x^4 + x^2y^2 + y^4$

$= (x^2)^2 + 2x^2y^2 + (y^2)^2 – x^2y^2$

$= (x^2 +y^2)^2 – (xy)^2$

$= (x^2 +y^2 + xy) (x^2 + y^2 - xy)$

$= (x^2 + xy + y^2) (x^2 – xy + y^2)$

L.C.M $= (x^2 + xy +y^2) \times (x -y) \times (x^2 – xy + y^2) \times (x + y)$

$= (x - y) (x + y) (x^2 + xy +y^2) (x^2 – xy + y^2)$

Exercise 8.2

1. Find the L.C.M. of:

(a) $2x^2$ and $3y$

(b) $6x^2y$ and $15xy^2z$

(c) a^2b, ab^3 and $2a^2b^4x$

(d) $15a^2b^3$, $40a^4b^5$ and $60a^3b^2c$

2. Find the L.C.M. of:

(a) $2x^2 - 8y^2$ and $x^2 - xy - 2y^2$ (b) $3m^2 - 27$ and $m^2 + m - 6$

(c) $x^3y - xy^3$ and $x^2 + 2xy + y^2$ (d) $a^2b - b^3$ and $a^3 - b^3$

3. Find the L.C.M. of:

(a) $x^3 - y^3$ and $x^4 + x^2y^2 + y^4$ (b) $a^3 - 1$ and $a^4 + a^2 + 1$

(c) $m^3 - \frac{1}{m^3}$ and $m^4 + 1 + \frac{1}{m^4}$ (d) $a^6 - b^6$ and $a^4 + a^2b^2 + b^4$

4. Find the L.C.M. of:

(a) $a^3 - 4a$, $a^4 + a^3 - 2a^2$ and $2a^3 - 16$ (b) $2x^2 - 8$, $x^2 - x - 2$ and $x^4 - 8x$

(c) $4y^3 - y$, $2y^3 - y^2 - y$ and $8y^4 + y$ (d) $x^3 + 1$, $x^4 + x^2 + 1$ and $x^4 + x^3 + x^2$

5. Find the L.C.M. of:

(a) $a^3 + 1$, $a^6 - 1$ and $a^4 + a^2 + 1$ (b) $x^4 - x$, $x^4 + x^2 + 1$ and $x^6 - 1$.

(c) $m^3 - \frac{1}{n^3}$, $m^6 - \frac{1}{n^6}$ and $m^4 + \frac{m^2}{n^2} + \frac{1}{n^4}$

6. Find the L.C.M. of:

(a) $a^3 + 2a^2 - a - 2$ and $a^3 + a^2 - 4a - 4$

(b) $x^6 - 1$, $x^4 + x^3 + x^2$ and $x^3 + 2x^2 + 2x + 1$

(c) $x^2 + 2xy + y^2 - z^2$, $y^2 + 2yz + z^2 - x^2$ and $z^2 + 2xz + x^2 - y^2$

(d) $2a^3 - 16$, $a^2 + 3a + 2$ and $2a^2 - 8$

Unit: 9

Radical and Surd

9.0 Review

Let's consider some numbers like: -2, 0, 5, 6.5, $-\frac{2}{3}, \frac{11}{9}$ etc.

Can you express all the above numbers in the form of $\frac{p}{q}$, where q≠o ?

Discuss your solution with your friends in class.

A number which can be expressed in the form of $\frac{p}{q}$, where p and q both are integers and q≠o is known as rational number. For example,- 4, 0, 7, -4.6, $\frac{5}{7}$, etc.

The rational numbers when expressed as decimal, it is either terminating or non-terminating recurring or repeating decimal. For examples, $\frac{4}{5} = 0.8$, $\frac{3}{4} = 0.75$, etc are terminating decimals.

$\frac{2}{3}$ = 0.666 …., $\frac{1}{6}$ = 0.1666 … etc are non- terminating repeating decimal.

$\frac{1}{7}$ = 0.14285714 … is non- terminating recurring decimal.

But some numbers cannot be expressed in the form of $\frac{p}{q}$. Such numbers are known as irrational numbers. In other words, a number which is not a rational number is called an irrational number. For examples, $\sqrt{2} = 1.41421 \ldots..$, $\sqrt{7} = 2.645751$ …., π = 3.14159265….

From the above examples, we can say that the irrational numbers are non-terminating decimal numbers.

9.1 Surd:

An irrational number of the form $\sqrt[x]{\text{rational number}}$ is called surd. For example, $\sqrt{3}, \sqrt{7}, \sqrt[3]{7}$ are surds because 3 and 7 are rational numbers but $\sqrt{\pi}$ is not surd because π is an irrational number.

Thus, a number which is irrational of rational number is known as a surd. In other words, the rational numbers whose root cannot be found exactly are called surds. For examples,

$\sqrt{2} = 1.41421 \ldots..$

$\sqrt{3} = 1.732205 \ldots..$

But $\sqrt{4}$, $\sqrt[3]{8}$, $\sqrt[5]{243}$, etc. are not surds because we can find the exact value of these root.

In general $\sqrt[n]{x}$, n^{th} root of the positive rational number 'x' is surd if $\sqrt[n]{x}$ is an irrational number.

Order → $\sqrt[n]{x}$ ← Radicand

↑

Radical sign

Pure and mixed surds:

A surds having coefficient one which is either + ve or –ve is called a pure surd. For, $\sqrt{3}$, $-\sqrt{7}$, $\sqrt[3]{9}$,$-\sqrt[4]{3}$ etc.

A surd having coefficient as a rational number other than one is called a mixed surd. For examples, $2\sqrt{3}$, -$4\sqrt[3]{7}$, $2x\sqrt{8}$ etc.

A pure surd can be expressed as a mixed surd and vice-versa.

Expressing the Surds:

A pure surds to mixed surds: Factories the radicand as a product of two or more than two factors and write factor in power form if possible equal to the order of the surd. For example, $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{2^2 \times 3}$ =$2\sqrt{3}$ is a mixed surd. $\sqrt[3]{250}$ = $\sqrt[3]{125 \times 2}$ = $\sqrt[3]{5^3 \times 2}$ = $5\sqrt[3]{2}$ is a mixed surd.

Mixed surds to pure surds: Write the coefficient of surd under the radical sign with power equal to the order of the surd and then multiply the factors under the radical sign. For example, $3\sqrt{2}$ = $\sqrt{3^2 \times 2}$ = $\sqrt{9 \times 2}$ = $\sqrt{18}$ is a pure surd.

-2 $\sqrt[3]{7}$ = $-\sqrt[3]{2^3 \times 7}$ = $-\sqrt[3]{8 \times 7}$ = $-\sqrt[3]{56}$ is a pure surd.

Different order surds to the same order surds: Find the L.C.M of the orders of different order surds and then make the order of each surds equal to the L.C.M of orders.

For example, $\sqrt{2}$ and $\sqrt[3]{3}$ are different order surds.

Order of $\sqrt{2}$ is 2 and order of $\sqrt[3]{3}$ is 3.

So, the L.C.M of 2 and 3 is 6.

Now, $\sqrt{2}$ = $\sqrt[6]{2^3} = \sqrt[6]{8}$

$\sqrt[3]{3} = \sqrt[6]{3^2} = \sqrt[6]{9}$

Hence, $\sqrt[6]{8}$ and $\sqrt[6]{9}$ are same order surds.

Comparison of surds:

Two or more than two surds of the same order can be easily compared by just comparing their radicands. The surds having the greatest and smallest radicand are the greatest and the smallest surds respectively. For example, $\sqrt[3]{8}$ and $\sqrt[3]{12}$ are two surds having the same order 3. Both surds have the radicand 8 and 12 respectively where 8 < 12. So, $\sqrt[3]{8}$ is smallest surd and $\sqrt[3]{12}$ is greatest surd.

If there are two or more than two surds having the different order then at first the surds should be converted into the same order. After that we can use the same process as the surds having the same order.

Like and unlike surds:

Let's consider the surds as

$\sqrt{7}$, $5\sqrt{7}$, $-4\sqrt{7}$.

All these surds have the same order and equal radicand. These surds are called like surds.

Let's again consider the surds as

$\sqrt{7}$, $\sqrt[3]{7}$, $\sqrt{15}$

Above surds have the different order and different radicands. These surds are called unlike surds.

Operation of Surds

Addition and subtraction of on surds:

Two or more than two surds can be added and subtracted if the given surds are like surds. For example,

Add $3\sqrt{7}$ and $5\sqrt{7}$

$3\sqrt{7} + 5\sqrt{7} = (3+5)\sqrt{7} = 8\sqrt{7}$

Subtract $2\sqrt[3]{5}$ from $5\sqrt[3]{5}$

$5\sqrt[3]{5} - 2\sqrt[3]{5} = (5-2)\sqrt[3]{5} = 3\sqrt[3]{5}$.

Multiplication and division of surds: Two surds can be multiplied and divided if the surds have the same order. For example,

$3\sqrt{2} \times 2\sqrt{5} = 3 \times 2\sqrt{2 \times 5} = 6\sqrt{10}$

$6\sqrt{15} \div 2\sqrt{5} = \frac{6\sqrt{15}}{2\sqrt{5}} = 3\sqrt{\frac{15}{5}} = 3\sqrt{3}$

The surds having different order can also be multiplied and divided by making their order same. For example,

$2\sqrt{3} \times \sqrt[3]{2} = 2\sqrt[6]{3^3} \times \sqrt[6]{2^2} = 2\sqrt[6]{27} \times \sqrt[6]{4} = 2\sqrt[6]{27 \times 4} = 2\sqrt[6]{108}$

$\sqrt[3]{6} \div \sqrt{2} = \frac{\sqrt[3]{6}}{\sqrt{2}} = \frac{\sqrt[6]{6^2}}{\sqrt[6]{2^3}} = \frac{\sqrt[6]{36}}{\sqrt[6]{8}} = \sqrt[6]{\frac{36}{8}} = \sqrt[6]{\frac{9}{2}}$

Example 1:

Express the surds $2\sqrt{3}$, $\sqrt[3]{2}$ and $\sqrt[4]{5}$ in the same order.

Solution:

Here, The given surds are $2\sqrt{3}$, $\sqrt[3]{2}$ and $\sqrt[4]{5}$

L.C.M of the orders 2, 3 and 4 is 12.

Now,

$2\sqrt{3} = 2\sqrt[2\times 6]{3^6}$ $=2\sqrt[12]{729}$

$\sqrt[3]{2} = \sqrt[3\times 4]{2^4}$ $=\sqrt[12]{16}$

$\sqrt[4]{5} = \sqrt[4\times 3]{5^3}$ $=\sqrt[12]{125}$

Hence, the required surds of the same order are $2\sqrt[12]{729}$, $\sqrt[12]{16}$ and $\sqrt[12]{125}$

Examples 2:

Arrange the surds in ascending order: $\sqrt{5}$, $\sqrt[3]{7}$, $\sqrt[4]{6}$

Solution:

Here, The given surds are $\sqrt{5}$, $\sqrt[3]{7}$, $\sqrt[4]{6}$

Now, L.C.M of the orders 2, 3 and 4 is 12

$\sqrt{5} = \sqrt[2\times 6]{5^6} = \sqrt[12]{15625}$

$\sqrt[3]{7} = \sqrt[3\times 4]{7^4} = \sqrt[12]{2401}$

$\sqrt[4]{6} = \sqrt[4\times 3]{6^4} = \sqrt[12]{216}$

216 < 2401 <15625

$\sqrt[12]{216} < \sqrt[12]{2401} < \sqrt[12]{15625}$

i.e. $\sqrt[4]{6} < \sqrt[3]{7} < \sqrt{5}$

Hence the required ascending order of the given surds is $\sqrt[4]{6}$, $\sqrt[3]{7}$, $\sqrt{5}$

Example 3:

Add: $3\sqrt{12} + 2\sqrt{27}$

Solution:

Here, $3\sqrt{12} + 2\sqrt{27}$

$= 3\sqrt{4 \times 3} + 2\sqrt{9 \times 3}$

$= 3 \times 2\sqrt{3} + 2 \times 3\sqrt{3}$

$= 6\sqrt{3} + 6\sqrt{3}$

$= (6 + 6)\sqrt{3}$

$= 12\sqrt{3}$

Example 4:

Subtract $3\sqrt{18}$ from $4\sqrt{50}$.

Solution:

Here, $4\sqrt{50} - 3\sqrt{18}$

= $4\sqrt{25 \times 2} - 3\ \sqrt{9 \times 2}$

= $4 \times 5\sqrt{2}\ - 3 \times 3\sqrt{2}$

= $20\sqrt{2} - 9\sqrt{2}$

= (20 - 9) $\sqrt{2}$

= 11 $\sqrt{2}$

Example 5:

Simplify: $\sqrt{45} - 3\sqrt{20} + 5\sqrt{5}$

Solution:

Here, $\sqrt{45} - 3\sqrt{20} + 5\sqrt{5}$

= $\sqrt{9 \times 5} - 3\sqrt{4 \times 5} + 5\sqrt{5}$

= $3\sqrt{5}\ - 3 \times 2\sqrt{5} + 5\sqrt{5}$

= (3- 6 + 5) $\sqrt{5}$ = $2\sqrt{5}$

Example 6:

Multiply: $3\sqrt{6}$ by $4\sqrt[3]{4}$

Solution:

Here, $3\sqrt{6}\ \times\ 4\ \sqrt[3]{4}$

= $12\ \sqrt[2\times3]{6^3}\ \times\ \sqrt[3\times2]{4^2}$

= $12\sqrt[6]{216}\ \times\ \sqrt[6]{16}$

= $12\ \sqrt[6]{216 \times\ 16}$

= $12\ \sqrt[6]{3456}$

Example 7:

Simplify $(2\sqrt{x} + 3\sqrt{y})\ (2\sqrt{x} - \sqrt{y})$

Solution:

Here, $(2\sqrt{x} + 3\sqrt{y})\ (2\sqrt{x} - \sqrt{y})$

= $2\sqrt{x}\ (2\sqrt{x} - \sqrt{y}) + 3\sqrt{y}\ \left(2\sqrt{x} - \sqrt{y}\right)$

= $4\ \sqrt{x^2} - 2\sqrt{xy} + 6\ \sqrt{xy} - 3\sqrt{y^2}$

= $4x - (2-6)\ \sqrt{xy}$ - 3y

= $4x + 4\ \sqrt{xy}$ - 3y

Example 8:

Divide $150\sqrt[6]{72}$ by $20\sqrt{8}$

Solution:

Here, $150\sqrt[6]{72} \div 20\sqrt{8}$

$= \frac{150\sqrt[6]{72}}{20\sqrt{8}}$

$= \frac{15}{2}\frac{\sqrt[6]{72}}{\sqrt[6]{8^3}} = \frac{15}{2}\sqrt[6]{\frac{72}{8\times8\times8}}$

$= \frac{15}{2}\sqrt[6]{\frac{9}{64}} = \frac{15}{2}\sqrt[6]{\left(\frac{3}{8}\right)^2} = \frac{15}{2}\sqrt[3]{3/8} = \frac{15}{2}\frac{\sqrt[3]{3}}{\sqrt[3]{8}} = \frac{15\sqrt[3]{3}}{2\times2} = \frac{15\sqrt[3]{3}}{4}$

Example 9:

Simplify: $\sqrt{\mathbf{a^2 - b^2}} \div (\mathbf{a + b})$

Solution:

Here, $\frac{\sqrt{a^2-b^2}}{(a+b)}$

$= \frac{\sqrt{(a+b)(a-b)}}{\sqrt{(a+b)^2}}$

$= \sqrt{\frac{(a+b)(a-b)}{(a+b)^2}}$

$= \sqrt{\frac{a-b}{a+b}}$

Exercise 9.1

1. **Write the order of the following surds.**

   (a) $\sqrt[3]{4}$ (b) $\sqrt[7]{13}$ (c) $3\sqrt[p]{x}(p > 1)$

2. **Convert the following surds into the simplest form.**

   (a) $\sqrt{12}$ (b) $\sqrt{72}$ (c) $\sqrt[4]{9}$ (d) $\sqrt[6]{16x^4}$ (e) $\sqrt[4]{81xy^4}$

3. **Convert the following surds into the mixed surds.**

   (a) $\sqrt{18}$ (b) $\sqrt[3]{250}$ (c) $\sqrt[4]{32x^5}$

4. **Convert the following surds into the pure surds.**

   (a) $2\sqrt{5}$ (b) $3a\sqrt[3]{2}$ (c) $-4\sqrt[3]{5x}$ (d) $(x + y)\sqrt{\frac{(x-y)}{(x+y)}}$

5. **Express the following surds in the same order.**

(a) $\sqrt[3]{3}$ *and* $\sqrt{2}$ (b) $2\sqrt[3]{2}$, $\sqrt[4]{3}$ *and* $\sqrt[6]{4}$ (c) $\sqrt[3]{3}$, $\sqrt[6]{6}$ and $\sqrt[2]{5}$

6. **Compare the following surds.**

(a) $\sqrt{3}$ and $\sqrt[3]{2}$ (b) $\sqrt[6]{162}$, and $\sqrt{5}$ (c) $\sqrt[4]{12}$ and $\sqrt[6]{8}$

7. **Arrange the following surds in ascending order.**

(a) $2\sqrt{5}$, $\sqrt[3]{4}$ and $\sqrt[6]{27}$ (b) $2\sqrt[3]{4}$, $\sqrt[4]{8}$ and $3\sqrt[3]{2}$ (c) $\sqrt[3]{7}$, $\sqrt[4]{8}$ and $\sqrt[6]{6}$

8. **Add:**

(a) $2\sqrt{4x^2}+3\sqrt[3]{8x^3}$ (b) $\sqrt{12}+2\sqrt{75}+3\sqrt{108}$ (c) $\sqrt[3]{54a^3}+\sqrt[3]{128a^3}$

9. **Subtract:**

(a) $\sqrt{50}-\sqrt{32}$ (b) $\sqrt[3]{24x^3}$ - $\sqrt[3]{3x^3}$ (c) $\sqrt[4]{81x^4y}$ - $x\sqrt[4]{y}$

10. **Simplify:**

(a) $\sqrt{125}+\sqrt{5}-\sqrt{45}$ (b) $4\sqrt{2}-2\sqrt{8}+\frac{3}{\sqrt{2}}$ (c) $\sqrt{18}-3\sqrt{20}+4\sqrt{5}$

11. **Multiply:**

(a) $3\sqrt{6}\times 2\sqrt{5}$ (b) $4\sqrt[3]{3}\times 2\sqrt{3x^2}$ (c) $\sqrt[3]{(a-b)^{-8}}\times\sqrt[3]{(a-b)^4}$

12. **Divide:**

(a) $10\sqrt{15}\div 2\sqrt{5}$ (b) $120\sqrt[6]{72}\div 2\sqrt[3]{81}$ (c) $\sqrt[3]{8}\div 6\sqrt[6]{12}$

13. **Simplify:**

(a) $\sqrt[3]{3a^7b^8}\times\sqrt[3]{9a^2b}$ (b) $\sqrt{x^6y^{-2}z^4}\times\sqrt{x^{-8}y^{-8}z^4}$ (c) $\sqrt[4]{16xy^4}\div\sqrt[3]{8x^3y^3}$

14. **Simplify:**

(a) $(2\sqrt{a}+3\sqrt{b})(2\sqrt{a}-\sqrt{b})$ (b) $(5\sqrt{x}+3\sqrt{y})(5\sqrt{x}+3\sqrt{y})$

(c) $\frac{2x\sqrt{5}-\sqrt{80x^2}+\sqrt{45x^2}}{\sqrt{5x^2}}$

15. **Simplify:**

(a) $(a+b)\div\sqrt{a^2-b^2}$ (b) $(x-y)^{-2}\div\sqrt[3]{(x-y)^2}$ (c) $(x-y)\div\sqrt{x^2-y^2}$

9.2 Rationalization of Surds:

Let's consider a surd as $\sqrt{3}$. $\sqrt{3}$ x $\sqrt{3}$ = $\sqrt{9}$ =3 which is a rational number. The process of converting a surd to a rational number by multiplying the given surd with a suitable factor is called the rationalization of the surd. The suitable factor is called the rationalizing factor of the given surd. In the above example, $\sqrt{3}$ is a rationalizing factor of the given surd $\sqrt{3}$. Similarly, $\sqrt{a+b}$ is a surd. $\sqrt{a+b}$ x $\sqrt{a+b}$ = $\sqrt{(\mathrm{a}+\mathrm{b})^2}$ = (a + b) is a rational expression.

$\therefore$ $\sqrt{\mathrm{a}+\mathrm{b}}$ is a rationalizing factor of $\sqrt{\mathrm{a}+\mathrm{b}}$.

Conjugate of surd:

Let's consider a surd $(\sqrt{x}+\sqrt{y})$. When $(\sqrt{x}+\sqrt{y})$ is multiplied by $(\sqrt{x}-\sqrt{y})$, then we can write as

$(\sqrt{x}+\sqrt{y}) \times (\sqrt{x}-\sqrt{y}) = (\sqrt{x})^2 - (\sqrt{y})^2 = (x - y)$ is a rational expression.

$(\sqrt{x}-\sqrt{y})$ is a rationalizing factor of $(\sqrt{x}+\sqrt{y})$.

Hence, $(\sqrt{x}-\sqrt{y})$ is the conjugate of $\sqrt{x}+\sqrt{y}$ and vice versa.

So, two binomial surds which differ only in sign connecting their terms are said to be conjugate of each other.

Example 1:

Rationalize the denominator of $\frac{5}{3\sqrt{2}}$.

Solution:

Here, The given fraction is $\frac{5}{3\sqrt{2}}$

The denominator of $\frac{5}{3\sqrt{2}}$ is $3\sqrt{2}$ where $\sqrt{2}$ is a surd.

$\sqrt{2}$ is changed into a rational number when multiplied by $\sqrt{2}$.

So, $\frac{5}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{5\sqrt{2}}{3\times 2} = \frac{5\sqrt{2}}{6}$

Example 2:

Rationalize the denominator of $\frac{3-\sqrt{2}}{3+\sqrt{2}}$

Solution:

Here, $\frac{3-\sqrt{2}}{3+\sqrt{2}}$

$= \frac{3-\sqrt{2}}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$

$= \frac{(3-\sqrt{2})^2}{(3)^2-(\sqrt{2})^2}$

$= \frac{(3)^2-2\times3\times\sqrt{2}+(\sqrt{2})^2}{9-2}$

$= \frac{9-6\sqrt{2}+2}{7}$

$= \frac{1}{7}(11-6\sqrt{2})$

Example 3:

Simplify: $\frac{x+\sqrt{y}}{x-\sqrt{y}} - \frac{x-\sqrt{y}}{x+\sqrt{y}}$

Solution:

Here, $\frac{x+\sqrt{y}}{x-\sqrt{y}} - \frac{x-\sqrt{y}}{x+\sqrt{y}}$

$= \frac{x+\sqrt{y}}{x-\sqrt{y}} \times \frac{x+\sqrt{y}}{x+\sqrt{y}} - \frac{x-\sqrt{y}}{x+\sqrt{y}} \times \frac{x-\sqrt{y}}{x-\sqrt{y}}$

$= \frac{(x+\sqrt{y})^2}{(x)^2-(\sqrt{y})^2} - \frac{(x-\sqrt{y})^2}{(x)^2-(\sqrt{y})^2}$

$= \frac{x^2+2x\sqrt{y}\ \ +y}{x^2-y} - \frac{x^2-2x\sqrt{y}+y}{x^2-y}$

$= \frac{x^2+2x\sqrt{y}+y-x^2+2x\sqrt{y}-y}{x^2-y}$

$= \frac{4x\sqrt{y}}{x^2-y}$

Example 4:

Simplify: $\mathbf{\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1}}$

Solution:

Here, $\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2} + \frac{(2-\sqrt{3})^2}{(2)^2-(\sqrt{3})^2} + \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$

$= \frac{(2)^2+2\times2\times\sqrt{3}+(\sqrt{3})^2}{4-3} + \frac{(2)^2-2\times2\times\sqrt{3}+(\sqrt{3})^2}{4-3} + \frac{(\sqrt{3})^2-2\times\sqrt{3}\times1+(1)^2}{3-1}$

$= \frac{4+4\sqrt{3}+3}{1} + \frac{4-4\sqrt{3}+3}{1} + \frac{3-2\sqrt{3}+1}{2}$

$= 7 + 4\sqrt{3} + 7 - 4\sqrt{3} + \frac{4-2\sqrt{3}}{2}$

$= 14 + 2 - \sqrt{3}$

$= 16 - \sqrt{3}$

Example 5

If $\frac{5+\sqrt{3}}{5-\sqrt{3}} = x + y\sqrt{3}$, find the values x and y.

Solution:

Here, $\frac{5+\sqrt{3}}{5-\sqrt{3}} = x + y\sqrt{3}$

or, $\frac{5+\sqrt{3}}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}} = x + y\sqrt{3}$

or, $\frac{(5+\sqrt{3})^2}{(5)^2-(\sqrt{3})^2} = x + y\sqrt{3}$

or, $\frac{(5)^2+2\times5\times\sqrt{3}+(\sqrt{3})^2}{25-3} = x + y\sqrt{3}$

or, $\frac{25+10\sqrt{3}+3}{22} = x + y\sqrt{3}$

or, $\frac{28+10\sqrt{3}}{22} = x + y\sqrt{3}$

or, $\frac{14}{11} + \frac{5}{11}\sqrt{3} = x + y\sqrt{3}$

Comparing the like terms on both sides, we get

$x = \frac{14}{11}$ and $y = \frac{5}{11}$

Exercise 9.2

1. Find the rationalizing factor of:

   (a) $2\sqrt{5}$ (b) $2a\sqrt{x+1}$ (c) $2-\sqrt{3}$ d) $\sqrt{x}+\sqrt{a}$

2. Rationalize the denominator of:

   (a) $\frac{5}{\sqrt{3}}$ b) $\frac{12x}{\sqrt{8}}$ c) $\frac{15x\sqrt{2}}{2\sqrt{3}}$ d) $\frac{20ab}{\sqrt{72}}$

3. Rationalize the denominator of:

   (a) $\frac{4}{\sqrt{3}+\sqrt{2}}$ (b) $\frac{3\sqrt{5}}{\sqrt{5}+\sqrt{3}}$ (c) $\frac{\sqrt{11}-\sqrt{5}}{\sqrt{11}+\sqrt{5}}$ (d) $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

4. **Simplify:**

   (a) $\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}$ (b) $\frac{2+3\sqrt{2}}{2-3\sqrt{2}}$ (c) $\frac{x+\sqrt{y}}{x-\sqrt{y}} + \frac{x-\sqrt{y}}{x+\sqrt{y}}$

   (d) $\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}-\sqrt{a}} + \frac{\sqrt{x}-\sqrt{a}}{\sqrt{x}+\sqrt{a}}$ (e) $\frac{x+3\sqrt{y}}{x-3\sqrt{y}} - \frac{x-3\sqrt{y}}{x+3\sqrt{y}}$

5. **Simplify:**

   (a) $\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}} + \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$ (b) $\frac{a+\sqrt{a^2-1}}{a-\sqrt{a^2-1}} + \frac{a-\sqrt{a^2-1}}{a+\sqrt{a^2-1}}$

(c) $\frac{6}{2\sqrt{3}-\sqrt{6}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$ (d) $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{5}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

6.(a) If $\frac{5+\sqrt{2}}{5-\sqrt{2}}=a+b\sqrt{2}$, find the value of a and b.

(b) If $x=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ and $y=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$, find the value of $(x+y)^2$.

(c) If $m=5-\sqrt{24}$, find the values of $\left(m+\frac{1}{m}\right)$ and $m^2+\frac{1}{m^2}$.

7. Write the difference between the rational number and irrational number with examples. Are these two numbers opposite numbers? Does rationalization change these two numbers into each other? Write the short report about rationalization by using internet service.

9.3 Radical equation

Let's see the following equations.

$2x + 1 = 5$ - - - - - - - - - - - (i)

$3x + y = 7$ - - - - - - - - - - -(ii)

$x^2 - 5x + 6 = 0$ - - -- - - - (iii)

$\left.\begin{array}{l} x + y = 6 \\ 2x - y = 3 \end{array}\right\}$ - - - - - - -- - - -(iv)

$\sqrt{x}+3=7$ - - - - - - - - (v)

In the above equations, equation (v) is different than equations (i) (ii) (iii) and (iv). In equation (v), the variable x is in the form of surd. So, the equation (v) is called, the radical equation. It is also called an equation involving the surd. Some other examples of such equations are $\sqrt{x-2}=8$, $\sqrt[3]{x+1}=3$, $x - 6 =\sqrt{x}$, etc.

To solve an equation involving surd, we should follow the following steps.

(i) The term containing surd is kept in one side of the given equation.

(ii) If an equation contains two surds on the same sides then one surd should be transporsted to other side.

(iii) We have to raise the power equal to the order of the surd on the both sides of the equation.

(iv) We use algebraic simplification to solve the equation after removing the radical sign.

(v) After solving the equation sometimes the root which is obtained may not be satisfied in the original equation. So, we must check the root in the given equation and reject the root which does not satisfy the original equation.

Example 1:

Solve : $\sqrt{x+2}$ **= 3**

Solution:

Here, $\sqrt{x+2} = 3$

Squaring on both sides, we have

$\left(\sqrt{x+2}\right)^2 = (3)^2$

Or, $x + 2 = 9$

$\therefore \quad x = 7$

Checking

$\sqrt{x+2} = 3$

Or, $\sqrt{7+2} = 3$

Or, $\sqrt{9} = 3$

Or, 3 = 3, which is true.

Hence, the value of x is 7.

Example 2:

Solve: $\sqrt[4]{2x+1} - 3 = 0$

Solution:

Here, $\sqrt[4]{2x+1} - 3 = 0$

Or, $\sqrt[4]{2x+1} = 3$

Raising the power 4 to both sides, we have;

$\left(\sqrt[4]{2x+1}\right)^4 = (3)^4$

Or, $2x + 1 = 81$

Or, $2x = 80$

$\therefore x = 40$

Checking: $\sqrt[4]{2x+1} - 3 = 0$

Or, $\sqrt[4]{2 \times 40 + 1} - 3 = 0$

Or, $\sqrt[4]{81} - 3 = 0$

Or, $\sqrt[4]{3^4} - 3 = 0$

Or, 3 - 3 = 0

Or, 0 = 0 which is true

Hence, the value of x is 40.

Example 3:

Solve: $\sqrt{x-20}+\sqrt{x} = 10$

Solution:

Here, $\sqrt{x-20}+\sqrt{x} = 10$

Or, $\sqrt{x-20} = 10-\sqrt{x}$

Squaring on the both sides, we have

$\left(\sqrt{x-20}\right)^2 = \left(10-\sqrt{x}\,\right)^2$

Or, $x - 20 = 100 - 20\sqrt{x} + x$

Or, $20\sqrt{x} = 120$

Or, $\sqrt{x} = 6$

Again, squaring on the both sides, we have

$\left(\sqrt{x}\right)^2 = (6)^2$

$\therefore x = 36$

Checking

$\sqrt{x-20} + \sqrt{x} = 10$

Or, $\sqrt{36-20}+\sqrt{36} = 10$

Or, $\sqrt{16}+\sqrt{36} = 10$

Or, 4 + 6 = 10

Or, 10 = 10, which is true

Hence, the value of x is 36.

Example 4:

Solve: $\sqrt{2x-1}+3=0$

Solution:

Here, $\sqrt{2x-1}+3=0$

Or, $\sqrt{2x-1} = -3$

Squaring on the both sides, we have;

$\left(\sqrt{2x-1}\right)^2 = (-3)^2$

Or, $2x - 1 = 9$

Or, $2x = 10$

$x = 5$

Checking

$\sqrt{2x-1}+3=0$

Or, $\sqrt{2\times5-1}+3=0$

Or, $\sqrt{9}+3=0$

Or, 3 + 3 = 0

Or, 6 = 0, which is false

x = 5 does not satisfy the original equation. So, x = 5 is rejected. Hence, the given equation has no real solution.

Example 5:

Solve: $x - 2\sqrt{x} = 3$

Solution:

Here, $x - 2\sqrt{x} = 3$

Or, $x - 3 = 2\sqrt{x}$

Squaring on the both sides, we have;

$(x - 3)^2 = (2\sqrt{x})^2$

Or, $x^2 - 6x + 9 = 4x$

Or, $x^2 - 10x + 9 = 0$

Or, $x^2 - 9x - x + 9 = 0$

Or, $x(x - 9) - 1(x - 9) = 0$

Or, $(x - 9)(x - 1) = 0$

Either,

$x - 9 = 0$ or, $x - 1 = 0$

$\therefore x = 9$ $\therefore x = 1$

Checking

If $x = 9$, then $x - 2\sqrt{x} = 3$

Or, $9 - 2\sqrt{9} = 3$

Or, 9 – 6 = 3

Or, 3 = 3, which is true.

If x = 1, then

$x - 2\sqrt{x} = 3$

or, $1 - 2\sqrt{1} = 3$

or, 1 -2 = 3

or, -1 = 3, which is false

$\therefore x = 1$ is rejected

Hence, $x = 9$ is the required solution of the given equation.

Example 6:

Solve: $\mathbf{2\sqrt{y} - \sqrt{4y-3} = \frac{1}{\sqrt{4y-3}}}$

Solution:

Here, $2\sqrt{y} - \sqrt{4y-3} = \frac{1}{\sqrt{4y-3}}$

Or, $2\sqrt{y}\,\sqrt{4y-3} - \left(\sqrt{4y-3}\right)^2 = 1$

Or, $2\sqrt{4y^2-3y}$ - 4y + 3 = 1

Or, $2\sqrt{4y^2-3y}$ = 4y – 2

Or, $\sqrt{4y^2-3y}$ = 2y – 1

Squaring on the both sides, we have

$$\left(\sqrt{4y^2-3y}\right)^2 = (2y-1)^2$$

Or, $4y^2 - 3y = 4y^2 - 4y + 1$

$\therefore$ y = 1

Checking

$2\sqrt{y} - \sqrt{4y-3} = \frac{1}{\sqrt{4y-3}}$

Or, $2\sqrt{1} - \sqrt{4\times1-3} = \frac{1}{\sqrt{4\times1-3}}$

Or, $2 \cdot \sqrt{1} - \sqrt{1} = \frac{1}{\sqrt{1}}$

Or, 2 – 1 = 1

Or, 1 = 1, which is true

Hence, y = 1 is the required solution.

Example 7:

Solve $\mathbf{\frac{x-1}{\sqrt{x}+1} = \frac{\sqrt{x}+1}{2} + 4}$

Solution:

Here, $\frac{x-1}{\sqrt{x}+1} = \frac{\sqrt{x}-1}{2} + 4$

Or, $\frac{(\sqrt{x})^2-(1)^2}{\sqrt{x}+1} = \frac{\sqrt{x}-1+8}{2}$

Or, $\frac{(\sqrt{x}+1)(\sqrt{x}-1)}{(\sqrt{x}+1)} = \frac{(\sqrt{x}+7)}{2}$

Or, $2\sqrt{x} - 2 = \sqrt{x} + 7$

Or, $\sqrt{x} = 9$

Squaring on the both sides, we have;

Checking

$\frac{x-1}{\sqrt{x}+1} = \frac{\sqrt{x}-1}{2} + 4$

Or, $\frac{81-1}{\sqrt{81}+1} = \frac{\sqrt{81}-1}{2} + 4$

Or, $\frac{80}{9+1} = \frac{9-1}{2} + 4$

Or, 8 = 4 + 4

Or, 8 = 8, which is true.

$(\sqrt{x})^2 = (9)^2$

$\therefore x = 81$

Hence, the value of x is 81.

Example 8:

Solve: $\frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}-\sqrt{3a}} + \frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}} = 5$

Solution:

Here, $\frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}-\sqrt{3a}} + \frac{\sqrt{x}-\sqrt{3a}}{\sqrt{x}+\sqrt{3a}} = 5$

Or, $\frac{(\sqrt{x}+\sqrt{3a})^2}{(\sqrt{x}-\sqrt{3a})} + \frac{(\sqrt{x}-\sqrt{3a})^2}{(\sqrt{x}+\sqrt{3a})} = 5$

Or, $\frac{x + 2\sqrt{x}.\sqrt{3a} + 3a + x - 2\sqrt{x}\,\sqrt{3a} + 3a}{(\sqrt{x})^2 - (\sqrt{3a})^2} = 5$

Or, $\frac{2x+6a}{x-3a} = 5$

Or, $5x - 15a = 2x + 6a$

Or, $3x = 21a$

$\therefore x = 7a$

Checking

$\frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}-\sqrt{3a}} + \frac{\sqrt{x}-\sqrt{3a}}{\sqrt{x}+\sqrt{3a}} = 5$

Or, $\frac{\sqrt{7a}+\sqrt{3a}}{\sqrt{7a}-\sqrt{3a}} + \frac{\sqrt{7a}-\sqrt{3a}}{\sqrt{7a}+\sqrt{3a}} = 5$

Or, $\frac{(\sqrt{7a}+\sqrt{3a})^2+(\sqrt{7a}-\sqrt{3a})^2}{(\sqrt{7a})^2-(\sqrt{3a})^2} = 5$

Or, $\frac{7a + 2\sqrt{7a}\sqrt{3a}+3a+7a-2\sqrt{7a}\sqrt{3a}+3a}{7a-3a} = 5$

Or, $\frac{20a}{4a} = 5$

Or, 5 = 5, which is true.

Hence, $x = 7a$ is the required solution.

Exercise 9.3

1. Solve:

(a) $\sqrt{x-2} = 3$ (b) $\sqrt{3x+1} - 2 = 5$ (c) $\sqrt{2x-3} + 2 = 5$

2. Solve:

(a) $\sqrt[3]{3x-1} - 2 = 0$ (b) $\sqrt[4]{y+1} - 2 = 0$ (c) $\sqrt[3]{4y+1} - \sqrt[3]{y+13} = 0$

3. Solve:

(a) $\sqrt{x+7} - 1 = \sqrt{x}$ (b) $\sqrt{x} - \sqrt{x-5} = 1$ (c) $\sqrt{y+12} - \sqrt{y} = 2$

4. **Solve:**

(a) $\sqrt{2x-1} + 1 = 0$ (b) $\sqrt{2z-3} + 5 = 0$ (c) $\sqrt[4]{3y+1} + 2 = 0$

5. **Solve:**

(a) $x - \sqrt{x} = 6$ (b) $\sqrt{y+4} + 2 = y$ (c) $y - \sqrt{y} = 2$

6. **Solve:**

(a) $\sqrt{x} + \sqrt{x+13} = \frac{21}{\sqrt{x+13}}$ (b) $\sqrt{2+z} + \sqrt{z+7} = \frac{15}{\sqrt{z+7}}$

(c) $\sqrt{x} + \sqrt{a+x} = \frac{3a}{\sqrt{x+a}}$ (d) $\sqrt{4x+5} - \sqrt{x+3} = \sqrt{x}$

7. **Solve:**

(a) $\frac{3x-4}{\sqrt{3x}+2} - 2 = \frac{\sqrt{3x}-2}{2}$ (b) $\frac{x-1}{\sqrt{x}+1} = 3 + \frac{\sqrt{x}-2}{2}$

(c) $\frac{x-9}{\sqrt{x}-3} = 4 - \frac{\sqrt{x}+2}{3}$ (d) $\frac{7y-16}{\sqrt{7y}-4} = \frac{\sqrt{7y}+4}{2} + 9$

8. **Solve:**

(a) $\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}-\sqrt{a}} + \frac{\sqrt{x}-\sqrt{a}}{\sqrt{x}+\sqrt{a}} = 4$ (b) $\frac{y+\sqrt{y^2-1}}{y-\sqrt{y^2-1}} + \frac{y-\sqrt{y^2-1}}{y+\sqrt{y^2-1}} = 98$

(c) $\frac{\sqrt{x}}{\sqrt{1-x}} + \frac{\sqrt{1+x}}{\sqrt{x}} = \frac{5}{2}$ (d) $\frac{\sqrt{x+3}-\sqrt{x-3}}{\sqrt{x+3}+\sqrt{x-3}} = \frac{1}{3}$

9. Show that the value of x in the equation $\sqrt{x^2-3x-3} = \sqrt{x^2-2x-4}$ -1 are 4 and $-\frac{4}{3}$. The value of $x = -\frac{4}{3}$ is not satisfied in the given equation, why? Give your suitable reason.

10. Show that x = 80 is the solution of $\frac{5x-16}{\sqrt{5x}+4} = \frac{\sqrt{5x}+4}{2} + 4$. Which value of x of this given equation is not solution of it, why? Give your suitable reason.

Unit: 10

Indices

10.0 Review

- We have to multiply a number by itself several times in mathematics.
- If a number x is multiplied by itself 4 times, we will write it as $x \times x \times x \times x = x^4$.
- Similarly, when x is multiplied by itself m times, we will write it x x x x x .... m times = x^m
- In x^m, x is a real number and m is a positive integer. In x^m, x occurs m times as a repeated factor. So, m is called the index of x and x is called the base. The index of a base is also called the exponent or power. The plural form of index is indices.

Laws of Indices:

There are some laws of indices which are as follows:

i) Product law

$a^m \times a^n = a^{m+n}$ where $a \neq 0$.

ii) Division law

$a^m \div a^n = a^{m-n}$, where $a \neq 0$.

iii) Power law

$(a^m)^n = a^{mn}$, where $a \neq 0$.

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $a \neq 0, b \neq 0$

$(a \times b)^m = a^m \times b^m$, where $a \neq 0, b \neq 0$

iv) $a^{-m} = \frac{1}{a^m}$, where $a \neq 0$

v) $\sqrt[m]{a} = a^{\frac{1}{m}}$, where $a \neq 0, m \in N$.

vi) $\sqrt[m]{a^n} = a^{n/m}$, where $a \neq 0$, m, n $\in$ N

vii) $a^0 = 1$, where $a \neq 0$.

10.1 Simplification of Indices:

We have to simplify the problems of indices by using the above laws of indices. Discuss the laws of indices in the group and solve the following problems in the index form.

$3 \times 3 \times 3 \times 3$ = ...

$2a \times 2a \times 2a \times 2a \times 2a \times 2a$ =......

$x^2 \times x^3 \times x^5 \times x^{-6} \times x^4$ = ...

$x^9 \div x^4$ = ...

$(-3)^2 \times (-3)^4 \times (-3)^7 \times (-3)^{-5} = \ldots\ldots$

$(x + 3) \times (x + 3)^3 \times (x + 3)^{-7} \times (x + 3)^{-4} = \ldots$

$x^3 \times x^2 \times x^{-5} = \ldots.$

Example 1:

Find the value of

(a) $\left(\frac{64}{125}\right)^{-2/3}$ (b) $(27)^{2/3} \times (8)^{\frac{1}{6}} \div (18)^{\frac{1}{2}}$

Solution:

(a) $\left(\frac{64}{125}\right)^{-2/3}$

$= \left[\left(\frac{4}{5}\right)^3\right]^{-2/3} = \left(\frac{4}{5}\right)^{-2} = \frac{4^{-2}}{5^{-2}} = \frac{5^2}{4^2} = \frac{25}{16}$

(b) $(27)^{2/3} \times (8)^{\frac{1}{6}} \div (18)^{\frac{1}{2}}$

$= (3^3)^{\frac{2}{3}} \times (2^3)^{\frac{1}{6}} \times \frac{1}{(18)^{\frac{1}{2}}}$

$= \frac{(3)^2 \times (2)^{1/2}}{(3^2 \times 2)^{1/2}}$

$= \frac{9 \times (2)^{1/2}}{(3^2)^{1/2} \times (2)^{1/2}}$

$= \frac{9 \times (2)^{1/2 - 1/2}}{3}$

$= 3 \times 2^0 = 3 \times 1 = 3$

Example 2: Simplify:

(a) $\frac{2^x \times 3 - 2^x}{2^{x+2} - 2^{x-1}}$ (b) $\left(\frac{a^x}{a^y}\right)^{x^2+xy+y^2} \times \left(\frac{a^y}{a^z}\right)^{y^2+yz+z^2} \times \left(\frac{a^z}{a^x}\right)^{z^2+zx+x^2}$

Solution: Here,

(a) $\frac{2^x \times 3 - 2^x}{2^{x+2} - 2^{x-1}}$

$= \frac{2^x \times 3 - 2^x}{2^x \times 2^2 - 2^4 \times 2^{-1}}$

$= \frac{2^x(3-1)}{2^x\,(2^2 - 2^{-1})} = \frac{2}{4 - \frac{1}{2}} = \frac{4}{7/2} = 2 \times \frac{2}{7} = \frac{4}{7}$

(b) $\left(\frac{a^x}{a^y}\right)^{x^2+xy+y^2} \times \left(\frac{a^y}{a^z}\right)^{y^2+yz+z^2} \times \left(\frac{a^z}{a^x}\right)^{z^2+zx+x^2}$

$= (a^{x-y})^{x^2+xy+y^2} \times (a^{y-z})^{y^2+yz+z^2} \times (a^{z-x})^{z^2+zx+x^2}$

$= a^{(x-y)(x^2+xy+y^2)} \times a^{(y-z)(y^2+yz+z^2)} \times a^{(z-x)(z^2+zx+x^2)}$

$= a^{x^3-y^3} \times a^{y^3-z^3} \times a^{z^3-x^3}$

$= a^{x^3-y^3+y^3-z^3+z^3-x^3}$

$= a^0$

$= 1$

Example 3: Simplify:

(a) $\dfrac{\left(x^2-\frac{1}{y^2}\right)^m\left(x-\frac{1}{y}\right)^{n-m}}{\left(y^2-\frac{1}{x^2}\right)^n\left(y+\frac{1}{x}\right)^{m-n}}$ (b) $\dfrac{1}{1+x^{l-m}+x^{l-n}} + \dfrac{1}{1+x^{m-n}+x^{m-l}} + \dfrac{1}{1+x^{n-m}+x^{n-l}}$

Solution:

(a) $\dfrac{\left(x^2-\frac{1}{y^2}\right)^m\left(x-\frac{1}{y}\right)^{n-m}}{\left(y^2-\frac{1}{x^2}\right)^n\left(y+\frac{1}{x}\right)^{m-n}}$

$= \dfrac{\left(x+\frac{1}{y}\right)^m\left(x-\frac{1}{y}\right)^m\left(x-\frac{1}{y}\right)^{n-m}}{\left(y-\frac{1}{x}\right)^n\left(y+\frac{1}{x}\right)^n\left(y+\frac{1}{x}\right)^{m-n}}$

$= \dfrac{\left(x+\frac{1}{y}\right)^m\left(x-\frac{1}{y}\right)^{m+n-m}}{\left(y-\frac{1}{x}\right)^n\left(y+\frac{1}{x}\right)^{n+m-n}}$

$= \dfrac{\left(x+\frac{1}{y}\right)^m\left(x-\frac{1}{y}\right)^n}{\left(y-\frac{1}{x}\right)^n\left(y+\frac{1}{x}\right)^m}$

$= \left(\dfrac{x+\frac{1}{y}}{y+\frac{1}{x}}\right)^m \left(\dfrac{x-\frac{1}{y}}{y-\frac{1}{x}}\right)^n$

$= \left(\dfrac{\frac{xy+1}{y}}{\frac{xy+1}{x}}\right)^m \left(\dfrac{\frac{xy-1}{y}}{\frac{xy-1}{x}}\right)^n$

$= \left(\dfrac{x}{y}\right)^m \left(\dfrac{x}{y}\right)^n = \left(\dfrac{x}{y}\right)^{m+n}$

(b) $\dfrac{1}{1+x^{l-m}+x^{l-n}} + \dfrac{1}{1+x^{m-n}+x^{m-l}} + \dfrac{1}{1+x^{n-m}+x^{n-l}}$

$= \dfrac{1}{1+\frac{x^l}{x^m}+\frac{x^l}{x^n}} + \dfrac{1}{1+\frac{x^m}{x^n}+\frac{x^m}{x^l}} + \dfrac{1}{1+\frac{x^n}{x^m}+\frac{x^n}{x^l}}$

$= \dfrac{1}{\frac{x^m.x^n+x^l.x^n+x^l.x^m}{x^m.x^n}} + \dfrac{1}{\frac{x^n.x^l+x^m.x^l+x^m.x^n}{x^n.x^l}} + \dfrac{1}{\frac{x^m.x^l+x^n.x^l+x^n.x^m}{x^m.x^l}}$

$= \dfrac{x^{m+n}}{x^{m+n}+x^{l+n}+x^{l+m}} + \dfrac{x^{n+l}}{x^{n+l}+x^{m+l}+x^{m+n}} + \dfrac{x^{m+l}}{x^{m+l}+x^{n+l}+x^{m+n}}$

$= \dfrac{x^{m+n}+x^{n+l}+x^{m+l}}{x^{m+n}+x^{n+l}+x^{l+m}} = 1$

Example 4:

If x + y + z = 0, prove that:

$$\frac{1}{1+a^x+a^{-y}}+\frac{1}{1+a^y+a^{-z}}+\frac{1}{1+a^z+a^{-x}}=1$$

Solution:

L.H.S $=\frac{1}{1+a^x+a^{-y}}+\frac{1}{1+a^y+a^{-z}}+\frac{1}{1+a^z+a^{-x}}$

$=\frac{1}{1+a^x+a^{-y}}+\frac{a^{-y}}{a^{-y}(1+a^y+a^{-z})}+\frac{a^x}{a^x(1+a^z+a^{-x})}$

$=\frac{1}{1+a^x+a^{-y}}+\frac{a^{-y}}{a^{-y}+1+a^{-y-z}}+\frac{a^x}{a^x+a^{x+z}+1}$

$=\frac{1}{1+a^x+a^{-y}}+\frac{a^{-y}}{a^{-y}+1+a^x}+\frac{a^x}{a^x+a^{-y}+1}$ $[x+y+z=0]$

$=\frac{1+a^{-y}+a^x}{1+a^x+a^{-y}}$

= 1 R.H.S proved

Exercise 10.1

1. **Evaluate the following:**

(a) $\left(\frac{8}{27}\right)^{-2/3}$ (b) $(4)^{\frac{1}{3}}\times\left(3^{\frac{1}{3}}\times3^{\frac{1}{2}}\right)^7\div9^{\frac{3}{4}}$ (c) $\left(\frac{64}{216}\right)^{\frac{2}{3}}\times\left(\frac{16}{36}\right)^{\frac{-3}{2}}$

(d) $4^{\frac{5}{2}}-2\times7^0-\left(\frac{1}{16}\right)^{-1/2}$ (e) $\frac{3^6\times7^{-4}\times5^{-3}\times9^2}{27^3\times35^{-3}\times49^{-1}}$

2. **Simplify**

(a) $(x^2-y^{-2})\div(x^{-1}-y^{-1})$ (b) $\frac{5^{x+1}+5^x}{5^{x+3}-5^{x+1}}$ (c) $\frac{3^{x+1}-3^x}{3^x\times4-3^{x-1}}$

(d) $\frac{7^{x+1}+3\times7^x}{7^x\times3-7^{x-1}}$ (e) $\frac{a^{m(n-p)}}{a^{n(m-p)}}\div\left(\frac{a^n}{a^m}\right)^p$

3. **Simplify**

(a) $\left(\frac{x^{a+b}}{x^c}\right)^{a-b}\times\left(\frac{x^{b+c}}{x^a}\right)^{b-c}\times\left(\frac{x^{c+a}}{x^b}\right)^{c-a}$

(b) $\left(\frac{x^m}{x^n}\right)^{m^2+mn+n^2}\times\left(\frac{x^n}{x^p}\right)^{n^2+np+p^2}\times\left(\frac{x^p}{x^m}\right)^{p^2+pm+m^2}$

(c) $\sqrt[mn]{\frac{x^m}{x^n}}\times\sqrt[np]{\frac{x^n}{x^p}}\times\sqrt[pm]{\frac{x^p}{x^m}}$ (d) $\sqrt[a-c]{x^{\frac{1}{a-b}}}\times\sqrt[b-a]{\frac{1}{x^{b-c}}}\times\sqrt[c-b]{\frac{1}{x^{c-a}}}$

4. **Simplify:**

(a) $\frac{\left(x+\frac{1}{y}\right)^a\left(x-\frac{1}{y}\right)^a}{\left(y+\frac{1}{x}\right)^a\left(y-\frac{1}{x}\right)^a}$ (b) $\frac{\left(1+\frac{x}{y}\right)^{\frac{a}{a-b}}\left(1-\frac{x}{y}\right)^{\frac{b}{a-b}}}{\left(\frac{y}{x}+1\right)^{\frac{a}{a-b}}\left(\frac{y}{x}-1\right)^{\frac{b}{a-b}}}$

(c) $\frac{\left(a^2-\frac{1}{b^2}\right)^x\left(a-\frac{1}{b}\right)^{y-x}}{\left(b^2-\frac{1}{a^2}\right)^y\left(b+\frac{1}{a}\right)^{x-y}}$ (d) $\frac{p^2}{(p-y)^y} - \frac{2p}{(p-y)^{y-1}} + \frac{1}{(p-y)^{y-2}}$

5. **Simplify:**

(a) $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{c-b}}$

(b) $\frac{1}{1+x^{a-b}+x^{a-c}} + \frac{1}{1+x^{b-c}+x^{b-a}} + \frac{1}{1+x^{c-a}+x^{c-b}}$

(c) $\frac{1}{1+a^{m-n}+a^{p-n}} + \frac{1}{1+a^{n-p}+a^{m-p}} + \frac{1}{1+a^{p-m}+a^{n-m}}$

6. If p $=x^a$, q $=x^b$ and r = x^c, prove that: $p^{b-c} \times q^{c-a} \times r^{a-b} = 1$

7. If abc = 1, prove that: $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = 1$

8. If a = $x^{q+r}y^p$, b = $x^{p+r}.y^q$ and c= $x^{p+q}.y^r$, prove that:
$a^{q-r}.\ b^{r-p}.\ c^{p-q} = 1$

9. If 2xyz = 1 and $x^3 + y^3 + z^3 = 1$. Prove that:
$a^{x2\ y\text{-}1\ z\text{-}1} \times a^{y2.z\text{-}1.x\text{-}1} \times a^{z2.\ x\text{-}1.y\text{-}1} = a^2$

10. Make 2/2 questions similar to Q.No. 1, 2 and 3 and find the solution of those questions discussing in the group.

10.2. Exponential Equation:

Let's see the following equations:

$2x = 8$ and $2^x = 8$.

What are the value of x in both equations?

Are both the equations satisfied by the same value of x?

In equation $2x = 8$, x is in the form of base. But in equation $2^x = 8$, x is the index of 2. The equation $2^x = 8$ is an exponential equation. Some more examples of the exponential equations are $2^{x+1} - 2^x = 4$, $7^x - 7^{-x} = 0$, $2^{x-4} = 4^{x-6}$, etc.

An exponential equation can be solved by using the following axioms.

If the bases of left and right hand sides of an equation are the same, their index must be equal.

i.e. if $a^x = a^y$, then $x = y$.

if $a^x = 1$, then $x = 0$.

Example 1: Solve:

(a) $3^{x-1} = 1$ (b) $3^{x+3} + 3^{x} - 28 = 0$

Solution:

(a) $3^{x-1} = 1$

Or, $3^{x-1} = 3^{0}$

$\therefore x - 1 = 0$

i.e. $x = 1$

(b) $3^{x+3} + 3^{x} - 28 = 0$

Or, $3^{x}.3^{3} + 3^{x} = 28$

Or, $3^{x}(27 + 1) = 28$

Or, $3^{x} \times 28 = 28$

Or, $3^{x} = 1$

Or, $3^{x} = 3^{0}$

$\therefore x = 0$

Example 2 : Solve: $4^{x} - 6\times 2^{x-1} + 2 = 0$

Solution: Here,

$4^{x} - 6\times 2^{x-1} + 2 = 0$

Or, $(2^{2})^{x} - 6 \times 2^{x} \times 2^{-1} + 2 = 0$

Or, $(2^{x})^{2} - 6 \times 2^{x} \times \frac{1}{2} + 2 = 0$

Or, $(2^{x})^{2} - 3 \times 2^{x} + 2 = 0$

Let 2^{x} = a, then equation is

$a^{2} - 3a + 2 = 0$

Or, $a^{2} - 2a - a + 2 = 0$

Or, $a(a - 2) - 1(a - 2) = 0$

Or, $(a - 2)(a - 1) = 0$

Either,	Or,
$a - 2 = 0$	$a - 1 = 0$
or, $a = 2$	or, $a = 1$
or, $2^{x} = 2^{1}$	or, $2^{x} = 2^{0}$
$\therefore x = 1$	$\therefore x = 0$

$\therefore x = 0, 1$

Example 3

Solve: $\mathbf{4^{x} + 4^{-x} = 16\frac{1}{16}}$

Solution: Here,

$4^{x} + 4^{-x} = 16\frac{1}{16}$

$4^{x} + \frac{1}{4^{x}} = \frac{257}{16}$

Let, 4^{x} = y, then equation is $y + \frac{1}{y} = \frac{257}{16}$

Or, $16y^2 + 16 = 257y$

Or, $16y^2 - 257y + 16 = 0$

Or, $16y^2 - 256y - y + 16 = 0$

Or, $16y(y - 16) - 1(y - 16) = 0$

Or, $(y - 16)(16y - 1) = 0$

Either, $y - 16 = 0$

Or, $4^x = 16$

Or, $4^x = 4^2$

$\therefore x = 2$

Or, $16y - 1 = 0$

Or, $y = \frac{1}{16}$

Or, $4^x = \frac{1}{4^2}$

Or, $4^x = 4^{-2}$

$\therefore x = -2$

$\therefore x = \pm 2$

Exercise 10.2

1. Solve:

(a) $3^{x-1} = 27$ (b) $5^{2x-1} = 1$ (c) $a^x \div a^2 = 1$

(d) $2^{x^2} = 16$ (e) $2^{x-1} = (\sqrt{2})^x$

2. Solve:

(a) $2^{x+1} + 2^x = 3$ (b) $2^{x+2} + 2^{x-1} = 9$ (c) $3^x + 3^{x+2} = \frac{10}{3}$

(d) $2^{y-2} + 2^{3+y} = 66$ (e) $2^{x-4} = 4a^{x-6}$

3. Solve:

(a) $4^x - 10 \times 2^{x-1} + 4 = 0$ (b) $9^x - 12 \times 3^{x-1} + 3 = 0$

(c) $9^x - 6 \times 3^{x-1} = 3$ (d) $16^y - 5 \times 4^{y+1} + 64 = 0$

(e) $3^{2x} - 4 \times 3^{x+1} + 27 = 0$ (f) $2^{2y+3} - 9 \times 2^y + 1 = 0$

4. Solve:

(a) $2^x + 2^{-x} = 2\frac{1}{2}$ (b) $4^x + \frac{1}{4^x} = 4\frac{1}{4}$ (c) $7^x + 343 \times 7^{-x} = 56$

(d) $3^x + 3^{-x} = 9\frac{1}{9}$ (e) $5^x + 2\frac{1}{5^x} = 25\frac{1}{25}$ (f) $5^{x+1} + 5^{2-x} = 126$

5. Make three equations of indices and give to your friends to find the solution. After that present the solution to the class.

Unit: 11

Algebraic Fraction

11.0 Review:

- $\frac{4x}{x-y}$ is a fraction where 4x is a numerator and (x - y) is a denominator 4x and (x - y) both are the algebraic expression. So, $\frac{4x}{x-y}$ is an example of an algebraic fraction. An expression of the form $\frac{f(x)}{g(x)}$, where f(x) and g(x) are polynomials and g(x) $\neq$ 0 is called an algebraic fraction. For examples $\frac{3x}{7}, \frac{2x-y}{a+b}$, etc.
- An algebraic fraction is also known as a rational fraction or rational expression.
- Write 2/2 algebraic fractions having the same denominator and different denominator.
- An algebraic fraction $\frac{4m}{x-7}$ is undefined when x= 7. Discuss about it in the group.

11.1 Simplification of Algebraic fractions:

We have already discussed about the simplification of algebraic fractions in the previous grades 7 and 8. In the grade 10, we will study the simplification of algebraic fractions which contains not more than four algebraic fractions. For the simplification of algebraic fractions we need the knowledge of H.C.F and L.C.M of algebraic expressions.

What are the H.C.F and L.C.M of $x^2 - y^2$ and $x^2 + 2xy + y^2$?

Discuss it in the group.

Write the same denominator of the following fractions and discuss in the group.

(a) $\frac{a}{b}, \frac{1}{b}, \frac{1}{ab},$ (b) $\frac{a}{x+y}, \frac{b}{x-y}, \frac{1}{x^2-y^2}$

Example 1:

Simplify: $\frac{2a-b}{a+b} + \frac{2a+b}{a-b}$

Solution:

Here, $\frac{2a-b}{a+b} + \frac{2a+b}{a-b}$

$= \frac{(2a-b)(a-b)+(2a+b)(a+b)}{(a+b)(a-b)}$

$= \frac{2a^2-2ab-ab+b^2+2a^2+2ab+ab+b^2}{a^2-b^2}$

$= \frac{4a^2+2b^2}{a^2-b^2}$

$= \frac{2(2a^2+b^2)}{a^2-b^2}$

Example 2:

Simplify: $\frac{a^2+3ab-4b^2}{a^2-16b^2} - \frac{2ab}{2a^2-8ab}$

Solution:

Here, $\frac{a^2+3ab-4b^2}{a^2-16b^2} - \frac{2ab}{2a^2-8ab}$

$= \frac{a^2+4ab-ab-4b^2}{(a)^2-(4b)^2} - \frac{2ab}{2a(a-4b)}$

$= \frac{a(a+4b)-b(a+4b)}{(a+4b)(a-4b)} - \frac{b}{(a-4b)}$

$= \frac{(a+4b)(a-b)}{(a+4b)(a-4b)} - \frac{b}{(a-4b)}$

$= \frac{a-b-b}{(a-4b)}$

$= \frac{a-2b}{a-4b}$

Example 3:

Simplify: $\frac{1}{a+2b} + \frac{1}{a-2b} + \frac{a}{4b^2-a^2}$

Solution:

Here, $\frac{1}{a+2b} + \frac{1}{a-2b} + \frac{a}{4b^2-a^2}$

$= \frac{a-2b+a+2b}{(a+2b)(a-2b)} - \frac{a}{a^2-4b^2}$

$= \frac{2a}{a^2-4b^2} - \frac{a}{a^2-4b^2}$

$= \frac{a}{a^2-4b^2}$

Example 4:

Simplify: $\frac{1}{x^2-x-6} - \frac{3}{x^2-2x-8} + \frac{2}{x^2-7x+12}$

Solution:

Here, $\frac{1}{x^2-x-6} - \frac{3}{x^2-2x-8} + \frac{2}{x^2-7x+12}$

$= \frac{1}{x^2-3x+2x-6} - \frac{3}{x^2-4x+2x-8} + \frac{2}{x^2-4x-3x+12}$

$= \frac{1}{x(x-3)+2(x-3)} - \frac{3}{x(x-4)+2(x-4)} + \frac{2}{x(x-4)-3(x-4)}$

$= \frac{1}{(x+2)(x-3)} - \frac{3}{(x-4)(x+2)} + \frac{2}{(x-4)(x-3)}$

$= \frac{(x-4)-3(x-3)+2(x+2)}{(x+2)(x-3)(x-4)}$

$= \frac{x-4-3x+9+2x+4}{(x+2)(x-3)(x-4)}$

$= \frac{9}{(x+2)(x-3)(x-4)}$

Example 5:

Simplify: $\frac{1}{1+a+a^2} - \frac{1}{1-a+a^2} + \frac{2a}{1+a^2+a^4}$

Solution:

Here, $\frac{1}{1+a+a^2} - \frac{1}{1-a+a^2} + \frac{2a}{1+a^2+a^4}$

$= \frac{(1-a+a^2)-(1+a+a^2)}{(1+a+a^2)(1-a+a^2)} + \frac{2a}{1+a^2+a^4}$

$= \frac{1-a+a^2-1-a-a^2}{(1+a^2)^2-(a)^2} + \frac{2a}{1+a^2+a^4}$

$= \frac{-2a}{1+2a^2+a^4-a^2} + \frac{2a}{1+a^2+a^4}$

$= \frac{-2a}{1+a^2+a^4} + \frac{2a}{1+a^2+a^4}$

$= \frac{-2a+2a}{1+a^2+a^4}$

$= \frac{0}{1+a^2+a^4}$

$= 0$

Example 6:

Simplify: $\frac{x+y}{x^2+xy+y^2} - \frac{x-y}{x^2-xy+y^2} + \frac{2x^3}{x^4+x^2y^2+y^4}$

Solution:

Here, $\frac{x+y}{x^2+xy+y^2} - \frac{x-y}{x^2-xy+y^2} + \frac{2x^3}{x^4+x^2y^2+y^4}$

$= \frac{(x+y)(x^2-xy+y^2)-(x-y)(x^2+xy+y^2)}{(x^2+xy+y^2)(x^2-xy+y^2)} + \frac{2x^3}{x^4+x^2y^2+y^4}$

$= \frac{(x^3+y^3)-(x^3-y^3)}{(x^2+y^2)^2-(xy)^2} + \frac{2x^3}{x^4+x^2y^2+y^4}$

$= \frac{x^3+y^3-x^3+y^3}{x^4+2x^2y^2+y^4-x^2y^2} + \frac{2x^3}{x^4+x^2y^2+y^4}$

$= \frac{2y^3}{x^4+x^2y^2+y^4} + \frac{2x^3}{x^4+x^2y^2+y^4}$

$= \frac{2y^3+2x^3}{x^4+x^2y^2+y^4}$

$= \frac{2(x^3+y^3)}{x^4+x^2y^2+y^4}$

$= \frac{2(x+y)(x^2-xy+y^2)}{(x^2+xy+y^2)(x^2-xy+y^2)}$

$= \frac{2(x+y)}{x^2+xy+y^2}$

Example 7.

Simplify: $\frac{a^2-(b-c)^2}{(a+c)^2-b^2} + \frac{b^2-(a-c)^2}{(a+b)^2-c^2} + \frac{c^2-(a-b)^2}{(b+c)^2-a^2}$

Solution:

Here, $\frac{a^2-(b-c)^2}{(a+c)^2-b^2} + \frac{b^2-(a-c)^2}{(a+b)^2-c^2} + \frac{c^2-(a-b)^2}{(b+c)^2-a^2}$

$= \frac{(a+b-c)(a-b+c)}{(a+b+c)(a+c-b)} + \frac{(b+a-c)(b-a+c)}{(a+b+c)(a+b-c)} + \frac{(c+a-b)(c-a+b)}{(b+c+a)(b+c-a)}$

$= \frac{a+b-c}{a+b+c} + \frac{b-a+c}{a+b+c} + \frac{c+a-b}{a+b+c}$

$= \frac{a+b-c+b-a+c+c+a-b}{a+b+c}$

$= \frac{a+b+c}{a+b+c} = 1$

Example 8.

Simplify: $\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4+1}$

Solution:

Here, $\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4+1}$

$= \frac{(x+1)-(x-1)}{(x-1)(x+1)} - \frac{2}{x^2+1} - \frac{4}{x^4+1}$

$= \frac{x+1-x+1}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4+1}$

$= \frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)} - \frac{4}{x^4+1}$

$= \frac{2x^2+2-2x^2+2}{x^4-1} - \frac{4}{x^4+1}$

$= \frac{4(x^4+1)-4(x^4-1)}{(x^4-1)(x^4+1)}$

$= \frac{4x^4+4-4x^4+4}{x^{8-1}}$

$= \frac{8}{x^{8-1}}$

Example 9.

Simplify: $\frac{1}{1+a}+\frac{2a}{1+a^2}+\frac{4a^3}{1+a^4}+\frac{8a^7}{a^8-1}$

Solution:

Here, $\frac{1}{1+a}+\frac{2a}{1+a^2}+\frac{4a^3}{1+a^4}-\frac{8a^7}{a^8-1}$

$= \frac{1}{a+1}+\frac{2a}{a^2+1}+\frac{4a^3}{a^4+1}-\frac{8a^7}{(a^4+1)(a^4-1)}$

$= \frac{1}{a+1}+\frac{2a}{a^2+1}+\frac{4a^3(a^4-1)-8a^7}{(a^4+1)(a^4-1)}$

$= \frac{1}{a+1}+\frac{2a}{a^2+1}+\frac{4a^7-4a^3-8a^7}{(a^4+1)(a^4-1)}$

$= \frac{1}{a+1}+\frac{2a}{a^2+1}-\frac{4a^7+4a^3}{(a^4+1)(a^4-1)}$

$= \frac{1}{a+1}+\frac{2a}{a^2+1}+\frac{4a^3(a^4+1)}{(a^4+1)(a^4-1)}$

$= \frac{1}{a+1}+\frac{2a}{a^2+1}-\frac{4a^3}{(a^2+1)(a^2-1)}$

$= \frac{1}{a+1}+\frac{2a(a^2-1)-4a^3}{(a^2+1)(a^2-1)}$

$= \frac{1}{a+1}+\frac{2a^3-2a-4a^3}{(a^2+1)(a^2-1)}$

$= \frac{1}{a+1}-\frac{2a^3+2a}{(a^2+1)(a^2-1)}$

$= \frac{1}{a+1}-\frac{2a(a^2+1)}{(a^2+1)(a^2-1)}$

$= \frac{1}{a+1}-\frac{2a}{(a+1)(a-1)}$

$= \frac{a-1-2a}{(a+1)(a-1)}$

$= \frac{-a-1}{(a+1)(a-1)}$

$= \frac{-(a+1)}{(a+1)(a-1)}$

$= \frac{1}{(1-a)}$

Exercise 11.1

1. Simplify:

(a) $\frac{2}{x+3}-\frac{3}{x-2}$ (b) $\frac{y}{y-6}+\frac{1}{y-2}$ (c) $\frac{x-y}{x+y}+\frac{x+y}{x-y}$

(d) $\frac{a-3}{a+5}-\frac{a-5}{a+3}$ (e) $\frac{2x-y}{x+y}+\frac{x+2y}{x-y}$ (f) $\frac{a}{ab-b^2}+\frac{b}{ab-a^2}$

2. Simplify:

(a) $\frac{a-2}{a^2-1}-\frac{a+1}{a^2-2a+1}$ (b) $\frac{m^2+mn+n^2}{m+n}+\frac{m^2-mn+n^2}{m-n}$ (c) $\frac{x^2-4a^2}{x^2-2ax}-\frac{x^2+2ax-8a^2}{x^2-4a^2}$

(d) $\frac{y^2}{y-y^3}-\frac{y}{1+y^2}$ (e) $\frac{2x^2-xy-6y^2}{4x^2-9y^2}-\frac{2xy}{4x^2-6xy}$ (f) $\frac{4x^2+y^2}{4x^2-y^2}-\frac{2x-y}{2x+y}$

3. Simplify:

(a) $\frac{1}{x+y}+\frac{1}{x-y}-\frac{x}{y^2-x^2}$ (b) $\frac{x-y}{xy}+\frac{y-z}{yz}+\frac{z-x}{zx}$ (c) $\frac{1}{2a+5}+\frac{1}{2a-5}-\frac{10}{4a^2-25}$

(d) $\frac{x-y}{x+y}+\frac{x+y}{x-y}+\frac{4xy}{y^2-x^2}$ (e) $\frac{1}{2(m-n)}-\frac{1}{2(m+n)}-\frac{n}{(n^2-m^2)}$

(f) $\frac{x}{x-2a}+\frac{x}{x+2a}+\frac{2x^2}{x^2-4a^2}$

4. Simplify:

(a) $\frac{1}{x^2-5x+6}-\frac{1}{x^2-4x+3}-\frac{1}{x^2-3x+2}$ (b) $\frac{a-3}{a^2-a-6}-\frac{2a+5}{a^2+5a+6}+\frac{2a-1}{2a^2+5a-3}$

(c) $\frac{4}{x^2-3x+2}+\frac{3}{x^2-5x+6}-\frac{2}{4x-x^2-3}$ (d) $\frac{x-1}{x^2-3x+2}+\frac{x-2}{x^2-5x+6}+\frac{x-5}{x^2-8x+15}$

5. Simplify:

(a) $\frac{1}{1+x+x^2}-\frac{1}{1-x+x^2}+\frac{2x}{1+x^2+x^4}$ (b) $\frac{y+1}{1+y+y^2}+\frac{y-1}{1-y+y^2}-\frac{2y^2}{1+y^2+y^4}$

(c) $\frac{1}{1+p+p^2}-\frac{1}{1-p+p^2}-\frac{2p}{1+p^2+p^4}$ (d) $\frac{2x^4}{1+x^2+x^4}+\frac{1}{1-x+x^2}+\frac{1}{1+x+x^2}$

6. Simplify:

(a) $\frac{a-b}{a^2-ab+b^2}+\frac{a+b}{a^2+ab+b^2}-\frac{2b^3}{a^4+a^2b^2+b^4}$

(b) $\frac{x-2}{x^2-2x+4}+\frac{x+2}{x^2+2x+4}-\frac{16}{x^4+4x^2+16}$

(c) $\frac{a+x}{a^2+ax+x^2}+\frac{a-x}{x^2-ax+x^2}+\frac{2x^3}{a^4+a^2x^2+x^4}$

7. Simplify:

(a) $\frac{(a-b)^2-c^2}{a^2-(b+c)^2}+\frac{(b-c)^2-a^2}{b^2-(c+a)^2}+\frac{(c-a)^2-b^2}{c^2-(a+b)^2}$ (b) $\frac{x^2-(y-z)^2}{(x+z)^2-y^2}+\frac{y^2-(x-z)^2}{(x+y)^2-z^2}+\frac{z^2-(x-y)^2}{(y+z)^2-x^2}$

(c) $\frac{b-c}{a^2-(b+c)^2}+\frac{c+a}{b^2-(c+a)^2}+\frac{a+b}{(a+b)^2-c^2}$

8. Simplify:

(a) $\frac{x}{(x-y)(x-z)}+\frac{y}{(y-z)(y-x)}+\frac{z}{(z-x)(z-y)}$ (b) $\frac{a+b}{(p-q)(p-r)}+\frac{a+b}{(q-r)(q-p)}+\frac{a+b}{(r-p)(r-q)}$

(c) $\frac{ax^2+b}{2x-1}+\frac{ax^2-b}{2x+1}+\frac{4ax^3}{1-4x^2}$

(d) $\frac{1}{8(1-\sqrt{x})}-\frac{1}{8(1+\sqrt{x})}+\frac{2\sqrt{x}}{8(1-x)}$

9. Simplify:

(a) $\frac{1}{1-x}+\frac{1}{1+x}+\frac{2}{1+x^2}+\frac{4}{1+x^4}$

(b) $\frac{1}{a-b}+\frac{1}{a+b}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$

(c) $\frac{2}{x+1}+\frac{1}{x-1}+\frac{3x}{1-x^2}+\frac{x}{1+x^3}$

10. Simplify:

(a) $\frac{1}{1+x}+\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}-\frac{8x^7}{x^8-1}$

(b) $\frac{a}{a-b}+\frac{b}{a+b}+\frac{2ab}{a^2+b^2}-\frac{4a^3b}{a^4-b^4}$

(c) $\frac{1}{a+1}+\frac{2}{a^2+1}+\frac{4}{a^4+1}-\frac{8}{1-a^8}$

11. Write the roles of factorization, H.C.F and L.C.M. while simplifying the algebraic fractions. Work in groups and present your answer to the class.

12. Make one/one question of algebraic fractions containing two fractions and three fractions, and give to your friend to find the solution.

Unit: 12

Equations

12.0 Review

Let's see the following equations

$2x - 6 = 0$, $x + y = 7$, $x^2 - 4 = 0$, $2x + y = 5$ and $x + 3y = 5$ etc.

Discuss the above equations in groups and find the solution. Equations $2x + y = 5$ and $x + 3y = 5$ are two linear equations of two variables. These two equations are satisfied by only $x = 2$ and $y = 1$. So, the straight lines formed from these two equations intersect each other at only one point which is (2, 1). These equation are called simultaneous linear equations of two variables. The process to solve the simultaneous linear equations are already discussed in grade nine (IX). Can you say the process of solving the simultaneous linear equations of two variables? Discuss it.

12.1 Word Problems Based on Simultaneous Linear Equations:

Simultaneous linear equations with two variables are involved in our day to day life. So, we can express the given problems in the equation form and solve them by using appropriate methods. The following steps are used for solving the simple problems.

1. Read the given problem carefully with full concentration as many times as needed to understand and identify the two unknowns.
2. Suppose the unknown quantities by two variables like x and y with their proper unit.
3. Construct two equations with variables x and y from the given information.
4. Solve the equations by any one convenient method to find the values of x and y.

Let's study the following examples which are solved by using the above steps.

Example 1:

The sum of two numbers is 36 and their difference is 8. Find the numbers.

Solution:

Let the two numbers be x and y. Then,

$x + y = 36$ .... (1)

$x - y = 8$ .... (2)

Adding equation (1) and equation (2)

$$\begin{array}{l} x + y = 36 \\ x - y = 8 \\ \hline 2x \;\; = 44 \end{array}$$

or, $x = 22$

Substituting the value of x in equation (1), we get

$22 + y = 36$

or, $y = 36 - 22$

or, $y = 14$

Hence, the required two numbers are 22 and 14.

Example 2:

The perimeter of a rectangular ground is 154m. If the length of the ground is 7m. longer than its breadth, find the area of the ground.

Solution:

Let the length and breadth of the rectangular ground be xm and ym respectively. Then,

$2(x + y) = 154$m. [$\therefore$ perimeter of the rectangle = $2(l + b)$]

or, $x + y = 77$m. .... (1)

Again, length = breadth + 7m.

or, $x = y + 7$m .... (2)

From equations (1) and (2),

y + 7m. + y = 77m

or, 2y = 70m.

$\therefore$ y = 35 m.

Substituting the value of y in equation (1), we get

x + 35m. = 77m.

$\therefore$ x = 42m.

Now, Area of the ground (A) = $l \times b$

$= x \times y$

= 42m. x 35m.

= 1470m^2.

Example 3:

7 pens and 5 pencils cost Rs. 375. Again 4 pens and 7 pencil cost Rs. 264. Find the unit price for each pen and pencil.

Solution:

Let the unit price for each pen and pencil be Rs. x and Rs. y respectively. Then, according to the question,

$7x + 5y = 375$ .... (1)

$4x + 7y = 264$ .... (2)

Multiplying equation (1) by 7 and equation (2) by 5 and then subtracting, we have;

$$\begin{array}{l} 49x + 35y = 2625 \\ 20x + 35y = 1320 \\ -\quad -\qquad - \\ \hline 29x \qquad = 1305 \end{array}$$

$\therefore x = \frac{1305}{29} = 45$

Substituting the value of x in equation (1), we get;

$7 \times 45 + 5y = 375$

or, $315 + 5y = 375$

or, $5y = 60$

$\therefore y = \frac{60}{5} = 12$

Hence, the unit price for each pen and pencil is Rs. 45 and Rs. 12 respectively.

Example 4:

6 years ago, a man's age was six times the age of his daughter. After 4 years, thrice his age will be equal to eight times of his daughter's age. What are their present ages? Find it.

Solution:

Let the present ages of a man and his daughter be x years and y years respectively. Then, according to the questions;

$x - 6 = 6(y - 6)$

or, $x - 6 = 6y - 36$

or, $x = 6y - 30$ .... (1)

Again,

$3(x + 4) = 8(y + 4)$

or, $3x + 12 = 8y + 32$

or, $3x = 8y + 20$

or, $x = \frac{8y+20}{3}$ .... (2)

From equation (1) and equation (2), we get

$6y - 30 = \frac{8y+20}{3}$

or, $18y - 90 = 8y + 20$

or, $10y = 110$

$\therefore y = 11$

Substituting the value of y in equation (1), we get

$x = 6 \times 11 - 30 = 36$

Hence, the present ages of the man and his daughter are 36 years and 11 years respectively.

Example 5:

The sum of the digits of a two digit number is 13. If 27 is added to the number, the places of the digits are reversed. Find the number.

Solution:

Let a two digit number be 10x + y where x and y are the digits of ten's place and unit place respectively. Then, according to question,

$x + y = 13$ .... (1)

Again, $10x + y + 27 = 10y + x$

or, $9x = 9y - 27$

or, $x = y - 3$ .... (2)

Substituting the value of x in equation (1) from equation (2), we get

$y - 3 + y = 13$

or, $2y = 16$

$\therefore y = 8$

Again, substituting the value of y in equation (2), we get

$x = 8 - 3 = 5$

Now, the required two digit number is

$10x + y = 10 \times 5 + 8 = 58$

Example 6:

If 2 is added to the numerator of a fraction, the fraction becomes 4/5. If 1 is subtracted from the denominator, the fraction becomes 3/4. Find the original fraction.

Solution:

Let the original fraction be $\frac{x}{y}$. Then, according to the question;

$$\frac{x+2}{y} = \frac{4}{5}$$

or, $5x + 10 = 4y$

or, $5x - 4y = -10$ .... (1)

Again, $\frac{x}{y-1} = \frac{3}{4}$

or, $4x = 3y - 3$

or, $4x - 3y = -3$ ....(2)

Multiplying equation (1) by 4 and equation (2) by 5 and then subtracting. We have,

$$
\begin{array}{rl}
20x - 16y &= -40 \\
\underset{(-)}{20x} \underset{(+)}{-} 15y &\underset{(+)}{=} -15 \\
\hline
-y &= -25
\end{array}
$$

$\therefore$ y = 25

Substituting the value of y in equation (1), we get,

$5x - 4 \times 25 = -10$

or, $5x = -10 + 100$

or, $5x = 90$

or, $x = \frac{90}{5}$

or, $x = 18$

Hence, the original fraction is $\frac{18}{25}$.

Example 7:

A bus started its Journey from Kathmandu to Nepalgunj at 3pm. with a uniform speed of 40km/hr. After 1 hour, another bus also started its Journey from Kathmandu to the same destination with a uniform speed of 50km/hr. At what time would they meet each other? Find it.

Solution:

Let the first bus travels x hrs. and the second bus travels y hrs. from their starting time. Then, according to the question,

$x - y = 1$ .... (1)

Since the speed of the first bus is 40 km/hr. So, the distance covered by the first bus in x hrs. is 40x km.

Since the speed of the second bus is 50km/hr. So, the distance covered by the second bus in y hrs. is 50 y km. They meet after certain times. so, the distance covered by both buses are equal.

$40x = 50y$

or, $x = \frac{5}{4}y$ .... (2)

Substituting the value of x in equation (1) from equation (2), we get,

$\frac{5}{4}y - y = 1$

or, $5y - 4y = 4$

$\therefore$ y = 4

Then, $x = \frac{5}{4}y = \frac{5}{4} \times 4 = 5$

After traveling 5 hrs. by the first bus, they meet each other. It means they meet each other at 3 PM. + 5 hrs = 8 PM.

Exercise 12.1

1. Solve the following simultaneous equations:

(a) $x + y = 17$
$x - y = 3$

(b) $2x - 5y = 1$
$7x + 3y = 24$

(c) $\frac{x}{4} + \frac{y}{5} = 2$
$\frac{x}{2} + y = -3$

(d) $\frac{4}{x} + \frac{3}{y} = \frac{29}{30}$
$\frac{3}{x} + \frac{5}{y} = 1$

(e) $3^{x+y} = 9$
$2^{x-y} = 1$

2.(a) The sum of two numbers is 29 and their difference is 5. Find the numbers.

(b) The sum of two angles of a triangle is 105° and their difference is 15°. Find the angles.

(c) A number is thrice the other. If their difference is 18, find the numbers.

3.(a) The perimeter of a rectangular field is 150m. If the length of the field is 5m. longer than its breadth, find the area of the field.

(b) The length of a rectangular pond is 15m. less than twice of its width. If the perimeter of the pond is 330m., find its area.

(c) The perimeter of a rectangular piece of a land is 140m. The size of the land is decreased due to the expansion of the road and the new length and breadth of the land are equal to $\frac{17}{20}$ and $\frac{13}{15}$ times of the original length and breadth respectively. If the new perimeter of the land is 120m, find the original length and breadth of the land.

4.(a) The total cost of 2 tables and 3 chairs is Rs. 5100. If the chair is cheaper than table by Rs. 800, find the cost of their unit items.

(b) The total cost of 3kg apples and 5 kg. oranges is Rs. 1080. If the cost of 3 kg. apples is the same as the cost of 7kg. oranges, find the cost of each kg. of both fruits.

(c) A pair of trousers of a school uniform is more expensive than a shirt by Rs. 300. If the total cost of the trousers and the shirt is Rs. 1200, find the cost of each.

5.(a) Two years ago, the father's age was nine times his son's age. But three years later, the father's age will be five times his son's age only. Find their present ages.

(b) Three years ago, the ratio of the ages of two boys was 4:3. Three years hence, the ratio of their ages will be 11:9. Find their present ages.

(c) Two years ago the age of mother was six times as old as her son. Three years hence, she will be 11 years older than 2 times the age of her son. Find their present ages.

6.(a) The sum of the digits of a two digit number is 9. If 27 is subtracted from the number, the digits are reversed. Find the number.

(b) The sum of the digits of a two digit number is 11. The number formed by interchanging the digits of that number will be 45 more than the original number. Find the number.

(c) A number of two digit is 3 more than 6 times the sum of the digits. If the digits are interchanged, the number is decreased by 18. Find the number.

7.(a) The denominator of a fraction is 2 more than its numerator. If 9 is added to the numerator and 3 is subtracted from the denominator, it becomes 2. Find the original fraction.

(b) If 1 is added to the denominator of a fraction, the fraction becomes ½. If 1 is added to the numerator of the fraction, the fraction becomes 1. Find the original fraction.

(c) If the numerator of a fraction is multiplied by 6 and the denominator is reduced by 3, the result becomes 6. If the numerator of the fraction is increased by 12 and 3 is subtracted from the thrice of the denominator, the result becomes 8/9. Find the original fraction.

8.(a) A bus started its journey from Birgunj to Biratnagar at 6 am. with a uniform speed of 50km/hr. After 1 hour, another bus also started its journey from Birgunj to the same destination with a uniform speed of 60km/hr. At what time would they meet each other?

(b) Two runners start from the same point at the same time. They will be 8km. apart at the end of two hours if running in the same direction and they will be 26km. apart at the end of one hour if running in opposite direction. Find their speed..

9. Observe the unit price of any two items in a departmental store. Then make two word problems of simultaneous equations on the basis of that unit price and solve them. After that present your work in the group.

12.2 Quadratic Equation

Let's consider an equation $2x + 6 = 0$. This equation is a linear equation with variable x. But in this topic, we are going to discuss the quadratic equation that means the highest power of the variable contained in the equation is 2. The quadratic equation is also called second degree equation. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b, c $\in$ R, a $\neq$ o and R is the set of real numbers.

Let's consider two quadratic equations.

$x^2 - 25 = 0$ and $2x^2 + 5x + 3 = 0$

The equation $x^2 - 25 = 0$ is called pure quadratic equation. The equation $2x^2 + 5x + 3 = 0$ is called adfected quadratic equation. The quadratic equation can be solved by different methods. The some usual method among them are factorization method, completing square method and using formula. In this topic we will deal the verbal problems which will come in quadratic equation. Let's study the some examples which deal with the word problem related to the quadratic equation.

Example 1:

If 8 is added to the square of a number, the sum is 33. Find the number.

Solution:

Let the number be x, Then, according to question,

$x^2 + 8 = 33$

or, $x^2 = 25$

or, $(x)^2 = (\pm5)^2$

$\therefore x = \pm5$

Hence, the required number is ±5.

Example 2:

If the product of any two consecutive natural numbers is 182, find the numbers.

Solution:

Let two consecutive natural numbers be x and $x + 1$.

Then, according to question

$x(x+1) = 182$

or, $x^2 + x - 182 = 0$

or, $x^2 + (14-13)x - 182 = 0$

or, $x^2 + 14x - 13x - 182 = 0$

or, $x(x + 14) - 13(x + 14) = 0$

or, $(x + 14)(x - 13) = 0$

Either, $x + 14 = 0$ or, $x - 13 = 0$

$\therefore x = -14$ $\therefore x = 13$

According to question, the number is natural number.

So, $x = - 14$ is rejected.

If $x = 13$, then $x + 1 = 13 + 1 = 14$

Hence, the required two consecutive natural numbers are 13 and 14.

Example 3:

The present ages of elder and younger sisters are 15 years and 13 years respectively. In how many years will the product of their ages be 323? Find it.

Solution:

Let after x years the product of their ages will be 323.

Then, $(15 + x)(13 + x) = 323$

or, $195 + 15x + 13x + x^2 - 323 = 0$

or, $x^2 + 28x - 128 = 0$

or, $x^2 + (32 - 4)x - 128 = 0$

or, $x^2 + 32x - 4x - 128 = 0$

or, $x(x + 32) - 4(x + 32) = 0$

or, $(x + 32)(x - 4) = 0$

Either, $x + 32 = 0$ or, $x - 4 = 0$

$\therefore x = -32$ $\therefore x = 4$

Since, x denotes years, it is never negative.

$\therefore x = -32$ is rejected.

Then $x = 4$ is the required solution.

Hence, after 4 years the product of their ages will be 323.

Example 4:

In a two digit number, the product of two digits is 12. If 36 is subtracted from the number, the number will be reversed. Find the number.

Solution:

Let a two digit number be $10x + y$, where x and y are the digits of ten's place and unit place respectively.

Then, according to question,

$xy = 12$ .... (1)

Again, $10x + y - 36 = 10y + x$

or, $9x = 9y + 36$

or, $x = y + 4$ .... (2)

Substituting the value of x in equation (1) from equation (2), we get

$(y + 4).y = 12$

or, $y^2 + 4y - 12 = 0$

or, $y^2 + 6y - 2y - 12 = 0$

or, $y(y + 6) - 2(y + 6) = 0$

or, $(y + 6)(y - 2) = 0$

Either, $y + 6 = 0$ or, $y - 2 = 0$

$\therefore y = -6$ $\therefore y = 2$

The digit of a number is always positive. So, y = -6 is rejected. If y = 2, then $x = 2 + 4 = 6$

Hence, the required two digit number is $10x + y = 10 \times 6 + 2 = 60 + 2 = 62$.

Example 5:

The length of a room is 5ft. longer than its breadth and the area of the room is 150 sq.ft., calculate the perimeter of the room.

Solution:

Let the length and breadth of a room be x ft. and y ft. respectively. Then, according to question,

$x = y + 5$ft. .... (1)

Again, area of the room = 150 sq.ft.

or, $x \times y = 150$ sq.ft. ....(2)

Substituting the value of x in equation (2) from equation (1), we get;

$(y + 5)y = 150$

or, $y^2 + 5y - 150 = 0$

or, $y^2 + 15y - 10y - 150 = 0$

or, $y(y + 15) - 10(y + 15) = 0$

or, $(y + 15)(y - 10) = 0$

Either, $y + 15 = 0$ or, $y - 10 = 0$

$\therefore y = -15$ $\therefore y = 10$

Since, the measurement is never negative. So, y = -15 is rejected.

y = 10 is the solution.

If y = 10, then,

$x = y + 5 = 10 + 5 = 15$

$\therefore$ length of the room = x = 15 ft.

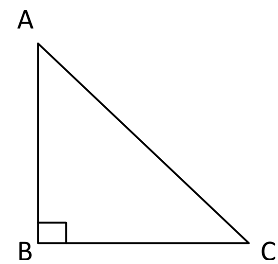
breadth of the room = y = 10 ft.

perimeter of the room = $2(x + y) = 2(15 + 10)$ft. = 50 ft.

Example 6:

The length of hypotenuse of a right angled triangle exceeds the length of the base by 2cm and exceeds twice the length of altitude by 1cm. Find the length of each sides of the triangle.

Solution:



Let ABC be a right angled triangle where ∠B = 90°.

Then hypotenuse (h) = AC, base (b) = BC and altitude (p) = AB

Let, AC = x cm. Then, according to question,

AC = BC + 2 cm

or, x = BC + 2 cm

∴ BC = (x - 2) cm

Again, AC = 2AB + 1 cm

or, x = 2AB + 1 cm

∴ AB = $\frac{x-1}{2}$ cm

Now, by using the Pythagoras theorem,

$h^2 = p^2 + b^2$

or, $AC^2 = AB^2 + BC^2$

or, $x^2 = \left(\frac{x-1}{2}\right)^2 + (x - 2)^2$

or, $x^2 = \frac{x^2-2x+1}{4} + x^2 - 4x + 4$

or, $16x - 16 = x^2 - 2x + 1$

or, $x^2 - 18x + 17 = 0$

or, $x^2 - 17x - x + 17 = 0$

or, $x(x-17) - 1(x - 17) = 0$

or, $(x - 17)(x - 1) = 0$

Either, $x - 17 = 0$ or, $x - 1 = 0$

∴ $x = 17$ ∴ $x = 1$

Hypotenuse = x = 1 is not possible because hypotenuse is greater than base and attitude. So, x = 1 is rejected.

x = 17 is the required solution.

∴ hypotenuse = x = 17 cm

base = x - 2 = 17 -2 = 15 cm

perpendicular = $\frac{x-1}{2} = \frac{17-1}{2} = 8cm$

Exercise 12.2

1. **Solve:**

   (a) $2x^2 - 72 = 0$ (b) $x^2 - 9x + 8 = 0$ (c) $6x^2 - x - 2 = 0$ (d) $2x^2 = 4x+1$
   (e) $5x^2 + 2x - 3 = 0$

2. (a) If 7 is added to the square of a natural number, the sum is 32. Find the number.

   (b) If 5 is added to the half of the square of a natural number, the sum is 37. Find the number.

   (c) The difference of two numbers is 19 and their product is 120. Find the number.

3. (a) If the product of any two consecutive number is 56, find the numbers.

   (b) If the product of two consecutive odd numbers is 143, find the numbers.

   (c) y and y + 2 are two number. If the sum of their reciprocal is 5/12, find the numbers.

4. (a) The present age of two sisters is 14 years and 10 years respectively. In how many years will the product of their ages be 285? Find it.

   (b) The present age of the father and his son is 32 years and 7 years respectively. Find how many years ago the product of their age was 116.

   (c) The product of the present age of two brothers is 160. Four years ago, elder brother was twice as old as his younger brother. Find their present age.

5. (a) In a two digit number, the product of two digits is 20. If 9 is added to the number, the number will be reversed. Find the number.

   (b) The product of digits in a two digit number is 20. The number formed by inter changing the digits of the number will be 9 more than the original number. Find the original number.

   (c) The product of the digits in a two digit number is 15. When 18 is subtracted from the number, the digits interchange their place. Find the number.

6. (a) The length of a rectangular room is 5ft. longer than its breadth. If the area of the room is 66 sq.ft., calculate the perimeter of the room.

   (b) Calculate the length and breadth of a basket ball court whose area is 750 sq.ft. and perimeter is 110 ft.

   (c) The breadth of a rectangular ground is 2m. shorter than its length. If the area of the ground is 48 sq.m., find the perimeter of the ground.

7. (a) The hypotenuse of a right angled triangle is 13 cm. If the sum of its other two sides is 17cm., find the length of the remaining sides.

   (b) The hypotenuse of a right angled triangle is 20cm. If the ratio of the remaining two sides is 1:3, find the length of two sides.

(c) If the sides of a right angled triangle are $(x - 2)$cm, x cm and $(x + 2)$cm, find the length of it each side.

8. Write the perimeter and area of your class room by your amassing. From this imagination, calculate the length and breadth of your class room. compare the actual length and breadth with your calculation from amessination.

Unit: 13

Area of Triangles and Quadrilaterals

13.0 Review

You have studied many things about the triangles and quadrilaterals in the previous classes. In this class you will study about triangles and quadrilaterals having equal areas. The formulas that we use for calculating the areas of the triangles and different quadrilaterals like rectangle, square, parallelogram, rhombus etc. have already been taught to the students in the previous classes. We will use these formulas as well as different other properties of triangles and quadrilaterals. To recall all these things, it is better to start by replying the following questions

(i) What do you mean by a triangle?

(ii) Define a quadrilateral.

(iii) How many types of triangles are there on the basis of sides?

(iv) How many types of triangles are there on the basis of angles?

(v) What is the formula to calculate the area of a triangle?

(vi) What formula do you use for to calculate area of a rectangle?

(vii) How do you calculate the area of a square?

(viii) What do you mean by altitude of a triangle?

(ix) What is the height of a parallelogram?

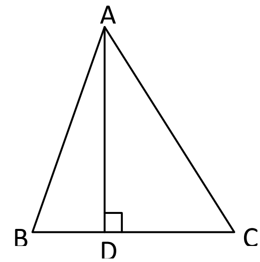
(x) Tell any three properties of a parallelogram.

Triangle:

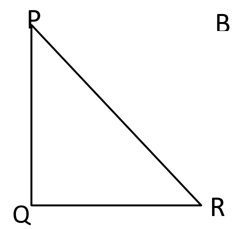
A triangle is a closed plane figure bounded by three line segments. The line segments are called the sides of the triangle. The points at which the two sides meet are called the vertices of the triangle. The horizontal side at which the triangle stands is called its base. The angle opposite to the base is called vertical angle. The perpendicular joining the vertex to the base is called the height or altitude of the triangle. The line segment joining the vertex to the mid-point of the base is called the median of the triangle A triangle has three vertical angles, three medians, three attitudes and three bases. The area of a triangle is generally denoted by Δ(delta)and is given by $\Delta = \frac{1}{2} \times$ height x base.

The areas of the triangles of different shapes are calculated as below:

(i) The area of $\Delta ABC = \frac{1}{2} \times AD \times BC$



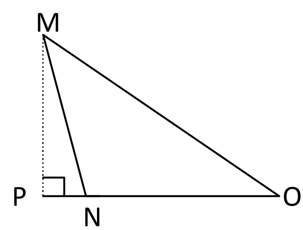
(ii) The area of $\Delta PQR = \frac{1}{2} \times PQ \times QR$



(iii) The area of the triangle MNO is given by

$A = \frac{1}{2} \times \text{base} \times \text{height}$

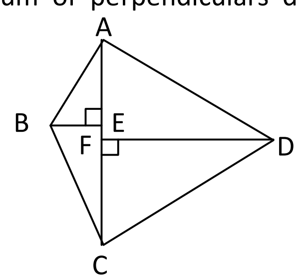
$= \frac{1}{2} \times NO \times MP$



Quadrilateral:

A quadrilateral is a closed plane figure bounded by four line segments. The area of the quadrilateral is given by A= $\frac{1}{2}$[a diagonal x sum of perpendiculars drawn from the opposite vertices to this diagonal].

In the adjoining figure,
the area of the quadrilateral
ABCD (A) = $\frac{1}{2}[AC \times (BE + DF)]$.

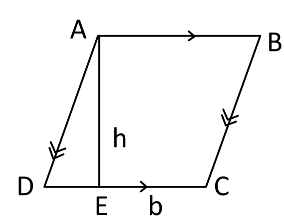


Parallelogram:

A quadrilateral whose opposite sides are parallel is called a parallelogram. The area of a parallelogram is given by:

A = Base $\times$ Height.

i.e A = b $\times$ h

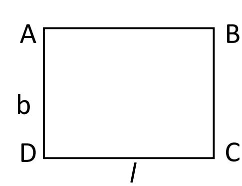


In the adjoining figure,

The area of the parallelogram ABCD = AE $\times$ DC = h $\times$ b
= b $\times$ h

Rectangle:

A rectangle is a parallelogram whose one angle is right angle i.e. 90^0. The area of a rectangle is given by the product of two adjacent sides, i.e. length and breadth.



In the given figure,

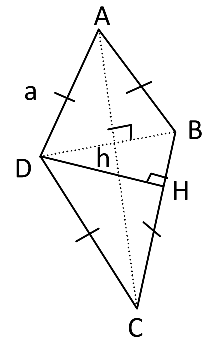
The area of the rectangle ABCD = AD × DC

$= b \times l$

$= l \times b$

Rhombus:

A rhombus is a quadrilateral whose all sides are equal and none of the angles is 90^0. The area of a rhombus is given by; A = height × one side.



In the given figure,

The area of the rhombus ABCD = AD × DH = BC × DH

= One side × height

$= a \times h$

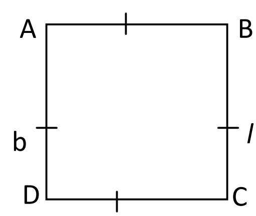
Also the area of a rhombus is one half of the product of its diagonals. In the given figure,

The area of the rhombus ABCD (A) = $\frac{1}{2}$ (AC × BD)

$= \frac{1}{2} d_1 \times d_2$, where d_1 and d_2 are the diagonals.

Square:

A square is a rectangle whose adjacent sides are equal. The area of a square is given by the square of a side.



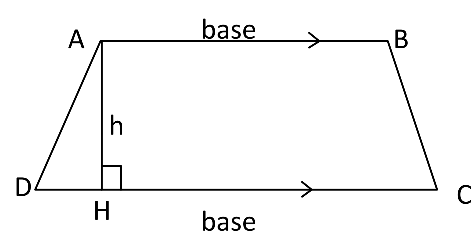
In the given figure, the area of the square ABCD = $(AB)^2 = (BC)^2 = l^2$

i.e. $A = l^2$

Trapezium:

A quadrilateral whose one pair of opposite sides are parallel is called a trapezium. The parallel sides are called its bases and the perpendicular distance between the parallel sides is called its height. The area of a trapezium is given by;

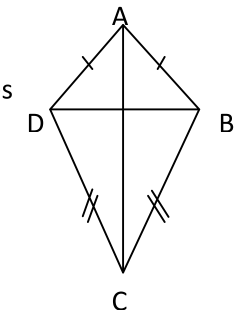
$A = \frac{1}{2} \times [\text{height} \times \text{sum of parallel sides (bases)}]$.



In the given figure, the area of the trapezium ABCD is given by $A = \frac{1}{2} \times AH \times (AB + CD)$

I.e. $A = \frac{1}{2} \times h$ (sum of bases)

Kite: A kite is a quadrilateral whose one of the diagonals separates it into two isosceles triangles of different length of sides.



The area of a convex kite is given by, A = product of its two diagonals divided by two

i.e. A = $\frac{1}{2}$ (product of its two diagonals)

In the given figure, the area of the kite ABCD

is given by, A = $\frac{1}{2} \times (AC \times BD)$

$= \frac{1}{2}$ [The product of diagonals]

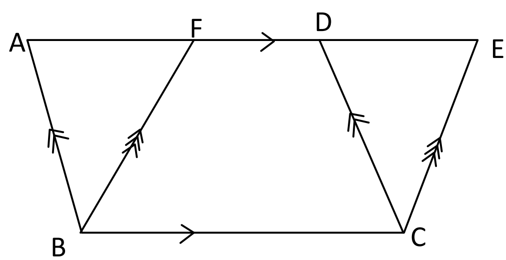
Note: *A kite can also be defined as the quadrilateral having two pairs of adjacent sides of equal lengths where each pair has different side lengths from the other.*

13.1 Theorems related to the areas of triangle and quadrilateral

Theorem - 1

Statement: Parallelograms standing on the same base and lying between the same parallels are equal in area.

Theoretical proof:



Given: Parallelograms ABCD and BCEF are standing on the same base BC and lying between the same parallels AE and BC as shown in the figure.

To prove: Parallelogram ABCD = parallelogram BCEF in area.

Proof:

S.N	Statements	S.N	Reasons
1.	In the ΔABF and CDE	1.	See the figure
(i)	AB = DC (s)	(i)	Opposite sides of parallelogram ABCD
(ii)	∠BAF = ∠CDE (A)	(ii)	Corresponding angles, AB∣∣DC

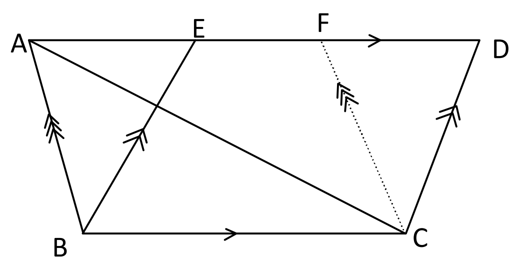
S.N	Statements	S.N	Reasons
(iii)	$\angle$BFA = $\angle$CED (A)	(iii)	Corresponding angles, BF$\|\|$CE
2.	$\therefore \Delta$ABF $\cong \Delta$CED	2.	S.A.A axiom
3.	ΔABF $= \Delta$CED	3.	Congruent triangles are equal in area
4.	ΔABF + Quadrilateral BCDF = ΔCED + quadrilateral BCDF	4.	Adding same quadrilateral BCDF to both sides of (3)
5.	Parallelogram ABCD = parallelogram BCEF	5.	Whole part axiom

Proved.

Theorem - 2

Statement: The area of a triangle is half of the area of a parallelogram standing on the same base and lying between the same parallels.

Theoretical proof:



Given: ΔABC and parallelogram BCDE are standing on the same base BC and lying between the same parallels AD and BC.

To prove: ΔABC = $\frac{1}{2}$ parallelogram BCDE

Construction: Draw CF$\|\|$BA

Proof:

S.N	Statements	S.N	Reasons
1.	AB$\|\|$FC	1.	From construction
2.	AF$\|\|$BC	2.	Given
3.	$\therefore$ ABCF is a parallelogram	3.	Form (1) and (2), opposite sides are parallel
4.	$\therefore$ ΔABC = $\frac{1}{2}$ parallelogram ABCF	4.	Diagonal AC bisects the parallelogram ABCF

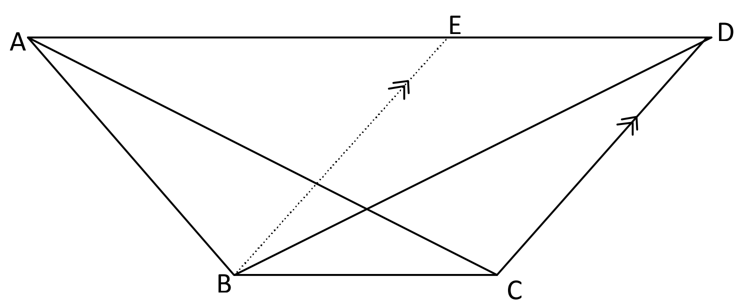
S.N	Statements	S.N	Reasons
5.	ABCF = parallelogram BCDE	5.	Both being on the same base BC and between the same parallels AD and BC (Theorem 1)
6.	∴ ΔABC = $\frac{1}{2}$ parallelogram. BCDE	6.	From statements (4) and (5)
7.	∴ ΔABC = $\frac{1}{2}$ parallelogram. BCDE in area	7.	From (6)

Proved.

Theorem - 3

Statement: Two triangles standing on the same base and lying between the same parallels are equal in area.

Theoretical proof:



1. **Given:** ΔABCand ΔBCD are standing on the same base BC and between the same parallels AD and BC.
2. **To prove:** ΔABC =ΔBCD in area.
3. **Construction:** Draw BE││CD

Proof:

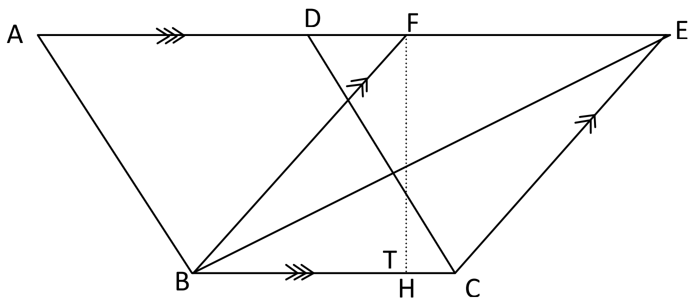
S.N	Statements	S.N	Reasons
1.	BCDE is a parallelogram.	1.	Opposite sides are parallel from construction and given.
2.	∴ ΔBCD =$\frac{1}{2}$ parallelogram BCDE	2.	Diagonal BD bisects the parallelogram. BCDE into two equal triangles.
3.	Also ΔABC =$\frac{1}{2}$ parallelogram BCDE	3.	A triangle is half of the parallelogram. On the same base and between the

S.N	Statements	S.N	Reasons
			same parallel (Theorem 2)
4.	∴ ΔABC =ΔBCD	4.	From statements (2) and (3)

Proved.

Note: The above theorems 1, 2 and 3 can also be proved by alternative methods as below.

Theorem - 1 (Alternative Method)



1. Given: Parallelogram ABCD and parallelogram BCEF are on the same base BC and between the same parallels AE and BC.
2. To prove: Parallelogram ABCD = Parallelogram BCEF in area.
3. Construction: Draw FH⊥BC.

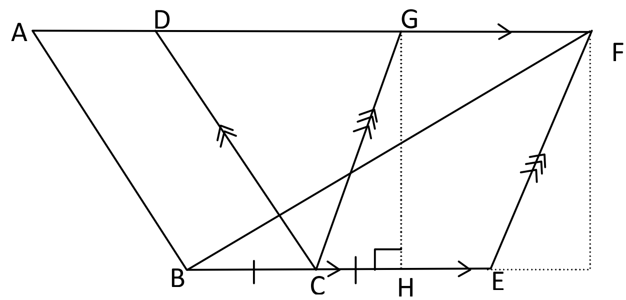
Proof:

S.N	Statements	S.N	Reasons
1.	FH is the height of the parallelogram BCEF	1.	By construction
2.	FH is also the height of the parallelogram ABCD.	2.	AE and BC are parallel and the parallelograms lie between them.
3.	∴ Area of parallelogram ABCD = BC x FH = base x height	3.	The area of the parallelogram is equal to base x height
4.	Area of the parallelogram BCEF = BC x FH	4.	The area of the parallelogram is equal to base x height
5.	∴ Parallelogram ABCD = Parallelogram BCEF	5.	From statements (3) and (4)

Proved.

Prove the theorems 2 and 3 by similar alternative methods yourselves.

Corollary: Parallelograms on the equal bases and between the same parallel are equal in area.



1. Given: Parallelogram ABCD and parallelogram CEFG are on equal bases BC and EC and between the same parallels AF and BE.
2. To prove: Parallelogram ABCD = parallelogram CEFG in area
3. Construction: Draw GH⊥BE.

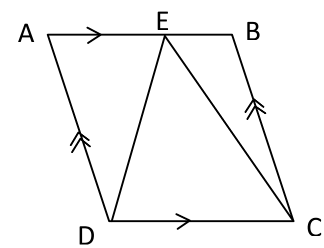
Proof:

S.N	Statements	S.N	Reasons
1.	GH is the height of parallelogram CEFG.	1.	By construction
2.	GH is also the height of the parallelogram ABCD.	2.	Because AF||BC and the parallelograms ABCD and CEFG are between the same parallels.
3.	∴ Area of the parallelogram CEFG = CE x GH	3.	∴ Area of the parallelogram = base x height
4.	Area of the Parallelogram CEFG = BC x GH	4.	From (3),CE = BC is given
5.	But parallelogram BCDA = BC x GH	5.	Area of the parallelogram is equal to base x height
6.	∴ Parallelogram ABCD = Parallelogram CEFG	6.	From (4) and (5)

Proved.

Example 1:

In the given figure, ABCD is a parallelogram whose area is $72cm^2$. If E is any point on AB, find the area of the triangle DEC.



Solution:

The parallelogram ABCD and the triangle DEC are on the same base DC and between the same parallels AB and DC. So the area of the triangle DEC must be half of the area of the parallelogram ABCD. Hence the required area of the triangle DEC = $\frac{1}{2}$ (area of the parallelogram ABCD) = $\frac{1}{2}$ x $72cm^2$ = $36cm^2$.

Example 2:

A triangle and a parallelogram are standing on the equal bases and between the same parallels. If the area of the triangle is 18 sq units, what will be the area of the parallelogram?

Solution:

The parallelogram and the triangle are standing on the equal bases and between the same parallels. So the area of the parallelogram will be twice the area of the triangle. Hence the required area of the parallelogram will be 2 x 18 = 36 sq units.

Example 3:

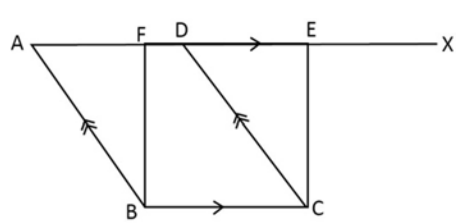
A triangle and a parallelogram are between the same parallels. If the base of the triangle whose area is $48cm^2$ is twice the base of the parallelogram, what will be the area of the parallelogram?

Solution:

Since the triangle and the parallelogram are between the base parallels and the base of the triangle is twice the base of the parallelogram, their areas will be equal. So, the required area of the parallelogram will be $48cm^2$.

Example 4:

Prove that the area of the parallelogram and the square on the same base and between the same parallels is equal.



Solution:

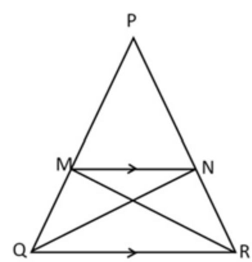
1. Given: Parallelogram ABCD and square BCEF are on the same base BC and between the same parallels A x and BC.
2. To prove: Parallelogram ABCD = Square BCEF in area.

Proof:

S.N	Statements	S.N	Reasons
1.	Area of the square BCEF = FB x BC = BC x BC = BC^2	1.	Formula for the area of the square
2.	FB $\perp$BC	2.	$\therefore$ $\angle FBC =$ 90^0, BCEF being the square.
3.	$\therefore$ Area of parallelogram ABCD = FB x BC = BC x BC = BC^2	3.	Area of the parallelogram = base x height and FB = BC being the sides of square.
4.	$\therefore$Parallelogram. ABCD =square BCEF	4.	From (1) and (3)

Proved.

Example 5:



In the given figure, MN||QR.
Prove that ΔPQN = ΔPRM

Solution:

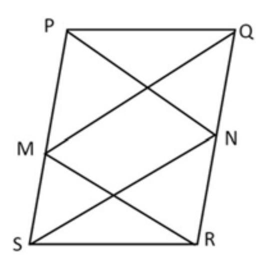
Given: In the triangle PQR, MN||QR

To prove: ΔPQN = ΔPRM

Proof:

S.N	Statements	S.V	Reasons
1.	ΔMNQ = ΔMNR	1.	Δ^son the same base MN and between the same parallels MN and QR.
2.	ΔPMN + ΔMNQ = ΔPMN + ΔMNR	2.	Adding same ΔPMN to both sides of (i)
3.	ΔPNQ = ΔPRM	3.	Whole part axiom

Example 6:



In the given figure, PQRS is a parallelogram.
M and N are any points on PS and QR respectively.
Prove that ΔPMQ + ΔMSR + ΔPNQ + ΔSNR = ▱PQRS

Solution:

1. Given: PQRS is a parallelogram with M and N any points of PS and QR.
2. To prove: ΔPMQ + ΔMSR + ΔPNQ + ΔSNR = parallelogram, PQRS

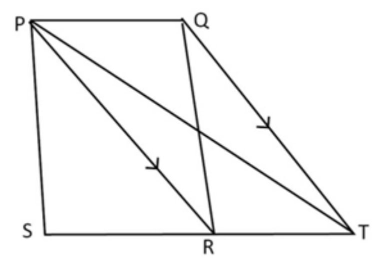
Proof:

S.N	Statements	S.N	Reasons
1.	ΔPNS = $\frac{1}{2}$□PQRS	1.	Both being on the same base PS and between the same parallels PS and QR.
2.	But ΔPNS + (ΔPQN + ΔSNR) = □PQRS	2.	Whole part axiom.
3.	∴ ΔPQN + ΔSNR = $\frac{1}{2}$□PQRS	3.	From (1) and (2)
4	Similarly ΔPMQ + ΔMSR = $\frac{1}{2}$□PQRS	4.	Δ MQR = $\frac{1}{2}$□PQRS and same as 1, 2, 3
5.	∴ ΔPQN + ΔSNR + ΔMSR + ΔPMQ = □PQRS	5.	From (3) and (4)

Proved.

Example 7:

In the given figure, QT||PR.
Prove that ΔPST = Quadrilateral PQRS.



Solution:

1. Given: QT||PR
2. To Prove: ΔPST = Quadrilateral PQRS

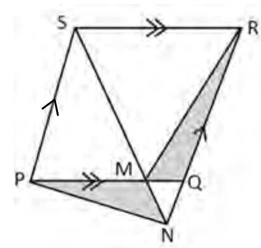
Proof:

S.N	Statements	S.N	Reasons
1.	PR||QT	1.	Given
2.	ΔPQR = ΔPRT	2.	Δ^son the same base PR and between the same parallels PR and QT
3.	∴ $\Delta PQR + \Delta PSR$ =ΔPRT + ΔPSR	3.	Adding the same ΔPSR to both sides
4	Quadrilateral PQRS = ΔPST	4.	Whole part axion

S.N	Statements	S.N	Reasons
	∴ ΔPST= Quadrilateral PQRS.		

Proved.

Example 8:



In the given figure, PQRS is a parallelogram in which RQ is produced upto N and SN and PN are joined where SN meets PQ at M. Prove that the shaded triangles are equal in area.

Solution:

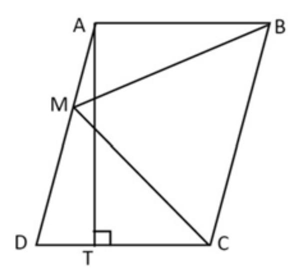
1. Given: PQRS is a parallelogram in which RQ is produced to N such that RN||SP.
2. To prove: ΔPMN = ΔRMQ in area.

Proof:

S.N	Statements	S.N	Reasons
1.	ΔSMR = $\frac{1}{2}$□PQRS	1.	Both being on the same base SR and between the same parallels SR and PQ.
2.	But ΔSMR + (ΔPMS + ΔRMQ) = □PQRS	2.	Whole part axiom.
3.	∴ ΔPMS + ΔRMQ = $\frac{1}{2}$□PQRS	3.	From (1) and (2)
4	But ΔPSN = $\frac{1}{2}$□PQRS	4.	Both being on the same base PS and between the same parallels PS and RN.
5.	∴ ΔPSN = ΔPMS + ΔRMQ	5.	From (3) and (4).
6.	∴ ΔPSN - ΔPSM = ΔPMS + ΔRMQ - ΔPSM	6.	Subtracting same ΔPSM from both sides of 5.
7.	∴ ΔPMN = ΔRMQ	7.	Whole part axiom.

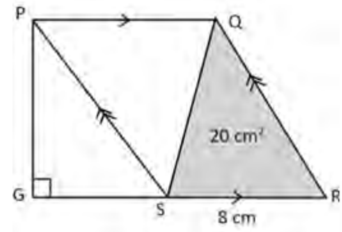
Proved.

Exercise 13

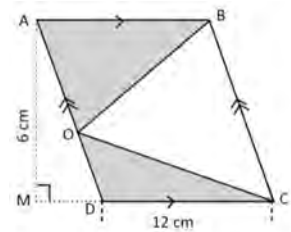


1. In the given figure, ABCD is a parallelogram in which AT⊥DC, AT = 5cm and AB= 4cm. If M be any point on AD, find the area of the triangle BMC.

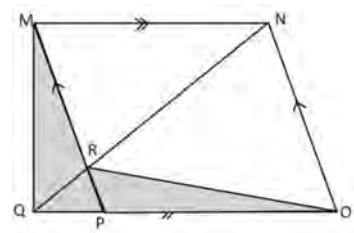
2. In the given figure, PQRS is a parallelogram with SR = PQ =8cm. If the area of the triangle QRS =20cm^2, find the value of PG where PG$\perp$SR.



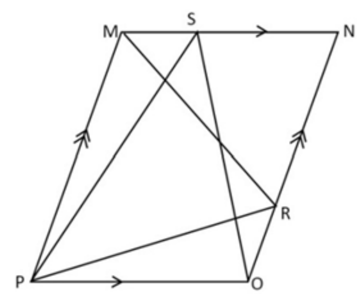
3. Find the height of a trapezium whose area is 100cm^2 and sum of its bases is 20cm.
4. The area of a parallelogram is 75cm^2. If the base of this parallelogram is thrice its height, find the length of the base.
5. If the area of a rhombus is 60cm^2 and one of its diagonals is 12cm, find the other diagonal.
6. Find the area of the shaded region in the given figure.



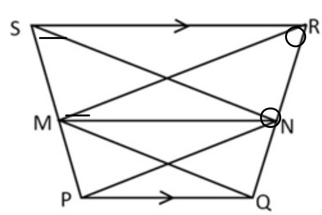
7. In the given figure, MNOP is a parallelogram. R is any point on MP. NR and OP produced meet at Q. Prove that the shaded triangles are equal in area.



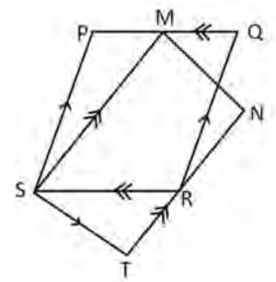
8. In the given figure, MNOP is a parallelogram and S and R are any two points on MN and NO. Prove that ΔPMS + ΔSMP = ΔPRO + ΔMNR



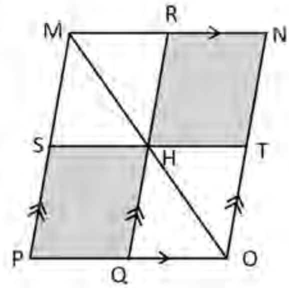
9. In the given figure, PQRS is a trapezium and MN is the median. Prove that ΔPNS = ΔQMR.



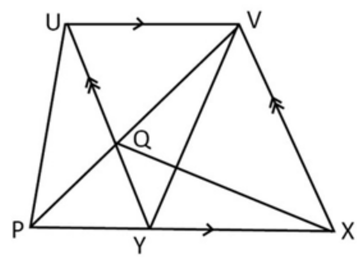
10. In the given figure, PQRS and MNTS are parallelograms. Prove that □PQRS = □MNTS



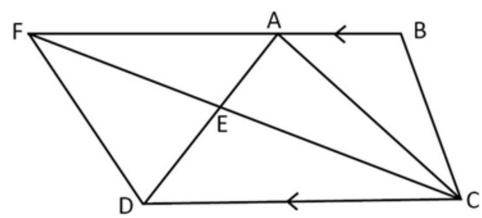
11. In the given figure, MNOP is a parallelogram. RQST and MO meet at H such that RQ||MP and ST||MN. Prove that □RNTH = □PQHS.



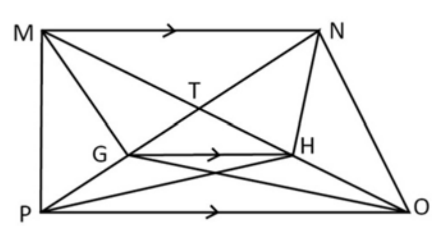
12. In the given figure, UVXY is a parallelogram where Q is any point on UY. Here VQ and XY produced meet at P and UP is joined. Prove that ΔUPY = ΔQPX = ΔVPY



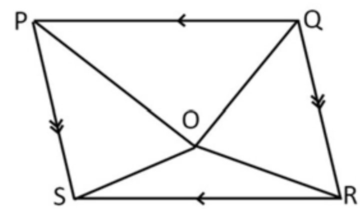
13. In the given figure, ABCD is a trapezium in which AB||CD. E is any point on AD. CE and BA produced meet at F where FD and AC are joined.
Prove that ΔFDC + ΔABC =Trapezium ABCD.



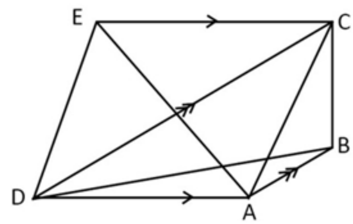
14. In the adjoining figure, MNOP is a trapezium in which MN||PO||GH where G and H are any points on the diagonals PN and OM respectively.
Prove that (i) ΔMGO =ΔNHP
(ii) ΔPTM = ΔTNO



15. In the adjoining figure, PQRS is a parallelogram and 'O' is any point inside it.
Prove that ΔPOS + ΔQOR = ΔPOQ + ΔSOR
$= \frac{1}{2}$□PQRS



16. In the adjoining figure,
EC || DA and DC || AB.
Prove that ΔEDA = ΔCDB



17. Prove the followings:

(a) A parallelogram and a rectangle standing on the same base and between the same parallels are equal in area.

(b) Area of a triangle is half of the area of a rectangle standing on the same base and between the same parallels.

18. BF and CE are the medians of a triangle ABC which meet at the point O. Prove that ΔBOC = Quadrilateral AEOF.

19. Prove that the areas of two triangles standing on the equal bases and between the same parallel lines are equal.

20. Suppose you have a plot of land at home which is not perfectly square, triangular, rectangular or any parallelogram. How will you find its area by using the known formulas of the areas of the triangles and quadrilaterals? Discuss among your friends and prepare a suitable diagram of the process that can be used for the required area.

Unit: 14

Construction

14.0. Review

In the previous unit, we studied about the triangles and quadrilaterals that are equal in area. We proved various theorems based on equality of areas of the triangles and quadrilaterals. Here, in this unit, we will study about the construction of the triangles and the quadrilaterals that are equal in area. For this work, we should have the concepts a fresh that we have got in the previous classes. Basically we should know the following concepts and facts:

(i) The parallelograms on the same base (or equal bases) and between the same parallels are equal in area.

(ii) A triangle is equal to one half of the parallelogram standing on the same base (or equal bases) and lying between the same parallels

(iii) Triangles standing on the same (or equal bases) and between the same parallels are equal in area.

Using these facts, we will perform the following construction of triangles and quadrilaterals that are equal in area.

14.1 Constructions of triangles and quadrilaterals that are equal in area. For the successful construction of the triangles and quadrilaterals having equal area, we need to adopt the following steps:

(i) At first, draw a tentative sketch of the triangle and quadrilateral according to the given data.

(ii) Analyse the given data and mark it on the rough sketch.

(iii) Construct a correct figure by using the given data and the previous concepts mentioned above. You should also know how to draw, triangles, quadrilaterals and parallelogram

(a) Construction of Parallelograms equal in area.

Example 1:

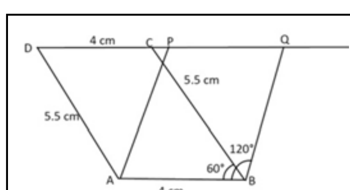
Construct a parallelogram ABCD with AB = 4cm, BC= 5.5cm and

$\angle ABC = 60^0$. Then construct a parallelogram PQBA whose one angle is 120^0 and area is equal to that of parallelogram. ABCD.

Solution:

Construction Process

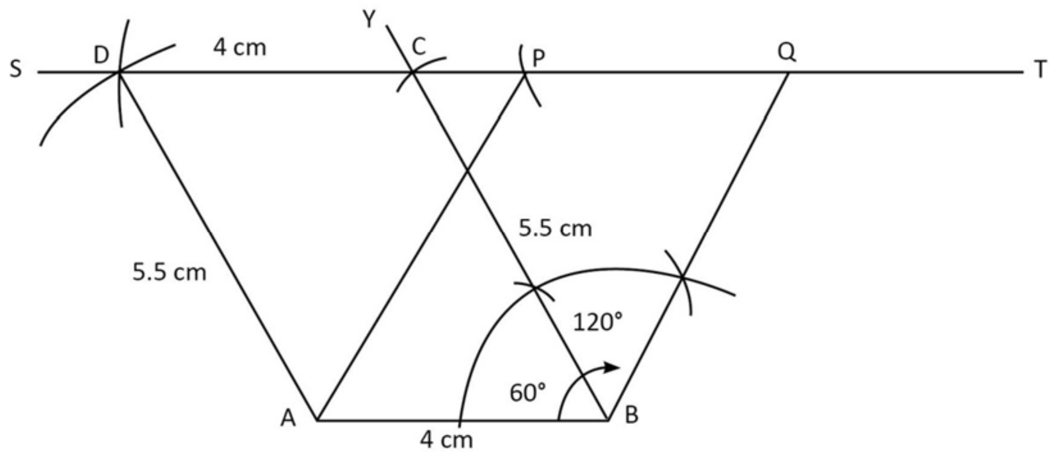
(i) Draw a rough sketch as shown in the figure.



Tentative sketch

(ii) Construct a parallelogram ABCD with the given data as shown in the figure.

(iii) Draw an angle of 120^0 at the point B meeting ST at Q.



(iv) Take an arc of radius AB from the point Q on the line ST cutting it at P.

(v) Join PA.

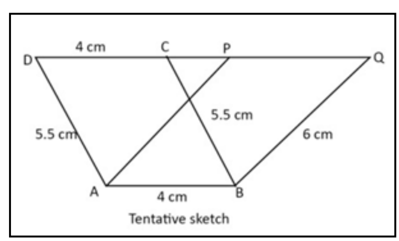
Here, parallelograms ABCD and PQBA are on the same base AB and between the same parallels ST and AB. So they must be equal. Thus, PQBA is the required parallelogram.

Example 2:

Construct a parallelogram ABCD having AB = 4cm, BC = 5.5 cm and $\angle ABC = 60^0$. Then construct another parallelogram PABQ equal to parallelogram ABCD and having one side 6cm.

Solution: Construction Process:

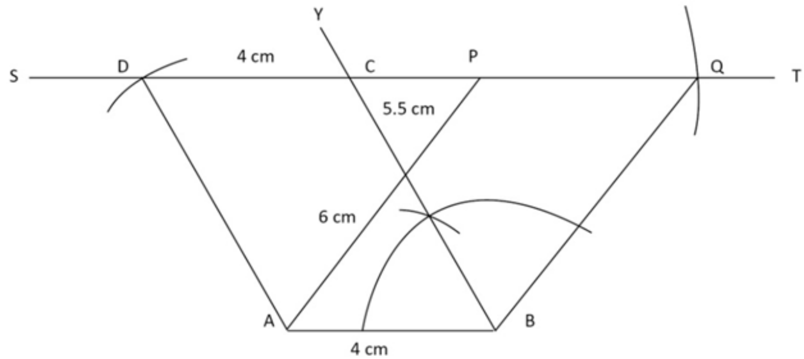
(i) Draw a tentative figure showing the given data.



(ii) Construct a parallelogram ABCD according to the given data.

(iii) Take an arc of radius 6cm from the point A cutting the line ST at P join PA.

(iv) Take an arc of radius AB=4cm from the point P on the line ST cutting it at the point Q. Join BQ.



Here, the parallelograms ABCD and PABQ are on the same base AB and between the same parallels AB and ST. So they must be equal in area. Hence PABQ is the required parallelogram.

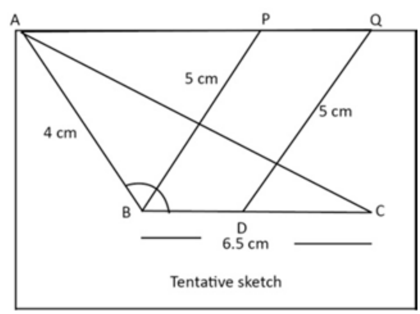
(b) Construction of triangles and parallelograms equal in area.

Example 3:

Construct a triangle ABC in which AB = 4cm, BC = 6.5cm and $\angle ABC = 120^0$. Hence construct a parallelogram PBDQ where PB = 5cm and which is equal to ΔABC in area.

Solution:

Construction Process:



(i) Construct a rough figure tentatively as in the give figure.

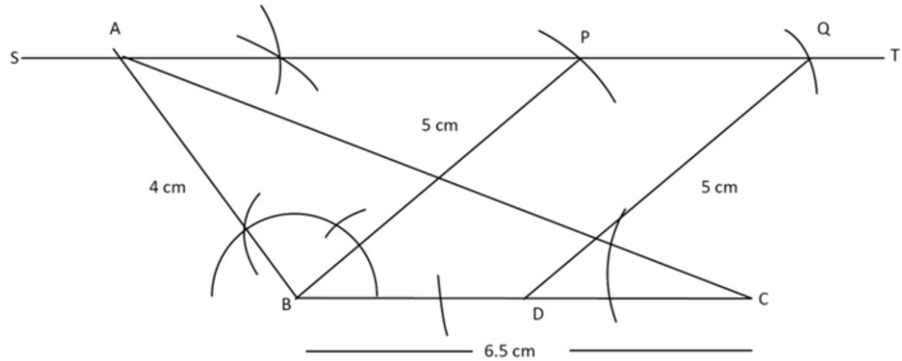
(ii) Construct a triangle ABC according to the given data. Draw ST || BC from A making ∠BCA = ∠CAT.

(iii) Divide BC at the point D.

(iv) Take an arc of radius 5cm from the point B cutting ST at P.

(v) Cut off PQ = BD on the line ST.

(vi) Join PB and BD.



Here, triangle ABC and the parallelogram PQDB are between the same parallels ST and BC. Also BD = DC. So the triangle ABC and the parallelogram PQDB must be equal in area. Thus, PQDB the required parallelogram.

Example 4:

Construct a parallelogram ABCD where AB = 5cm, BC = 4cm and ABC = $\angle 60^0$. Then construct a triangle PBE in which PB = 5.6cm. Such that ΔPBE = parallelogram ABCD.

Solution:

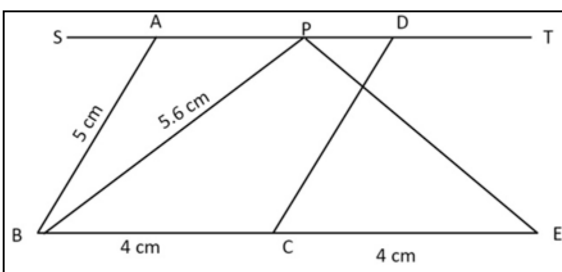
(i) Draw a rough figure according to the given data tentatively.

(ii) Draw a parallelogram ABCD according to the given data.

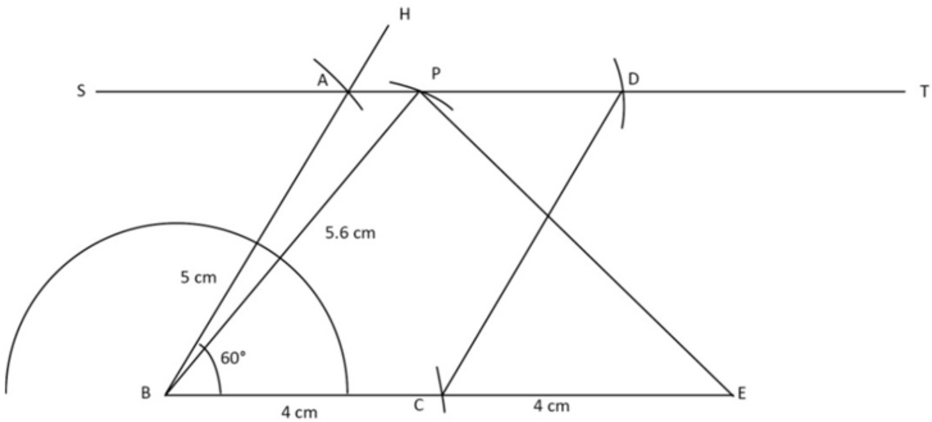
(iii) Produce BC to E making BC = CE.

(iv) Take an arc of radius 5.6cm from the point B cutting ST and P.

(v) Join PE and PB.



Tentative sketch



Here, the parallelogram ABCD (i) and the triangle PBE are between the same parallels ST and BE and BC = $\frac{1}{2}$BE. So they must be equal in area. Thus, PBE is the required triangle.

(c) Construction of triangles equal in area.

Example 5:

Construct a triangle ABC having $\angle ABC = 60^0$, BC = 4.4cm and AB = 5.2cm. Then construct and other triangle DBC equal to ΔABC and having DB = 6.2cm.

Solution:

Construction process:

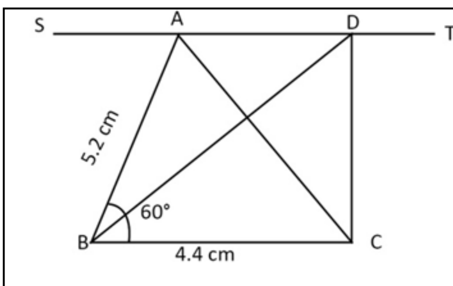
(i) First construct a rough figure tentatively as shown in the figure.

(ii) Construct the triangle ABC according to the given data as in previous classes.

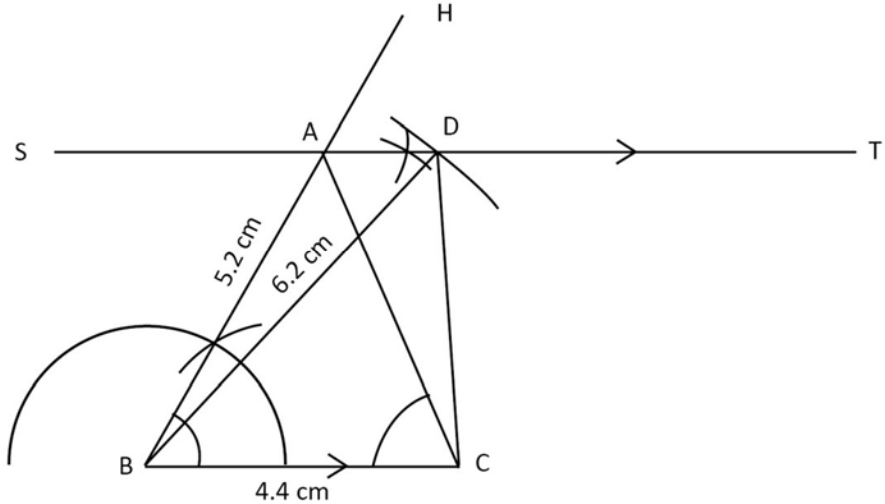
(iii) Draw a line ST || BC passing through the point A.

(iv) Take an arc of radius 6.2cm and centre B cutting ST at D

(v) Join DC.



Tentative sketch



Here, Δ^sABC and DBC are on the same base BC and between the same parallels BC and ST. So they must be equal in area. Thus DBC is the required triangle.

(D) Construction of a triangle and a quadrilateral equal in area.

Example 6:

Construct a quadrilateral PQRS in which PQ = 5cm, PS = 4cm, QR = 4.4 cm, RS = 3.6 cm and the diagonal PR = 6.4 cm. Then construct a triangle PSG equal in area to the quadrilateral PQRS.

Construction process:

(i) Draw a rough figure showing the given data tentatively.

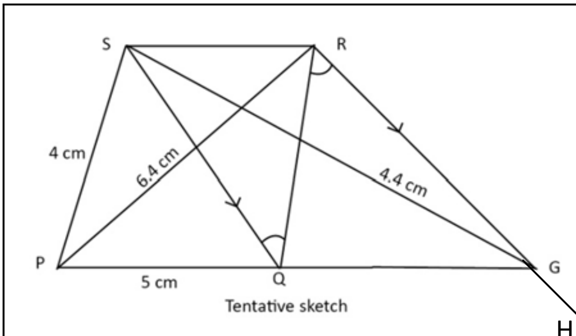
(ii) Construct a quadrilateral PQRS according to the given data as shown in the figure.

(iii) Join the diagonal QS.

(iv) Draw RH || SQ making $\angle$SQR = $\angle$QRH.

(v) Produce PQ to meet RH at G.

(vi) Join SG.



Tentative sketch

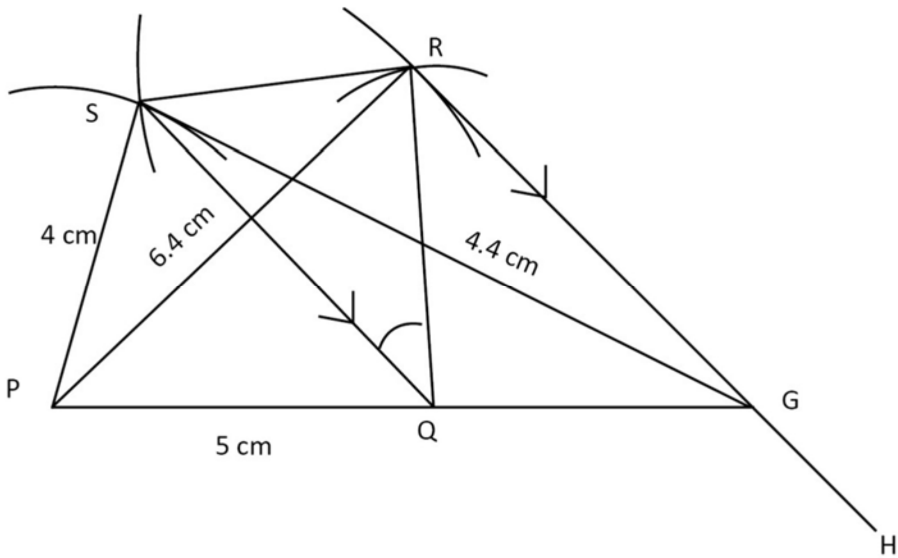
Here, (I) ΔSQG = ΔSQR

(ii) ΔSQR + ΔPSQ = Quadrilateral PQRS

(iii) ΔSQG + ΔPSQ = ΔPSG

(iv) $\therefore$ Quadrilateral PQRS = ΔPSG

Hence ΔPSG is the required triangle.



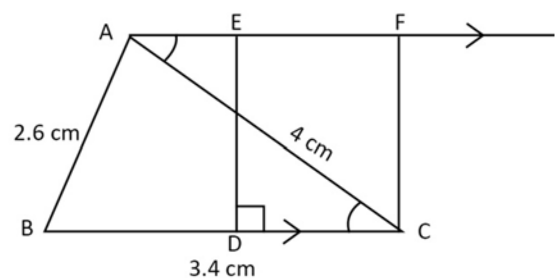
(D) Construction of a triangle and a rectangle equal in area.

Example 7:

Construct a triangle ABC having AB = 2.6cm, BC = 3.4cm and CA = 4cm. Then construct a rectangle equal to ΔABC in area.

Solution:

Construction process:



(i) Draw a rough figure according the given data tentatively.

(ii) Draw a triangle ABC according to the given data as shown in the figure.

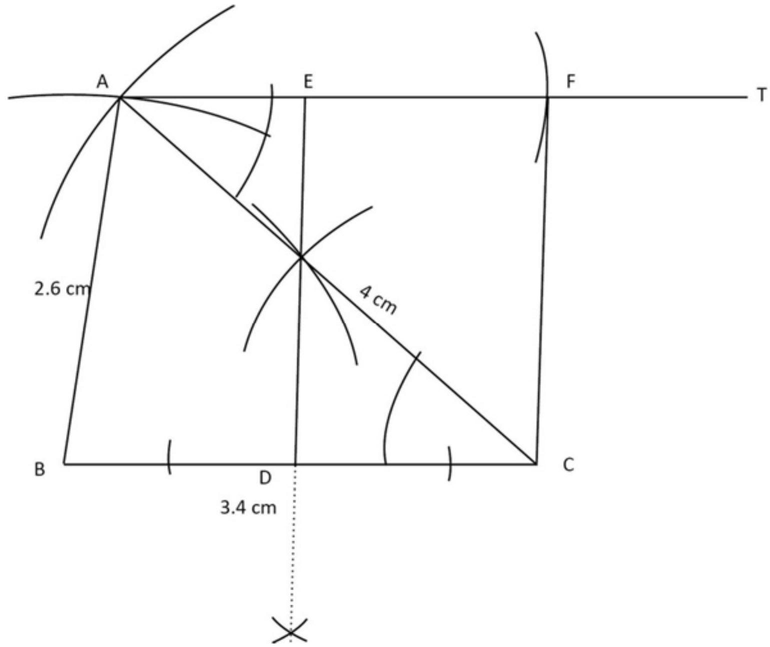
(iii) Bisect BC and mark the mid-point by D as shown in the figure.

(iv) Draw AT || BC through A making $\angle ACB = \angle CAT$.

(v) Draw 90^o angle at D cutting AT at E.

(vi) Cut off EF = DC on the line AT.

(vii) Join FC.



Here, the triangle ABC and the rectangle CDEF are on the bases BC and DC where BC = 2DC and they are lying between the same parallels. So they must be equal in area. Thus, CDEF is the required rectangle.

Exercise 14

1.(a) Construct a parallelogram ABCD in which AB=4.2 cm ,BC =5.4 cm, and $\angle ABC = 120°$ Then construct a parallelogram PABQ whose one angle is 60 and which is equal to parallelogram ABCD in area.

(b) Construct a parallelogram ABCD in which AB=4cm ,BC = 5cm, $\angle ABC = 45°$. Then construct another parallelogram PABQ having one side 6.4 cm and equal in area to the parallelogram ABCD.

(c) Construct a parallelogram ABCD in which AB=4cm, AD=6cm and $\angle BAD=45°$. Then construct another parallelogram having one angle 60 and equal area to that of the parallelogram ABCD.

2.(a) Construct a triangle ABC in which AB=3.8cm,BC=6.4cm and $\angle ABC=75°$. Then construct a parallelogram PBDQ having PB=5.4cm and equal in area to the triangle ABC.

(b) Construct a parallelogram PQRS in which PQ= 4.8 cm , QR=3.6cm and $\angle PQR = 45°$. Then construct a triangle AQE having AQ=5.2 cm and equal in area to the parallelogram PQRS.

(c) Construct a triangle ABC in which b = 5cm, c = 4.8cm and $\angle ABC = 45°$. Then construct a parallelogram having one side CD = 7.4cm and equal to the triangle ABC in area.

3.(a) Construct a triangle ABC having $\angle ABC = 30°$, BC = 4.6cm and AB = 5.6cm. The construct another triangle PBC equal to ΔABC and having PB = 6cm.

(b) Construct a triangle PQR in which PQ = 2.6cm, QR = 4cm and PR = 3.8cm. Then construct another triangle DPR having $\angle DPR = 75°$ and equal in area to ΔPQR.

(c) Construct a triangle EFG in which $\angle EFG = 60^o$, $\angle EGF = 30°$ and FG = 6cm. Then construct another triangle PFG having PF=7.4cm and equal in area to the triangle EFG.

4.(a) Construct a quadrilateral ABCD in which AB=4.8cm, BC=3.6cm,CD=4.2cm,DA=3cm and diagonal BD=6cm.Then construct a triangle DAG equal in area to the quadrilateral ABCD.

(b) Construct a quadrilateral PQRS in which PQ = 5cm, QR = 5.5cm, SP = 7cm and $\angle PQR = 75°$. Then construct a triangle PST equal to the quadrilateral PQRS in area.

(c) Construct a triangle DAE equal to a quadrilateral ABCD in which AB = 4.5cm, CD = 5.7cm, DA = 4.9cm and BD = 5.8cm.

5.(a) Construct a triangle QRT equal to the quadrilateral PQRS in which PQ = 5cm, QR = 9.6cm, Rs = 4.5cm, S.P=5.4cm and QS = 6.5cm.

(b) Construct a triangle with one angle 45° and equal in area to a rectangle having 6cm length and 4.4cm breadth.

(c) Construct a parallelogram with one angle 60° and equal in area to a rectangle having 6cm length and 4cm breadth.

(d) Construct a parallelogram having one side 5cm and equal in area to a triangle having sides 5cm, 6cm, and 8cm.

6. Collect cotton or nylon threads or any wooden pieces of uniform thickness and cardboard papers. Prepare the shapes of triangles, parallelograms and quadrilaterals on the cardboard by using the above low cost materials. Then prepare at least three samples of constructions showing two parallelograms, two triangles and one triangle and one parallelogram equal in areas. Also use rubber bands to prepare a sample of a triangle and a quadrilateral equal in area on the cardboard. [You may require pins and gum for this illustration.]

7. Collect different threads, juice pipes, wires, rubber bands and cardboard papers. Use all these low cost materials to prepare a sample of a quadrilateral ,a triangle and a parallelogram equal in area on the cardboard .Present your samples to the class in groups. Remember that the quadrilateral, the triangle and the parallelogram that are equal in area must be well highlighted.

Unit: 15

Circle

15.0 Review

We have studied many properties and formulas related to the circle. At the present text, we will study more about definitions, properties and theorems related to the circle. For all this, the following questions and their answers become relevant.

(i) What is a circle?

(ii) What are its different parts?

(iii) What are the relations between its different parts?

(iv) What is the formula for calculating the area of a circle?

Well, a geometrical path traced out by a moving point in a plane such that its distance from some fixed point always remains equal or constant is called a circle. The fixed point is called the centre of the circle and the constant distance is called the radius of the circle. The outer path is called the circumference.

A circle can also be defined as a closed plane figure bounded by a curve such that every point of the curve remains equidistant from a fixed point. The fixed point is called the centre of the circle, the constant distance is called the radius of the circle and the bounding curve is called the circumference of the circle.

15.1 Some definitions

Chord:

A line segment joining any two points of the circumference is called the chord of a circle.

Diameter:

The longest chord passing through the centre of a circle is called its diameter. It is twice the radius of the circle.

Semicircle:

One of two parts of a circle made by its diameter is said to be a semi-circle.

Segments:

Two unequal parts of a circle made by a chord are called its segments. The larger one is called major segment and the smaller one is called minor segment.

Sector:

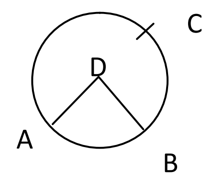
The area of a circle between any two radii is called its sector. The greater one is called major sector and the smaller one is called minor sector.

Central angle:

The angle subtended by an arc at the centre of a circle is called its central angle. It is the angle made by any two radii of the circle at its centre. The central angle and its opposite arc are equal in degree measure.

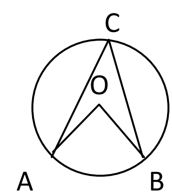
Arc:

A part of the circumference of a circle is called an arc. In the given figure, arc AB and arc ACB are written as $\widehat{AB}$ and $\widehat{ACB}$ respectively. An arc is measured in terms of the angle subtended by it at the centre of the circle. This is known as degree measure of an arc. In the given figure, $\widehat{AB} \doteq \angle AOB$, It means: arc AB is equal to $\angle AOB$ in degree measure. It is read as 'arc AB equals in degree to $\angle AOB$. The degree measure of an arc is also written by using ' $\equiv$' sign in place of '$\doteq$' sign as $\widehat{AB} \equiv \angle AOB$ in the above figure.



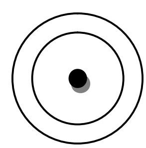
Inscribed angle:

An angle subtended by an arc of a circle at its circumference is called an inscribed angle. An inscribed angle is an angle made by any two chords of a circle at its circumference. The inscribed angle is also called an angle at the circumference. In the given figure, $\angle ACB$ is an inscribed angle and $\angle AOB$ is a central angle standing on the same arc AB. The inscribed angle is half of the opposite arc in degree measure. Here $\angle ACB = \frac{1}{2}\widehat{AB}$



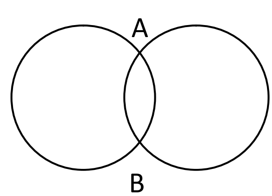
Concentric circles:

Two or more than two circles having same centre are called concentric circles.



Intersecting circle:

Two circles which intersect each other at two distinct points are called intersecting circles.



Con-cyclic points:

Two or more than two points which lie on the circumference of the same circle are called con-cyclic points.

Cyclic quadrilateral:

A quadrilateral whose all vertices lie on the circumference of a circle is called a cyclic quadrilateral.

In the given figure ABCD is a cyclic quadrilateral and A, B, C and D are the con cyclic points.

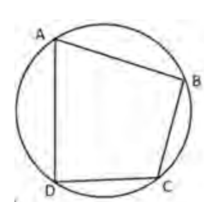
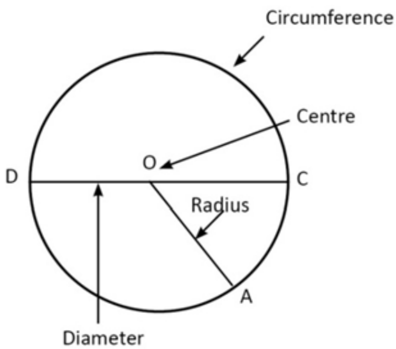
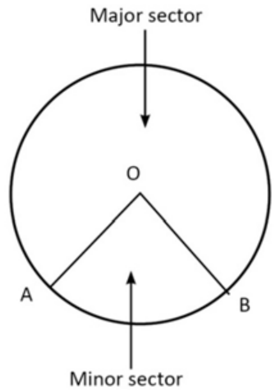
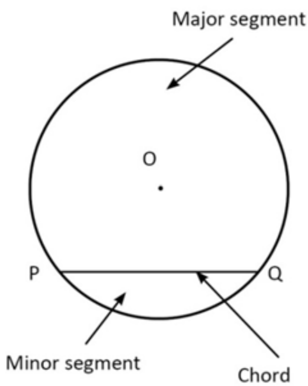
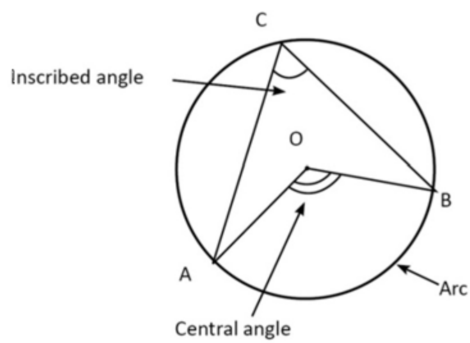


Illustration of Different parts of a circle.







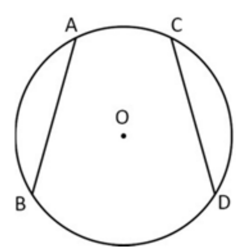


15.2 Concepts and experimental verifications of theorem

Theorem - 1

In a circle, equal chords cut off equal arcs.

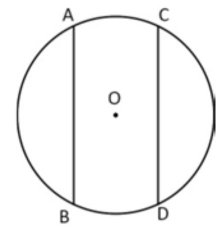
This theorem can be verified and proved experimentally and theoretically both. But in our course we have only its concept which can be understood well with the help of the given figure.



In the given figure, AB and CD are equal chords. They both make two parts of the whole circumference. The parts cut by them are equal correspondingly. That is to say that arc AB = arc CD i.e. $\widehat{AB}$ = $\widehat{AD}$. For more understanding, we can measure the lengths of $\widehat{AB}$ and $\widehat{CD}$ with the help of a thread. We can easily see that they are equal in length.

Theorem- 2: (Converse of theorem 1)

Two equal arcs of a circle subtend two equal chords in that circle. For this theorem also, we do not have to verify or prove experimentally or theoretically. What we need to do is to understand and retain the above statement with the help of the given figure.



In the given figure, arc AB and arc CD i.e. $\widehat{AB}$ and $\widehat{CD}$ are equal in length i.e. $\widehat{AB} = \widehat{CD}$. So both of them subtend equal chords AB and CD i.e. chord AB = chord CD.

Theorem- 3

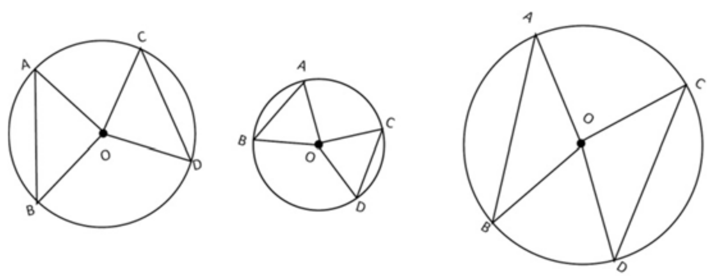
Two arcs, subtending equal angles at the centre of a circle, are equal in length.

Experimental verification

Procedure:

(i) Draw three circles of different radii as shown in the figure, mark the centre by O in each circle.

(ii) Draw two central angles ∠AOB and ∠COD in each circle such that ∠AOB = ∠COD



(iii) Measure the lengths of corresponding arcs AB and CD in each circle by using a thread.

(iv) Write above measurements in the following table.

Figure	Length of $\widehat{AB}$	Length of $\widehat{CD}$	Result
I			
Ii			
Iii			

Conclusion:

From the above table of measurement, we can conclude that the two arcs which subtend equal angles at the centre of a circle are equal in length.

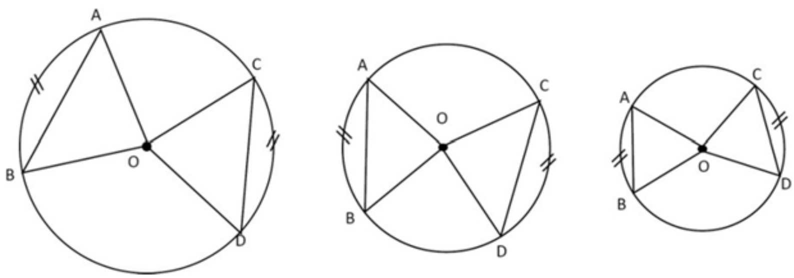
Theorem- 4: (Converse of theorem 3)

Two equal arcs of a circle subtend equal angles at the centre of the circle.

Experimental verification

Procedure:

(i) Draw three circles of different radii with 'O' as the centre as shown in the figure.



(ii) Cut off two equal arcs AB and CD in each circle by using a compass.

(iii) Join AO, BO, CO and DO in each of the circles.

(iv) Measure the central angles AOB and COD in each circle.

(v) Fill up the following table by the above measurement.

Figure	∠AOB	∠COD	Result
I			
Ii			
Iii			

Conclusion:

From the measurement shown in the above table, we can conclude that equal arcs of a circle subtend equal angles at the centre of the circle.

Theorem – 5

Inscribed angles of a circle standing on the same arc are equal.

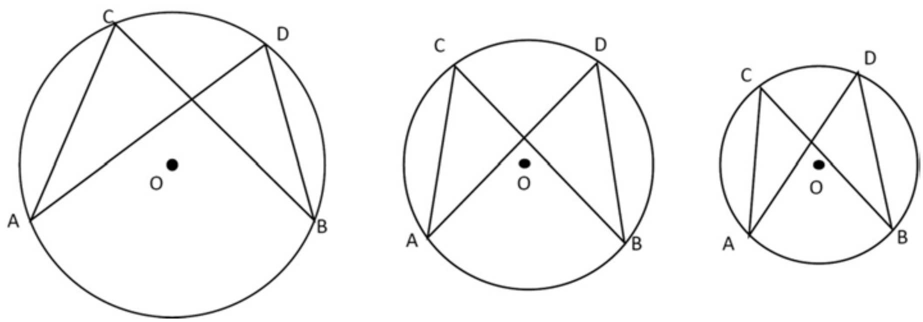
Or, In a circle, inscribed angles standing on the same arc are equal.

Or, Angles in the same segment of a circle are equal.

Experimental verification

Procedure:

(i) Draw three circles of different radii with centre 'O' as shown in the figure.



(ii) Draw two inscribed angles ACB and ADB in each circle standing on the same arc AB.

(iii) Measure the angles ACB and ADB with the help of a protractor.

(iv) Tabulate the above measurement as below:

Figure	$\angle ACB$	$\angle ADB$	Result
i.			
ii.			
iii.			

Conclusion:

From the above table, we can say that inscribed angles standing on the same arc are equal.

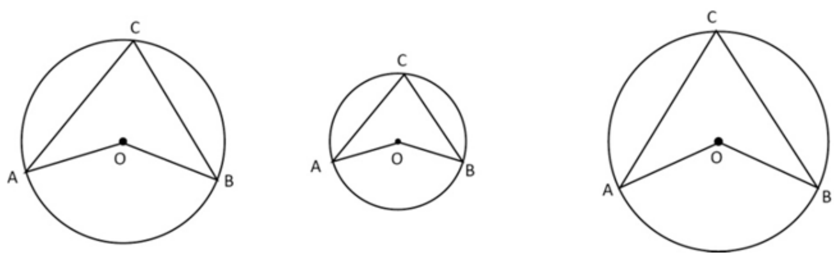
Theorem 6:

In a circle, the inscribed angle is half of the central angle standing on the same arc.

Experimental verification:

Procedure:

(i) Draw three circles of different radii with centre 'O'.



(ii) In each figure, draw an inscribed angle ACB and the central angle AOB standing on the same arc AB.

(iii) Measure the angles ACB and AOB in each figure by using protractor.

(iv) Tabulate the above measurement in the following table.

Figure	**∠ACB**	**∠AOB**	**Result**
I			
Ii			
Iii			

Conclusion:

From the above table, it has been verified that the inscribed angle is half of the central angle standing on the same arc.

Theorem – 7

The opposite angles of a cyclic quadrilateral are supplementary.

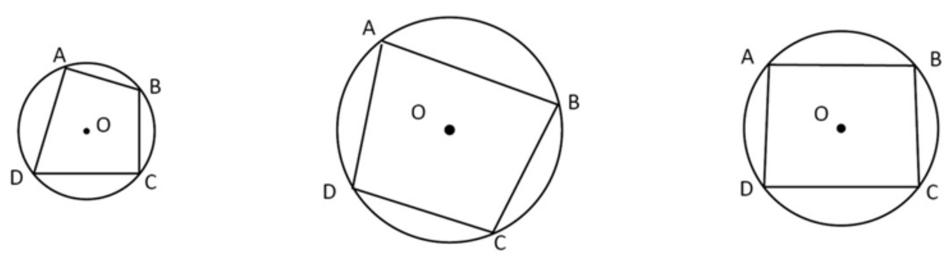
Or, the sum of opposite angles of a cyclic quadrilateral is 180^0.

Experimental verification

Procedure:

(i) Draw any three circles of different radii with centre 'O' as shown in the figure.

(ii) Draw a cyclic quadrilateral ABCD in each circle.



(iii) Measure the ∠S, ∠A, ∠B, ∠C and ∠D with the help of a protractor.

(iv) Prepare a table of above measurement as below:

Figure	∠A	∠B	∠C	∠D	∠A + ∠C	∠B + ∠D	Result
i							
ii							
iii							

Conclusion:

From the above table, it has been verified that the opposite angles of a cyclic quadrilateral are supplementary.

15.3 Theoretical proofs of the theorems

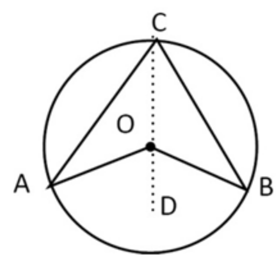
Theorem - 8

The central angle of a circle is double of the inscribed angle standing on the same arc

Or, In a circle, the angle at the circumference is half of the angle at the centre subtended by the same arc.

Given:

In the figure 'O' is the centre of the circle, ∠AOB is the central angle and ∠ACB is the inscribed angle standing on the same arc AB.



To prove: $\angle ACB = \frac{1}{2}\angle AOB$

Construction:

Join CO and produce it to any point D.

Proof:

S.N	Statements	S.N	Reasons
1.	In the triangle AOC, ∠CAO = ∠OCA	1.	OA = OC, base angles of isosceles triangle AOC
2.	In the triangle OCB, ∠OCB = ∠OBC	2.	OC = OB being the radii of the same circle and same as (1)
3.	But ∠AOD = ∠OAC + ∠ACO	3.	The relation of an external angle with the opposite interior angles of a triangle.
4.	Similarly ∠BOD = ∠OBC + ∠OCB	4.	Same as reason (3)
5.	∠AOD + ∠BOD = ∠OAC + ∠OCA +∠OBC + ∠OCB	5.	Adding (3) and (4)

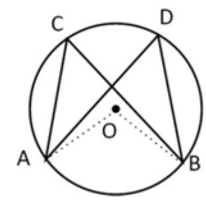
6.	$\angle AOB = \angle OAC + \angle OAC + \angle OCB + \angle OCB$	6.	From (i) and (4)
7.	$\angle AOB = 2\angle OAC + 2\angle OCB = (\angle OAC + \angle OCB)$	7.	From (6)
8.	But $\angle OAC + \angle OCB = \angle ACB$	8.	Whole part axiom
9.	$\therefore \angle AOB = 2\angle ACB$	9.	From (7) and (8)

Proved.

Theorem -9

The angles in the same segment of a circle are equal. Or

The inscribed angles standing on the same arc of a circle are equal



Given: In the figure, O is the center of a circle and $\angle ACB$ and $\angle ADB$ are the angles at the circumference standing on the same arc AB.

To prove: $\angle ACB = \angle ADB$

Construction: Join AO and BO.

Proof:

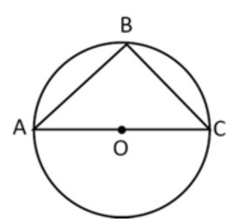
S.N	Statements	S.N	Reasons
1.	$\angle ACB = \frac{1}{2} \angle AOB$	1.	The relation of the central and inscribed angles subtended by the same arc.
2.	$\angle ADB = \frac{1}{2} \angle AOB$	2.	The inscribed angles is half of the central angle standing on the same arc.
3.	$\therefore \angle ACB = \angle ADB$	3.	From (1) and (2)

Proved.

Theorem - 10

An angle in a semi-circle is a right angle.

Or, An inscribed angle in a semi- circle is a right angle.



Given: In the figure alongside, 'O' is the centre, AC is the diameter and $\angle ABC$ is an angle in a semi – circle.

To prove: $\angle ABC = 90^0$

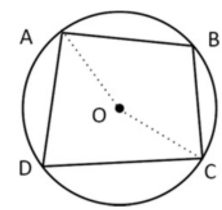
Proof:

S.N	Statements	S.N	Reasons
1.	$\angle ABC = \frac{1}{2} \angle AOC$	1.	Relation of inscribed angle and central angle standing on the same arc AC.
2.	$\angle AOC = 180^0$	2.	Straight angle, AC being the straight line
3.	$\therefore \angle ABC = \frac{1}{2} 180^0 = 90^0$	3.	From (1) and (2)

Proved.

Theorem - 11

The opposite angles of a cyclic quadrilateral are supplementary.



Given: In the drawn circle, O is the centre and ABCD is the cyclic quadrilateral.

To prove: $\angle DAB + \angle DCB = 180^0$ and $\angle ABC + \angle ADC = 180^0$.

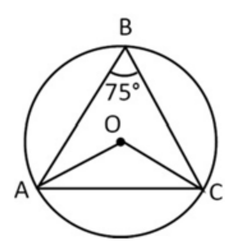
Construction: Join AO and CO.

Proof

S.N	Statements	S.N	Reasons
1.	$\angle ABC = \frac{1}{2}$ reflex $\angle AOC$ and $\angle ADC = \frac{1}{2}$ obtuse $\angle AOC$	1.	Relation of inscribed angle and central angle standing on the same arc AC.
2.	$\angle ABC + \angle ADC = \frac{1}{2}$ (reflex $\angle AOC$ + obtuse $\angle AOC$)	2.	Adding both sides of (1)
3.	$\therefore \angle B + \angle D = \frac{1}{2}(360^0) = 180^0$	3.	$\therefore \angle AOC$ + reflex $\angle AOC = 360^0$ i.e. the total angle at O = 360^0
4.	But $\angle A + \angle B + \angle C + \angle D = 360^0$	4.	The sum of all angles of a quadrilateral.
5	$\therefore \angle A + \angle C = 180^0$	5.	From (3) and (4)

Examples 1:

In the given figure, $\angle ABC = 75^0$ and 'O' is the centre of the circle. Find the values of (i) $\angle AOC$ (ii) $\angle OAC$ (iii) $\angle OCA$



Solution:

In the given figure,

$2\angle ABC = \angle AOC$ (Inscribed angle = $\frac{1}{2}$ central angle standing on the same arc)

Or, $2 \times 75^0 = \angle AOC$

$\therefore \angle AOC = 150^0$

Again, $\angle OAC = \angle OCA$ (Base angles of isosceles ΔAOC)

But, $\angle AOC + \angle OAC + \angle OCA = 180^0$.

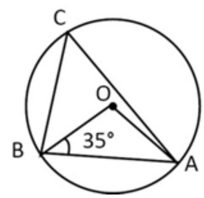
Or, $150° + 2\angle OCA = 180^0$

Or, $2\angle OCA = 180^0 - 150^0 = 30^0$

$\therefore \angle OCA = \frac{30^0}{2} =$

$\angle OAC = \angle OCA = 15°$

Example 2:



In the given figure, 'O' is the centre of the circle and $\angle OBA = 35^0$. Find the value of $\angle ACB$.

Solution:

In the figure,

$\angle OBA = \angle OAB$ ($\therefore$ OA = OB, base angles of isosceles Δ)

$\angle OAB = 35^0$

But, $\angle OAB + \angle OBA + \angle AOB = 180^0$ (Angle sum property of aΔ)

Or, $35^0 + 35^0 + \angle AOB = 180^0$

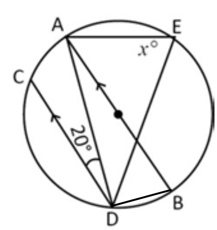
$\angle AOB = 180^0 - 70^0 = 110^0$

$\therefore \angle AOB = 110^0$

But, $\angle BCA = \frac{1}{2}\angle AOB$ (Inscribed angle = $\frac{1}{2}$ central angle subtended by the same arc)

$\therefore \angle BCA = \frac{1}{2}(110^0) = 55^0$

Example 3:



In the figure alongside, O is the centre of the circle; AB is the diameter and AB| |CD. DB is joined. If $\angle ADC = 20^0$, find the value of x^0.

Solution:

In the given figure $\angle ADC = \angle DAB = 20^0$ (AB| |CD alternate angles)

$\therefore \angle ADB = 90^0$ (AB is the diameter, angle in a semi circle)

$\therefore$ ABD $= 180^0 - 90^0$

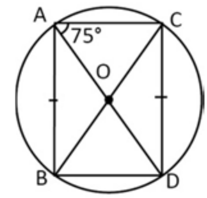
$= 180^o - 110^o = 70^o$

But, $\angle$ABD = $\angle$AED (Both being the angles in the same segment)

$\therefore \angle AED = 70^o = x^o$

$\therefore x = 70^o$

Example 4:



In the given figure, AB and CD are equal chords.
If $\angle DAC = 75^o$, find the values of $\angle$ADB, $\angle$ACB and $\angle$CBD.

Solution:

In the given circle, AB and CD are equal chords.

So they cut off equal arcs i.e. $\widehat{AB} = \widehat{CD}$

$\therefore \angle$CAD = $\angle$ADB [$\because$ Equal arcs subtend equal angles at the circumference]

$\therefore \angle ADB = 75^o$

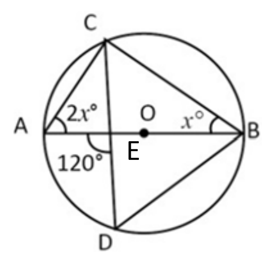
Again, AC| |BD [$\angle$CAD = $\angle$ADB]

$\therefore \angle$ACB = $\angle$CBD

But, $\angle DAC = \angle CBD = 75^o$ (Inscribed angles standing on the same arc DC)

$\therefore \angle ACB = \angle CBD = 75^o$

Example 5:



In the given figure, AB is the diameter of the circle with centre O. If $\angle AED = 120^0$,
Find the $\angle$s of the triangle ACB and ΔDCB.

or

Find the angles of ΔACB and ΔDCB.

Solution:

In the figure, $\angle ACB = 90^o$ (AB is the diameter)

But $x^o + 2x^o + 90^o = 180^o$ (Angle sum of the ΔACB)

Or, $3x^o = 90^o$

$\therefore x^o = 30^0$

Or, $2x^o = 60^0$

So the angles of the triangle ACB are 30^o, 60^o and 90^o. In the ΔEBC,

$\angle CEB + \angle ECB + \angle EBC = 180^0$ (Angle sum property of a Δ)

Again, $\angle$AED = $\angle$CEB (Vertically opposite angles) = 120^0

But , $\angle CEB + \angle ECB + \angle CBE = 180^0$

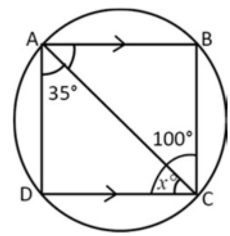
$\therefore 120^0 + \angle ECB + 30^0 = 180^0$

Or, $\angle ECB = 180^o - 150^o = 30^o$

$\therefore \angle DCB = \angle ECB = 30^0$

Example 6:

In the given figure, ABCD is a cyclic quadrilateral in which $\angle DCB = 100^o$, $\angle DAC = 35^o$ and AB || CD. Find the value of x and hence all the internal angles of cyclic quadrilateral ABCD.



Solution:

$\angle ACD = \angle CAB = x$ (Given)

But, $\angle DAB + \angle DCB = 180^o$ (Opposite angles of a cyclic quadrilateral)

$\angle DAB = 180^o - \angle DCB = 180^o - 100^o = 80^o$

Again, $\angle DAC + \angle CAB = \angle DAB$ (Hole part axiom)

$\therefore \angle CAB = \angle DAB - \angle DAC$

$= 80^o - 35^o = 45^o = x$

$\therefore x = 45^o$

Also, In ΔDAC, $35^o + x^o + \angle ADC = 180^o$ (Sum of all angles of aΔ)

Or, $\angle ADC = 180^o - 35^o - x^o = 180^o - 35^o - 45^o = 100^o$

$\therefore \angle ADC = 100^o$

But, $\angle ADC + \angle ABC = 180^o$ (Opposite angles of cyclic quadrilateral)

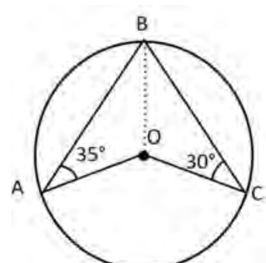
$\therefore \angle ABC = 180^o - \angle ADC = 180^o - 100^o = 80^o$

So the angles of the cyclic quadrilateral ABCD are

$\angle A = 80^o = \angle B$ and $\angle D = 100^o = \angle C$

Example 7:

In the given figure, O is the centre of the circle, $\angle BAO = 35^o$ and $\angle BCO = 30^o$.
Find the degree measure of $\widehat{AC}$ and $\widehat{ABC}$.



Solution:

Join BO.

Then, $\angle OBA = \angle OAB$ and $\angle OBC = \angle OCB$ ($\therefore$ OA = OB = OC)

$\therefore \angle ABO = 35^o$ and $\angle CBO = 30^o$

So, $\angle ABC = \angle ABO + \angle CBO = 35^o + 30^o = 65^o$

But, $\angle AOC = 2\angle ABC$ (Central angle is twice the inscribed angle at the same arc)

$= 2 \times 65^o = 130^o$.

We know that $\widehat{AC} \doteq \angle AOC = 130^o$

Again, the reflex $\angle AOC = 360^o - 130^o = 230^o$

$\therefore \widehat{ABC} \doteq$ reflex $\angle AOC \doteq 230^o$

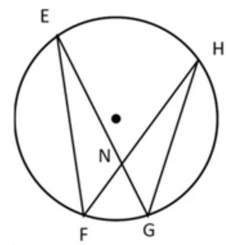
Hence, the degree measures of $\widehat{AC}$ and $\widehat{ABC}$ are 130^o and 230^o respectively.

Example 8:

In the given figure EF and HG are two equal chords

Prove that EG = HF and FN = GN

1. Given: Chord EF = chord HG
2. To prove: EG = HF and FN = GN



Proof

S.N	Statements	S.N	Reasons
1.	Chord EF = chord HG	1.	Given
2.	Arc EF = Arc HG	2.	Equal chords cut off equal arcs.
3.	$\widehat{EF} + \widehat{FG} = \widehat{HG} + \widehat{FG}$ or, $\widehat{EG} = \widehat{HF}$	3.	Adding same arc FG to both sides of 2.
4.	$\therefore$ Chord EG = chord HF	4.	Equal arcs subtend equal chords
5	$\therefore$ In Δ^s EFN and HGN	5.	
	(i) EF = HG (S)	(i)	Given
	(ii) $\angle$FEN = $\angle$GHN (A)	(Ii)	Inscribed angles on the same arc FG
	(iii) $\angle$ENF = $\angle$HNG (A)	(iii)	Vertically opposite angles
6.	$\therefore \Delta$EFN $\cong \Delta$FHG	6.	From (5) S.A.A statement .
7.	$\therefore$ FN = GN	7.	Corresponding sides of congruent triangles.

Proved.

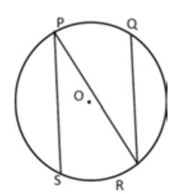
Example 9:

Prove that parallel chords of a circle include (Intercept) equal arcs.

Given: O is the centre of a circle with two parallel chords PS and QR.

To prove: Arc PQ = Arc SR i.e. $\widehat{PQ} = \widehat{SR}$

Construction: Join PR



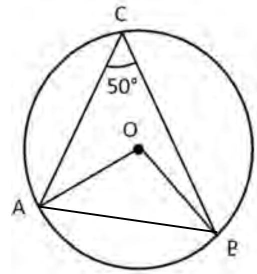
Proof:

S.N	Statements	S.N	Reasons
1.	$\angle$SPR = $\angle$PRQ	1.	PS$\|\|$QR, alternate angles made by a transversal with the parallel lines are equal.
2.	Arc SR = 2$\angle$SPR and Arc PQ = 2$\angle$PRQ	2.	Inscribed angles are half of the opposite arcs on which they stand.
3.	$\therefore$ Arc SR = Arc PQ i.e. $\widehat{SR} = \widehat{PQ}$	3.	From (i) and (2), twice of equals are also equal.

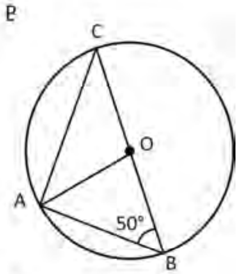
Proved.

Exercise 15.1

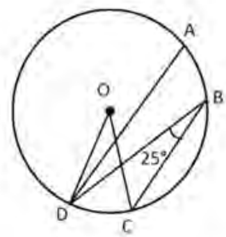
1.(a) In the given figure, $\angle ACB = 50^0$. If 'O' is the centre of the circle, find the angles of the triangle AOB.



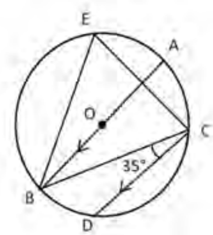
(b) In the given figure, 'O' is the centre of the circle, CB is the chord passing through the centre and $\angle ABO = 50^0$. Find the angles of a ΔCOA.



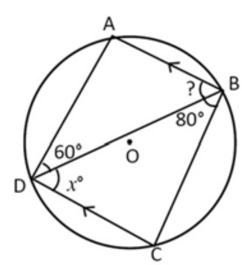
(c) In the given circle with centre 'O', $\widehat{AB} = \widehat{CD}$. If $\angle DBC = 25°$, find the value of $\angle ADB$ and $\angle DOC$.



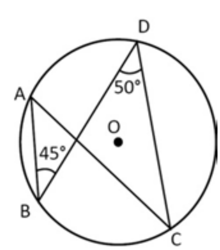
2.(a) In the given figure, AB||CD, 'O' is the centre and AB is the diameter of the circle. If $\angle BCD = 35^o$, find the value of $\angle BEC$.



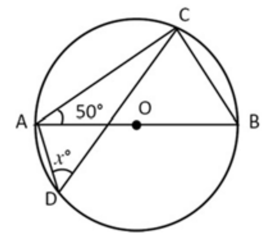
(b) In the given figure, ABCD is a cyclic quadrilateral in which AB||CD. If $\angle ABC = 80^o$, $\angle ADB = 60^o$ and $\angle ABD = \angle CDB = x^o$, find the value of x^o and hence all internal angles of the quadrilateral ABCD.



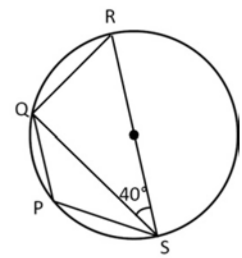
(c) In the given figure, O is the centre of the circle. If $\angle ABD = 45^o$ and $\angle BDC = 50^o$, find the degree measures of the arcs AD and BC.



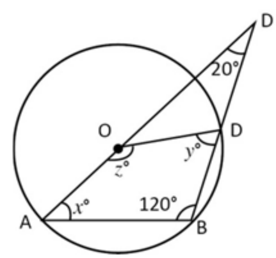
3. (a) In the given figure, 'O' is the centre and AB is the diameter. If $\angle CAB = 50^o$, find the value of x^o.



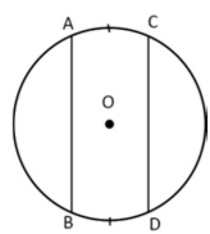
(b) In the adjoining figure, 'O' is the centre and RS is the diameter of the circle. If $\angle QSR = 40^0$, find the value of $\angle QPS$.



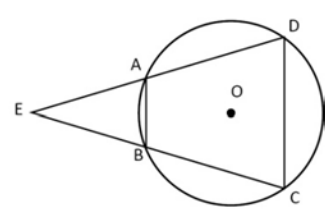
(c) In the figure alongside, find the values of x^o, y^o and z^o if $\angle ABD = 12^o$ and $\angle ADB = 20^o$ where O is the centre.



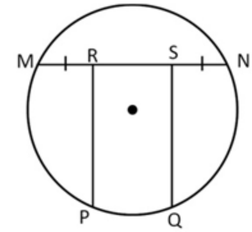
4. In the given figure, 'O' is the centre of the circle. If $\widehat{AC} = \widehat{BD}$, prove that AB||CD.



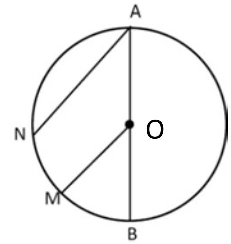
5. In the adjoining figure, ABCD is a cyclic quadrilateral whose sides DA and CB are produced to meet at E. If $\angle EAB = \angle EDC$, prove that EA = EB.



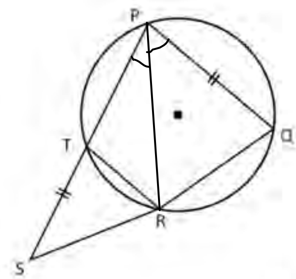
6. In the figure alongside, Arc MP = Arc NQ and MR = NS. Prove that PR = QS and $\angle PRM = \angle QSN$.



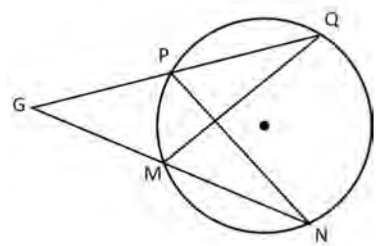
7. In the adjoining figure, 'O' is the center of the circle. If $\widehat{MB} = \widehat{MN}$, prove that AN$||$OM.



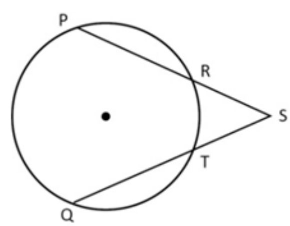
8. In the given figure, PQRT is a cyclic quadrilateral in which the side PT is produced to S such that TS = PQ. If $\angle QPR = \angle TPR$, prove that PR = SR.



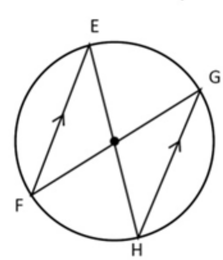
9. In the adjoining figure, the chords QP and NM are produced to meet at G. Prove that $\angle GMQ = \angle GPN$.



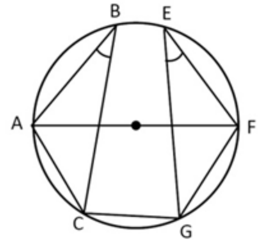
10. In the figure alongside, the chords PR and QT are produced to meet at S. Prove that $\angle RST \doteq \frac{1}{2}\,(\widehat{PQ} - \widehat{RT})$.



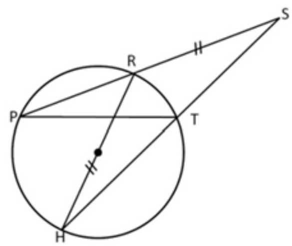
11. In the adjoining figure, chords EF and GH are parallel. Prove that EH = GF.



12. In the given figure, $\angle ABC = \angle FEG$ prove that ACGF is an isosceles trapezium.



13. In the adjoining figure, If RS = RH, prove that PT = TS.



14. Collect low cost materials like cotton and nylon threads, thin and flexible wires, rubber bands, tracing paper or cardboard paper and different colours. Draw the circles of different sizes on the tracing paper or cardboard paper by using the above materials. Highlight different parts of the circles by using the attractive colours. Also prepare different models of the circles showing the relationships of the central angles, inscribed angles and their corresponding arcs. By using the same materials, show that an angle in a semi-circle is a right angle. Present all these models to the class.

15. Circles and their parts have so many real life applications such as designing of tyres, camera lenses, spherical and cylindrical objects, rotating wheels, machinery parts, satellite's orbits, etc. Use the materials collected above and different wooden pieces of uniform thickness to prepare at lest two models of circles and their properties that are used in real life. Show all these models to your mathematics teacher and get the feedback.

15.4 The tangent and the secant of a circle

A straight line and a given circle may have three positional possibilities:

(i) The straight line may have no connection or intersection with the circle as shown in the figure (i)

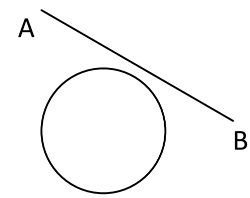


fig. (i)

(ii) The straight line may just touch the circle at a single point as shown in the figure (ii). Such a straight line which just touches a circle at a single point externally is called the tangent of a circle. In the figure (ii), PQ is the tangent of the circle. The point at which the tangent touches the circle is called the point of contact. It is the only point that is common to the circle and the tangent. In the figure (2), 'R' is the point of contact.

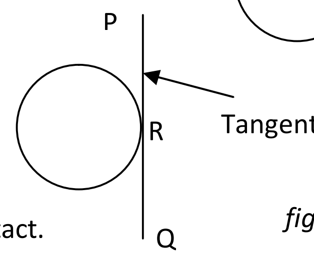


fig. (ii)

(iii) The straight line may intersect the circle at two distinct points as shown in the figure (3). Such a line which intersects the circle at two distinct points is called the secant of the circle. In the figure (ii), MN is the secant of the circle.

Sometimes, a straight line may touch two or more circles at a single point or different points. Such a line is called the common tangent. It is shown in the figures (iv) and (v) below.

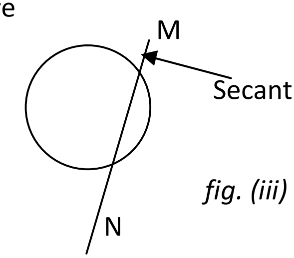


fig. (iii)

Common tangents

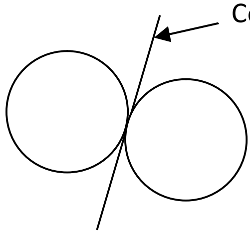


fig. (iv)

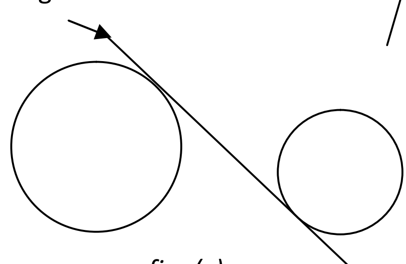
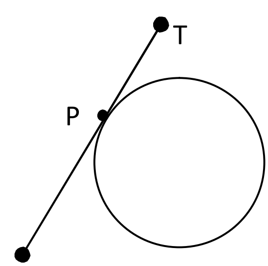


fig. (v)

15.5 The length of a tangent

Sometimes a tangent to a circle may be drawn from some external point. At that time the length of the line segment from the external point to the point of contact is called the length of the tangent. In the given figure, PT is the length of the tangent.

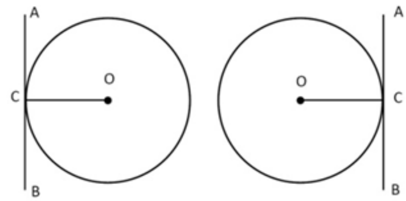


15.6 Some properties of a tangent

(a) A tangent to a circle and the radius of the circle are perpendicular to each other at the point of contact. (Concept only). According to the present curriculum, we do not have to prove this property theoretically as well as experimentally. But we must understand the concept well to retain it for longer time. This can be understood well by performing the following activity.

Activity:

Draw at least two circles of different radii with centre 'O'. Draw a tangent line AB at the point 'C' and join OC in each circle. Measure the angles ∠OCB and ∠OCA in each circle. Tabulate the obtained measurement in the table below. You will see that both of the angles OCA and OCB are 90^0. This shows that OC⊥AB.

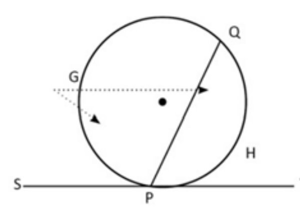


SN.	Measure of ∠OCA	Measure of ∠OCB	Result

From above result, we see that the radius and the tangent at the point of contact are perpendicular to each other.

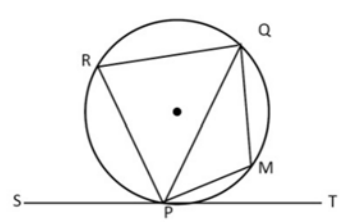
(b) Alternate segments of a circle

A tangent and a chord drawn at the point of contact create two segments of the circle which are known as the alternate segments. Specifically, the chord at the point of contact separates the circle at two parts which are called alternate segments of the circle with respect to the chord and the tangent. In the given figure, QGP and QHP are the alternate segments of the circle with respect to the angles QPT and QPS respectively.



(c) Angles In the alternate segments

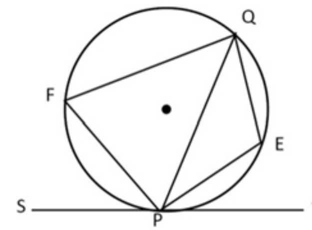
The angles subtended by the opposite arcs in the alternate segments of the circle are known as angles in the alternate segments. In the given figure ∠QRP and ∠QMP are alternate segment angles or angles in the alternate segments. The angle QRP



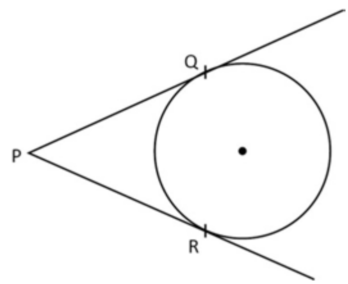
is subtended by the alternate arc ∠QMP and the angles ∠QMP is subtended by the alternate arc ∠QRP. That is why these angles are called alternate segment angles.

(d) The angles made by a tangent of a circle with a chord drawn at the point of contact are equal to the angles formed in the respective alternate segments of the circle. (Concept only)

We do not have to prove this property theoretically or experimentally. We make it clear with the help of the following figure. In the figure alongside, ∠QPT = ∠QFP and ∠QPS = ∠QEP. To get more confidence, we can measure these angles with the help of protractor. We will get them equal.

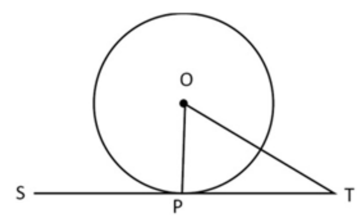


(e) Two tangents that can be drawn to a circle from some external point are equal in length. (Concept only) We make this concept clear with the help of alongside figure. Here PQ and PR are the tangents drawn from P to the circle. Q and R are the points of contact. According to the above statement PQ = PR. For more confidence you can measure their length and see the equal result.



Example 1:

In the given figure, 'O' is the centre of a circle, P is the point of contact, ST is the tangent. If TP = 4cm OP = 3cm, find the length of OT.



Solution:

Here, OP is the radius of the circle and P is the point of contact. So OP⊥ST

i.e. ∠OPS = ∠OPT = 90^o.

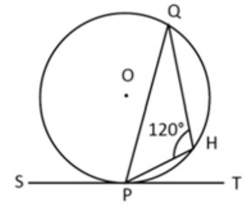
∴ΔOPT is a right angle triangle.

Now, by Pythagoras theorem, we get

OT = $\sqrt{OP^2 + PT^2}$ = $\sqrt{(4)^2 + (3)^2}$ = $\sqrt{25}$ = 5cm.

∴ OT = 5cm

Example 2:



In the figure alongside, $\angle QHP = 120^o$. Find the angles made by the tangent ST with the chord QP at the point of contact 'P'.

Solution:

$\angle QHP$ is the angle in the alternate segment for the anlge, so $\angle QPS = \angle QHP = 120^o$

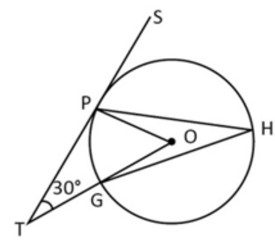
Again, $\angle QPS + \angle QPT = 180^o$ (Sum of adjacent angles)

$\therefore \angle QPT = 180^o - \angle QPS$

$= 180^o – 120^o = 60^o$

$\therefore \angle QPS = 120^o$ and $\angle QPT = 60^o$

Example 3:



In the given figure, 'O' is the centre and P is the point of contact of the tangent ST. If $\angle OTP = 30^o$, find the value of $\angle PHG$.

Solution:

$OP \perp ST$ (Radius and tangent at the point of contact are at 90^0.)

$\therefore \angle OPT = 90^o$

So, $\angle POT = 180^o – 30^o – 90^o$ - = $60^o = \angle POG$

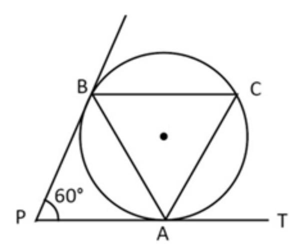
But $\angle PHG = \frac{1}{2} \angle POG$ (Relation of inscribed and central angles standing on the same arc)

$= \frac{1}{2}(60^o)$

$= 30^o$

$\therefore \angle PHG = 30^o$

Example 4:



In the adjoining figure, $\angle BPA = 60^o$, PA and PB are tangents to the circle at A and B respectively. Find the value of $\angle BCA$.

Solution:

PB = PA (Tangents drawn from external point to a circle are equal in length.)

$\therefore \angle PBA = \angle PAB$ ($\therefore$ PA = PB)

But, $\angle PBA + \angle PAB + \angle BPA = 180^o$

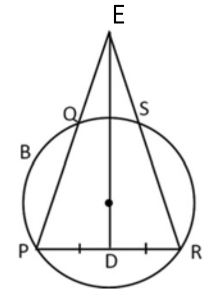
$\therefore 2\angle PBA + 60^o = 180^o$

Or, $2\angle PBA = 120^o$

Or, $\angle PBA = 60^o$

But ∠PBA = ∠BCA (The angle made by the tangent PB with the chord BA = ∠BCA)

∴ ∠BCA = 60°



Example 5:

In the adjoining figure, the equal chords PQ and RS are produced to meet at E. If D is the mid-point of PR, prove that ED⊥PR.

1. Given: PQ = RS and PD = DR. Also PQ and RS are produced to meet at E.
2. To prove: ED⊥PR

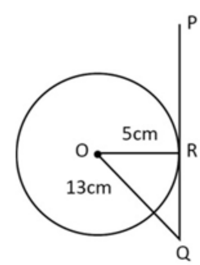
Proof:

S.N	Statements	S.N	Reasons
1.	Chord PQ = chord RS	1.	Given
2.	Arc PQ = Arc RS	2.	Equal chords cut of equal arcs
3.	Arc PQ + Arc QS = Arc RS + Arc QS	3.	Adding Arc QS to both insides of (2)
4	∴ Arc PS = Arc RQ i.e. $\widehat{PS} = \widehat{RQ}$	4.	From (3)
5.	∴ ∠PRS = ∠QPR	5.	Inscribed angles standing on the equal arcs.
6.	ΔEPR is an isosceles	6.	From (5), base angles are equal.
7.	∴ ED⊥PR	7.	The median of an isosceles triangle is perpendicular to the base.

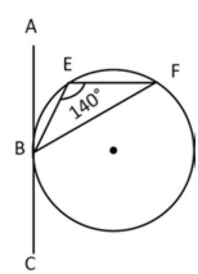
Proved.

Exercise 15.2

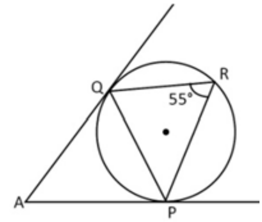
1.(a) In the adjoining figure, PQ is the tangent at R and O is the centre of the circle. If OR = 5cm, OQ= 13cm, find the value of QR.



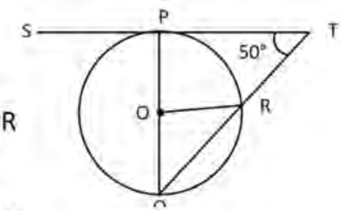
(b) In the adjoining figure, AC is the tangent to the circle at B and FB is the chord at the point of contact B. If ∠BEF = 140°, find the value of ∠ABF.



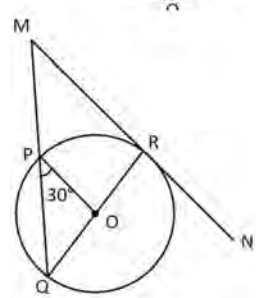
(c) In the adjoining figure, AP and AQ are the tangents from A to the circle and QP is the chord. If $\angle QRP = 55^\circ$, find the value of $\angle QAP$.



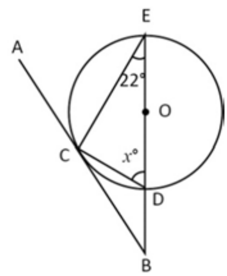
2.(a) In the adjoining figure, ST is the tangent to a circle at P and PQ is the diameter of the circle with centre O. If $\angle PTQ = 50^\circ$, find the value of $\angle POR$



(b) In the figure, alongside, MN is the tangent to a circle with centre 'O' at R. RQ is the diameter of the circle. If $\angle OPQ = 30^\circ$, find the value of $\angle PMR$.

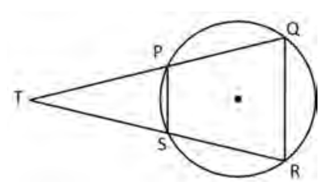


(c) In the adjoining figure, AB is the tangent to a circle with centre 'O' at the point 'C'. If CE = CB, $\angle CED = 22^\circ$ and BE is the secant of the circle, calculate the value of x and $\angle DCB$.



3. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

4. In the given figure, PQRS is a cyclic quadrilateral whose two sides QP and RS are produced to meet at T. If TP = TS, prove that PS||QR.



5. Prove that the exterior angle of a cyclic quadrilateral is equal to its opposite interior angle.

6. Prove that a line segment joining any external point to the centre of a circle bisects the angle between the tangents drawn from the same point to the same circle.

7. While a vehicle tyre rolls over a smoothly pitched road, any point on the rim of the tyre traces an important curve in space. Visualize the shape of this possible curve mentally. Make a rough sketch of this on the tracing paper. Make it attractive by colouring. Find out its actual name and definition if possible. Show this curve to your mathematics teacher. You can get a similar curve by rolling a coin vertically over a table. Perform this activity. Every point lying on the rim of the coin traces the same curve as of the tyre of a vehicle.

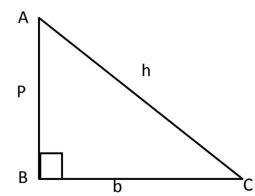
8. When a small circle rolls along the circumference of a larger circle internally in vertical position, it traces a geometrical curve. Find out its shape and name. Sketch this curve on the tracing paper. You can search on internet for this curve. Find its application in geometry.

9. Make groups of 4 students each. Try to find out the daily applications of all the geometrical concepts, properties and theorems that you have studied so far. Present your findings to the class group wise. Prepare an article of the applications of geometry in real life. Publish this article on your school magazine or in any news paper.

| Unit: 16 |

Trigonometry

16.0 Review



We have studied various properties of a right angled triangle in the previous classes. Right angled triangle is the origin of trigonometry. So it is bitter to start from the introduction of right angled triangle. A triangle having one angle 90° is called a right angled triangle. Figure ABC is a right angled triangle at the vertex B. In the given position the side BC is called the base, AB is called the perpendicular and AC is called the hypotenuse of the right angled triangle ABC. We denote the base, the perpendicular and the hypotenuse by b, P and h respectively. Taking anyone of the angles other than 90° as the reference angle, we can define the ratio of the sides of a right-angled triangle in six different ways. Mathematicians have given six names to these ratios as below.

i) Taking 'C' as the reference angle , $\frac{AB}{BC}=\frac{P}{b}$ is called the tangent of the angle C. It is written as tan C = $\frac{p}{b}$

ii) In the same position, $\frac{AB}{AC}=\frac{p}{h}$ is called of the sine of the angle C. It is written as sin C = $\frac{p}{h}$.

iii) Similarly, $\frac{BC}{AC}=\frac{b}{h}$ is called the cosine of the angle C. It is written as cos C = $\frac{b}{h}$

From the above definitions, it is easy to see that;

Tan C = $\frac{p}{b}=\frac{(\frac{p}{h})}{(\frac{b}{h})}=\frac{\sin C}{\cos C}$

These three ratios are called the fundamental trigonometric ratios.

The reciprocals of the above fundamental ratios give birth, to three more trigonometric ratios which are known as cotangent, cosecant and secant. The reciprocal of sine is called cosecant, that of cosine is called secant and that of tangent is called cotangent. These are written as cosec, sec and cot respectively. So in the above position of the right-angled triangle, cosec (C) = $\frac{1}{\sin c}=\frac{h}{p}$

Sec (C) = $\frac{1}{\cos c}=\frac{h}{b}$

and cot (C) = $\frac{1}{\tan c}=\frac{b}{p}$

Using these definitions, we can prepare a table of values of trigonometric ratios of some standard angles from 0° to 90° as we have done in class 9, The table is again shown below:

Angles / Ratios	0°	30°	45°	60°	90°	Remark
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	sin = $\frac{p}{h}$
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	cos = $\frac{b}{h}$
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	tan = $\frac{p}{b}$
Cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	cot = $\frac{1}{tan}$
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	sec = $\frac{1}{cos}$
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	cosec = $\frac{1}{sin}$

The above six ratios are also called trigonometric functions of given angle.

Concept and application in real life

Trigonometry is a branch of mathematics that studies the relationships between the sides and angles of a triangle. The word Trigonometry is derived from Greek words 'Trigonoh' means 'triangle' and 'matron' means 'measure'. So the word trigonometry means literally the measurement of the sides and the angles of a triangle. This word was coined by the astronomers during the 3rd BC in course of studying the astronomical bodies with the help of geometry.

The first mathematician who prepared the first trigonometric table of values of sine function was Hipparchus in 140 BC. This table is used at present also. So the mathematician Hipparchus is called the father of trigonometry. Trigonometry has been developed as very important branch of applied mathematics at present. It has wide range of applications in almost every field of science and technology. Basically, it is used in astronomy for navigation and location of celestial bodies. Besides this, it is used in pure and applied mathematics, physics, biology, chemistry, ecology, engineering, surveying, architecturing, designing, and analyses like Fourier transform, maclaurins series, Tayor's series, Laplace transform, etc. So trigonometry has been the part and parcel of our present activities. We will be using trigonometry for calculating the heights and distances in the succeeding chapter as well.

16.1 Areas of the triangles and quadrilaterals

In the previous classes and chapters, we have studied about how to find areas of the triangles and quadrilaterals by using the geometrical techniques and formulas. In this section, we will develop trigonometric techniques and formulas to find their areas. In these formulas, the trigonometric ratios of some reference angles will be involved. The derivations of the formulas are as below.

Let ABC be the triangle with base BC and height AD. Let AB = c, BC = a and CA = b be the length of the sides opposite to the vertices C, A and B respectively as shown in the figure. Let $\angle C = \theta$ be the given angle. Then in the right angled triangle ADC, we have, $\frac{AD}{AC} = \sin\theta$ (Definition of sine function)

or AD=AC sin θ = b sin θ ....(i)

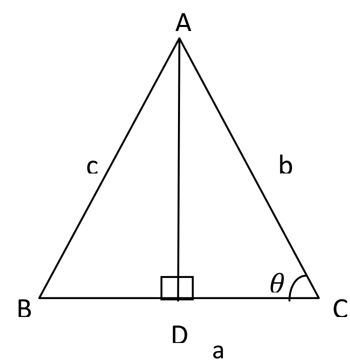
But We Know that area of the triangle ABC is given by;

A $= \frac{1}{2}$ height X base

$= \frac{1}{2}$ ADxBC

$= \frac{1}{2}$ b sinθ x a (Using (i))

$= \frac{ab\ sin\theta}{2}$



∴ Area of ΔABC $=\frac{1}{2}$ ab sinθ ....(ii)

We take $\angle B = \theta$ as the reference angle then in the right angled triangle ABD,

$\frac{AD}{AB}$ = sinB= sinθ

or AD=AB sinθ = c sinθ ....(iii)

So area if the triangle ABC = $\frac{1}{2}$ height X base

$= \frac{1}{2}$ AD xBC

$= \frac{1}{2}$ c sinθ a (using (iii))

Area of ABC = $\frac{1}{2}$ a.c sinθ .....(iv)

Similarly, drawing perpendicular from B to AC and taking $\angle A = \theta$, we get area of ΔABC = $\frac{1}{2}$ bc sinθ .... (v)

From (ii), (iv) and (v), we get,

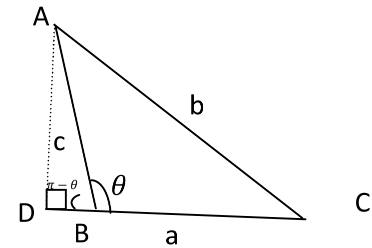
Area of ΔABC = $\frac{1}{2}$ absinθ = $\frac{1}{2}$ bc sinθ = $\frac{1}{2}$ ca sinθ

In general, we say area of the triangle ABC = $\frac{1}{2}$ ab sinC = $\frac{1}{2}$ bc sinA = $\frac{1}{2}$ ca sinB

i.e. Area of ΔABC = $\frac{1}{2}$ab sin C = $\frac{1}{2}$ bc sin A = $\frac{1}{2}$ca sin B

These formulas are equally valid for obtuse angle triangle as well as right-angled triangle as shown below.

Let $\angle ABC = \theta$ and $AD \perp BC$ be the height of the obtuse angled triangle ABC as shown in the figure



Then $\angle ABD = (\pi - \theta)$

Now, in the right angled triangle,

ADB, $\frac{AD}{AB}$ = sin $\angle ABD$

or, AD = AB sin $(\pi - \theta)$ = AB sinθ (sin $(\pi - \theta)$=sinθ)

$\therefore$ AD = c sinθ ...... (vii)

But, area of the triangle ABC $= \frac{1}{2}$ ADxBC

$= \frac{1}{2}$ c sinϑ x a (using (vii)) $= \frac{ca\ sin\theta}{2}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}$ ca sinθ $= \frac{1}{2}$ casinB

We write the above formulas in words as area of any triangle $= \frac{1}{2}$x product of two sides x sinθ where θ is the angle between the given sides.

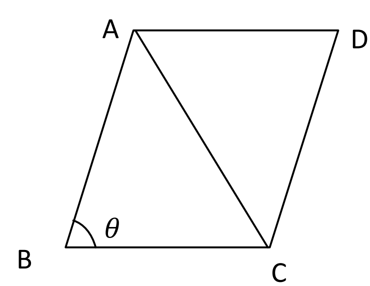
$\therefore$ Area of a triangle $= \frac{1}{2}$ (Product of two sides x sine of the angle between these sides.

For the above formula, we can take right-angled triangle ADC and get AD = AC sinC = b sinC.

C. In this case area of $\Delta ABC = \frac{1}{2}$ ab sin C.

Now, we find a trigonometric formula for the area of a parallelogram as below.

Let ABCD be a parallelogram with θ as the angle between AB and BC as shown in the figure. Join AC. Then area of ΔABC $=\frac{1}{2}$ AB x BC x sinθ



$\therefore$ Δ ABC $=\frac{1}{2}$ AB x BC x sinθ

But, we know that, Δ ABC $= \frac{1}{2}$ ▱ABCD

or ▱ ABCD = 2ΔABC = 2 ($\frac{1}{2}$ AB x BC x sinθ) = AB x BC x sinθ

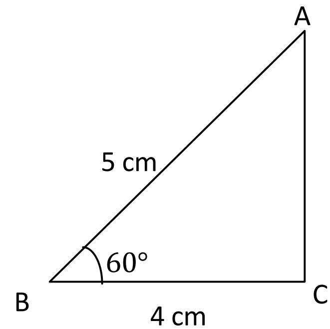
$\therefore$ ▱ ABCD = AB x BC x sinθ

We write this formula in words as:

The area of a parallelogram = product of two adjacent sides x sin of the angle between them.

Example 1:

Find the area of a given triangle.



Solution:

The area of Δ ABC = $\frac{1}{2}$ AB x BC x sinθ

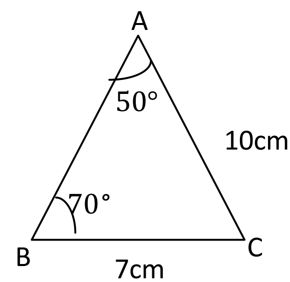
= $\frac{1}{2}$ 5 x 4 x sin60°

= $\frac{1}{2}$ x20 x $\frac{\sqrt{3}}{2}$ (∴ sin 60° = $\frac{\sqrt{3}}{2}$)

= 5$\sqrt{3}$ sq cm

Example 2:

Find the area of the given triangle.



Solution:

As the sides AC and BC are given, we need the angle ∠C

So ∠C = 180° - A - B = 180° - 50° - 70° = 180° - 120° = 60°

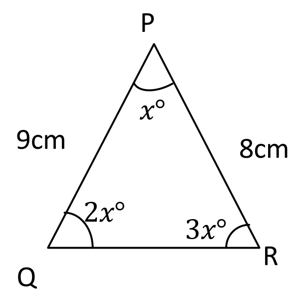
∴ The required area of the triangle ABC = $\frac{1}{2}$ AC x BC x sin 60°

= $\frac{1}{2}$ x 10 x 7 x $\frac{\sqrt{3}}{2}$ (∴ sin 60° = $\frac{\sqrt{3}}{2}$)

= $\frac{35\sqrt{3}}{2}$ sq cm

Example 3:

Find the area of a given triangle.



Solution:

Here, the sides PQ and PR are given. So we need ∠P

We know that $x° + 2x° + 3x° = 180°$

or, 6x° = 180°

∴ $x° = 30°$= ∠P

So the area of the triangle PQR = $\frac{1}{2}$ PQ x PR sin ∠P

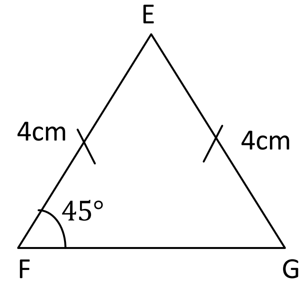
= $\frac{1}{2}$ x 9 x 8 x sin 30°

= $\frac{1}{2}$ x 9 x 8 x $\frac{1}{2}$

= 18 sq. cm.

Example 4:

Find the area of the ink scape triangle EFG given alongside.



Solution:

As EF = FG, $\angle F = \angle G$

$\therefore \angle G = 45°$

so $\angle E = 180° - \angle G - \angle F$

$= 180° - 45° - 45° = 90°$

$\therefore$ The area of the triangle EFG $= \frac{1}{2} \times EF \times EG \times \sin \angle E$

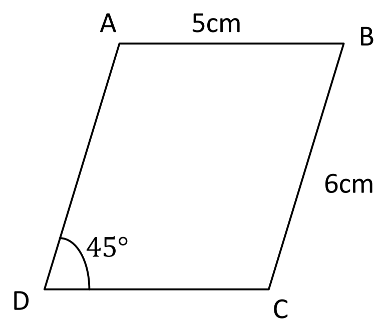
$= \frac{1}{2} \times 4 \times 4 \times \sin 90°$

$= \frac{16}{2} \times 1$ ($\therefore \sin 90° = 1$)

$= 8$ sq. cm

Example 5:

Find the area of the given parallelogram ABCD.



Solution:

Here, AD=BC= 6cm and DC=AB= 5cm

The area of the parallelogram ABCD

$= AD \times DC \times \sin 45°$

$= 6 \times 5 \times \sin 45°$

$= 30 \times \frac{1}{\sqrt{2}}$

$= 15\sqrt{2}$ sq. cm

Example 6:

In the adjoining figure, the area of the triangle PQS is $\frac{1}{3}$ of the area of the triangle PQR. Find the length of QS.

Solution:

By given, $\Delta PQS = \frac{1}{3} \Delta PQR$

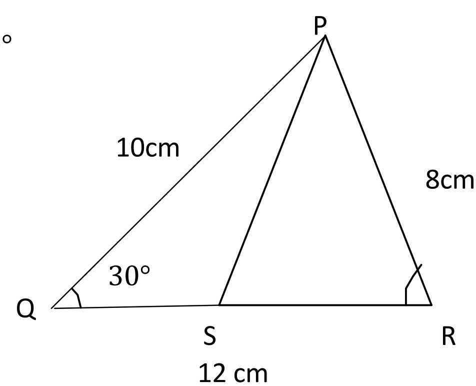
Or $\frac{1}{2} PQ \times QS \times \sin 30° = \frac{1}{3} \times \frac{1}{2} \times PR \times QR \times \sin 60°$

Or $\frac{1}{2} \times 10 \times QS \times \frac{1}{2} = \frac{1}{3} \times \frac{1}{2} \times 8 \times 12 \times \frac{\sqrt{3}}{2}$

Or $10\, QS = \frac{8x12}{\sqrt{3}}$

Or $QS = \frac{96}{10\sqrt{3}} = \frac{32x3}{10\sqrt{3}} = \frac{16x\sqrt{3}}{5} = \frac{16\sqrt{3}}{5}$

$\therefore QS = \frac{16\sqrt{3}}{5}$ cm



Example 7:

Find the height AD and base BC of the given triangle ABC if its area is 75 sq. cm

Solution:

The area of the triangle ABC $=\frac{1}{2}$ ABxBCxsinB

Or 72 = $\frac{1}{2}$ x 12 x BC x sin 45° = 6 x BC x $\frac{1}{\sqrt{2}}$ ($\therefore$ sin 45° = $\frac{1}{\sqrt{2}}$)

Or BC = $\frac{72\times\sqrt{2}}{6}$ $=12\sqrt{2}cm$

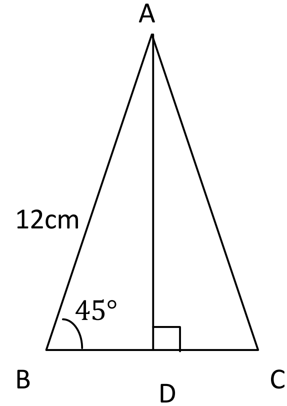
$\therefore$ BC = $12\sqrt{2}$ cm

Again, the area of the triangle ABC = $\frac{1}{2}$ x AD x BC

Or, 72 = $\frac{1}{2}$ x AD x $12\sqrt{2}$

Or, AD = $\frac{72}{6\sqrt{2}}$ = $\frac{12}{\sqrt{2}}$ = $6\sqrt{2}$cm

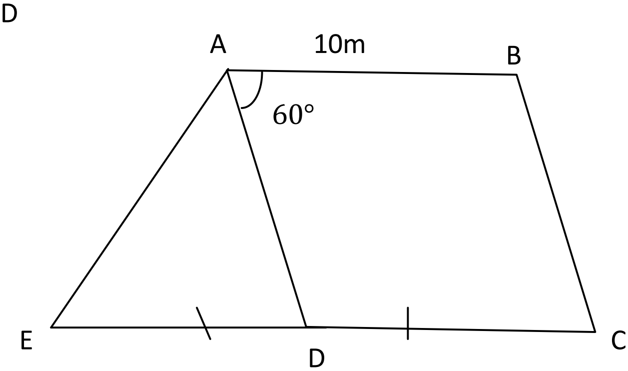
The height AD = $6\sqrt{2}$cm and base BC = $12\sqrt{2}$cm.



Example 8:

In the given figure, ABCD is a rhombus and D is the mid point EC. If AB =10cm and $\angle$BAD =60°, find the area of the triangle AED

Solution:



Here, ABCD is a rhombus. (Given)

AB = AD = 10cm

Also ED=DC (Given)

$\therefore \mathrm{ED} = \mathrm{DC} = \mathrm{AB} = 10\mathrm{cm}$

Again, AB ll DC or AB ll EC

(Opposite sides of a rhombus are ll)

$\therefore \angle\mathrm{BAD} = \angle\mathrm{ADE} = 60°$

So, area of ΔAED = $\frac{1}{2}$x AD $\times$ ED $\times \sin 60°$

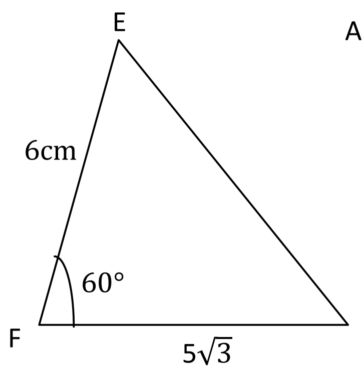
= $\frac{1}{2}$x 10 x 10 x $\frac{\sqrt{3}}{2}$

= $25\sqrt{3}$ sq. cm

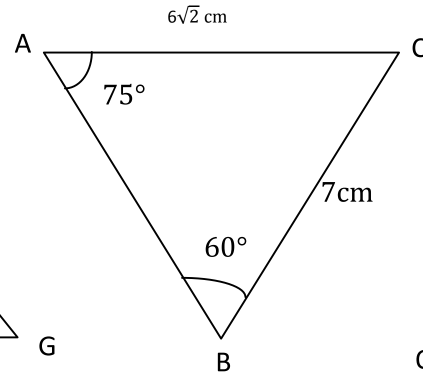
Exercise 16.1

1. Find the area of the following triangles.

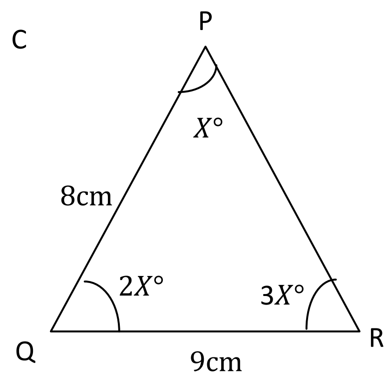
(a)



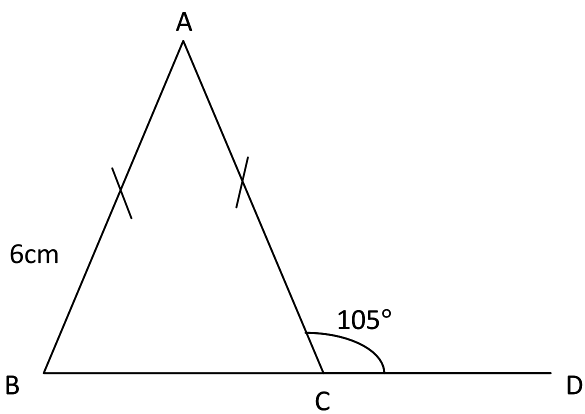
(b)



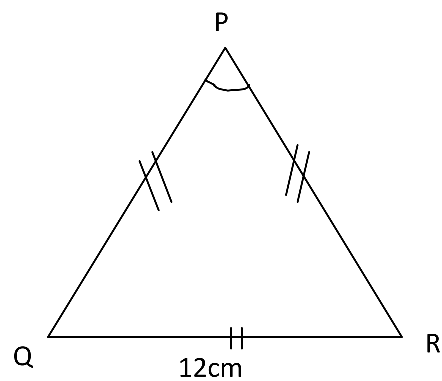
(c)



(d)



(e)



2. (a) Find the area of a triangle ABC if a = 4cm, b = 6cm and $\angle C = 30°$

(b) If the area of a triangle ABC = 30cm^2, a = 4cm and $\angle B = 45°$ find the length of the side AB.

(c) If the area of a triangle PQR = 32cm^2, PQ= 4cm, $\angle Q = 60°$, find the length of QR.

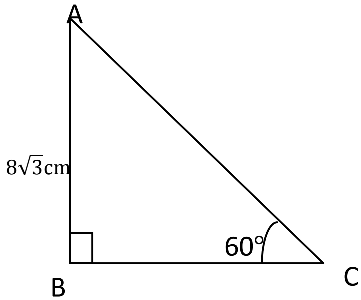
(d) Find the area of parallelogram with the adjacent sides 8cm and 10cm and including angle 60°.

(e) Find the area of a rhombus with a side 6cm and one of the angles 30°

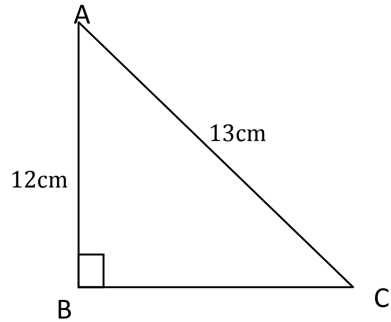
(f) If the area of a rhombus having a side length 6cm is 18cm^2, find its one of the angles.

3. Find the area of the given triangle ABC in each case:

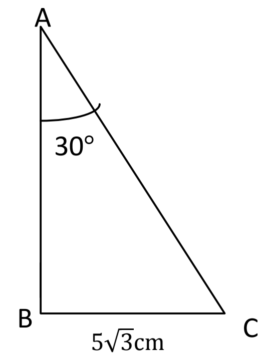
(a)



(b)



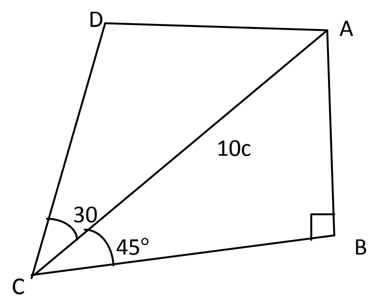
(c)



4.(a) In a triangle PQR, PR=6cm, PQ=7cm, ∠PRQ=100° and ∠PQR=50°, find the area of ∆PQR.

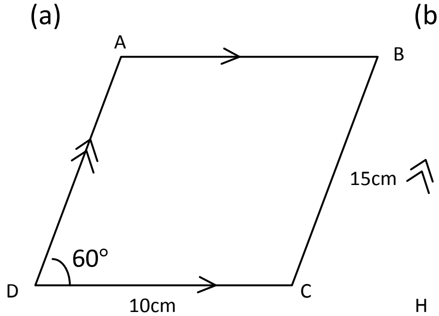
(b) Find the height AD and the base BC of the triangle ABC if its area is 64cm^2, AB = 8cm and ∠ABC =30°.

(c) In the given figure, the area of the triangle ADC is two third of the area of the triangle ABC. Find the length of the side DC.

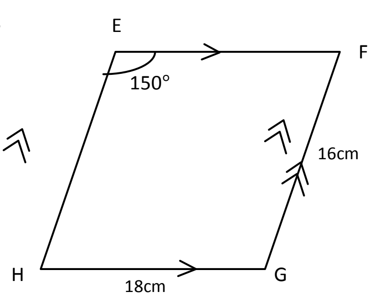


5. Find the area of the following parallelograms:

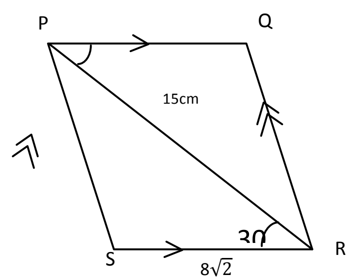
(a)



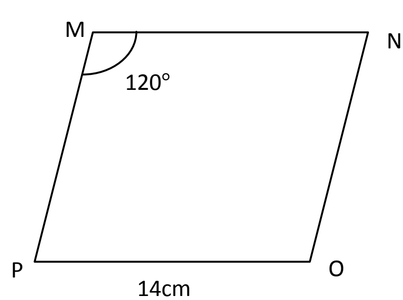
(b)



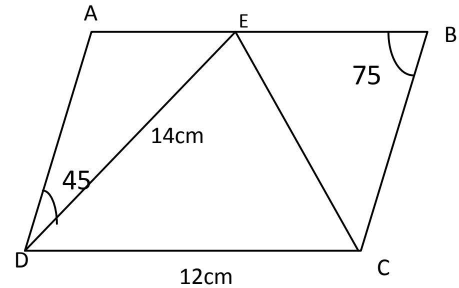
(c)



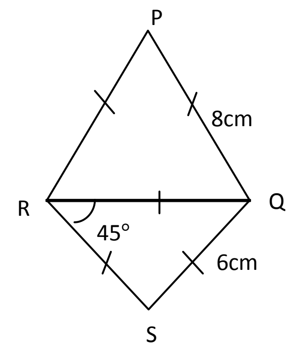
6.(a) In the adjoining figure, MNOP is a rhombus in which ∠PMN = 120° and PO = 14cm. Find its area.



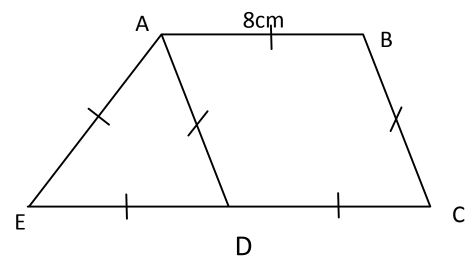
(b) In the given figure, find the area of the triangle DEC and the parallelogram ABCD.



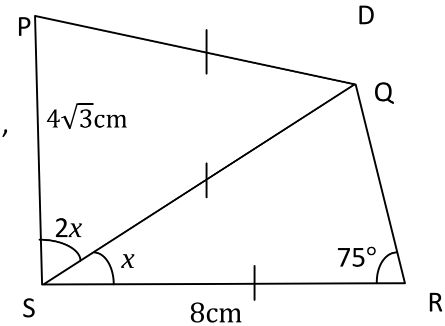
(c) In the given figure, PQR is an equilateral triangle and QRS is an isosceles triangle. Find the area of the quadrilateral PQSR.



(d) In the given figure, ABCD is a rhombus having AB = 8cm and ADE is an equilateral triangle. Find the area of the trapezium ABCE.



(e) In the given figure, PQRS is a quadrilateral in which SQ = SR = 8cm, $\angle QRS = 75°$, PS = $4\sqrt{3}$cm, and $\angle PSQ = 2\angle QSR$. Find the area of the quadrilateral PQRS.



7. Prove that area of a rhombus is given by square of its any one of the sides multiplied by sine of any one of the angles i.e. A= $(\text{side})^2$x(sine of its one angle).

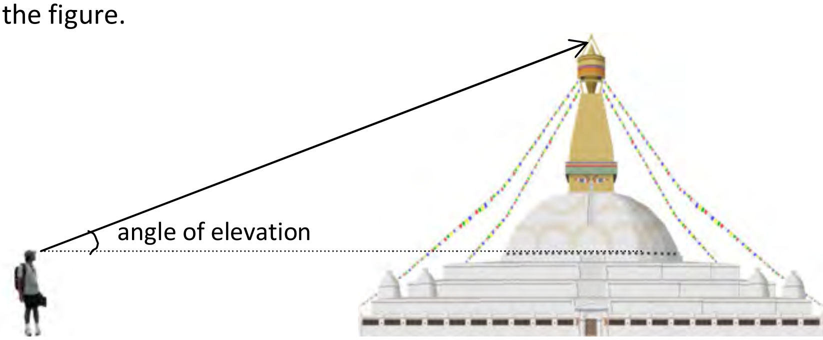
8. Make groups of three students each. Draw a regular hexagon having a side at least 10cm on the tracing paper. Find the area of this regular hexagon by using the formulas of the areas of the triangles, quadrilaterals and parallelograms. Explain the process that you have used for this calculation and show it to your mathematics teacher group wise.

16.2 Height and distance

This height and distance section is actually an example of one of the various applications of trigonometry. In our daily lives, we need to measure the height of different objects like towers, tall trees, high mountain peaks, telephone / telegraph posts, large buildings etc. For all this job, we use trigonometric formulas and concepts. In the same way, we need to calculate the distances between different objects, thickness (breadth / width) of various objects like the width of a river, the separation between two or more stationary or moving bodies, etc. For all these estimations, we use trigonometric formulas and methods. Basically, what we use here is the Pythagoras theorem and concept of solution of a right angled triangle. In a right angled triangle, if its two of the sides or one side and one angle other than the right angle are given, we can find out its remaining sides and angles. This process is called the solution of right angled triangle.. Besides this, we also need to know about two important angles, an angle of elevation and an angle of depression.

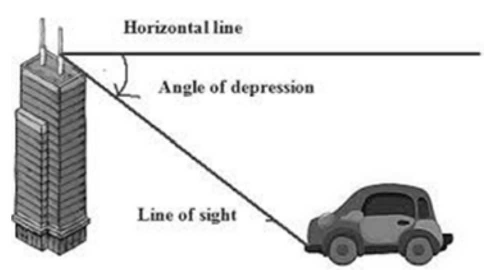
Angle of elevation:

Whenever we see the objects which are higher than our level of eyes, we have to bend our head backwards and our eyes get rotated upwards in the anticlockwise direction. The straight line joining our right eyes to the point of the object at which our eyes are focusing is called the line of eyes is left sight or observation. The angle made by this line of sight with the horizontal line through the level of our eyes is called the angle of elevation. The line of right is always in the upwards oblique direction and the angle of elevation increase in the upward direction as shown in the figure.

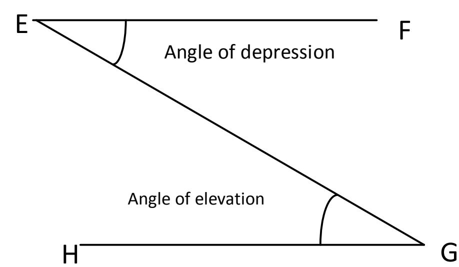


Angle of depression:

Whenever we see an object at a lower height than the level of our eyes, we bend our head downwards and our eyes get rotated downwards in the clock wise direction. At this time, the angle made by the line of observation with the horizontal line through the level of



our eyes is called the angle of depression. In this case the line of sight points in the downward oblique direction and the angle of depression increases in the downward (clock wise) direction as shown in the figure.

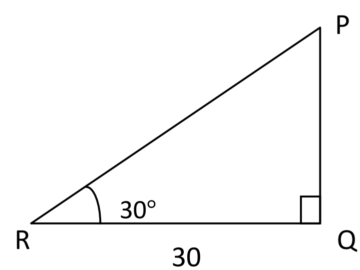


The angle of elevation and the angle of depression from the same two points for viewing the same object are always equal as shown in the figure.

Here, EF||GH. So $\angle FEG = \angle EGH$

Example 1:

The top of a telegraph post is attached to a horizontal plane at a distance of 30m from the foot of the post. If the angle of elevation of the post is 30° from that point, find the height of the post.



Solution:

Let PQ be the vertical post and RQ be the horizontal distance.

Join RP. Then by the question, $\angle PRQ=30°$ and RQ=30m

So in the right angled triangle PQR,

$\frac{PQ}{RQ}$ = Tan 30°

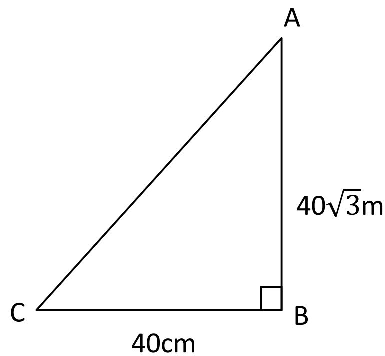
or PQ = RQ Tan 30° = 30 x $\frac{1}{\sqrt{3}}$ = $10\sqrt{3}$m

So, the height of the telegraph post is $10\sqrt{3}$m.

Example 2:

The top of a house which is $40\sqrt{3}$m high is observed from a point on the horizontal ground 40m away from the base of the house. What will be the angle of elevation of the house?

Solution:



Let AB be the height of the house and C be the point on the horizontal ground from where the top of the house is observed.

Then by the question, AB = $40\sqrt{3}$m and CB = 40m.

$\angle$ACB = ?

In the right angled triangle ACB,

$\frac{AB}{CB}$ = Tan $\angle$ACB

or Tan $\angle$ACB =$\frac{40\sqrt{3}}{40}$ =$\sqrt{3}$

or Tan $\angle$ACB = Tan 60°

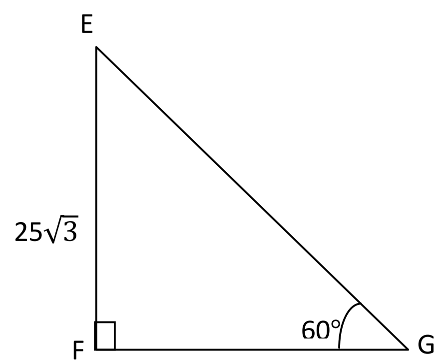
$\therefore$ $\angle$ACB = 60°

So, the angle of elevation of the house will be 60°

Example 3:

A tree of the height $25\sqrt{3}$m is situated on the edge of a river. If the angle of elevation of the tree observed from the opposite edge of the river is found to be 60°, what will be the breadth of the river?

Solution:



Let EF be the height of the tree and FG be the breadth of the river. Join EG.
Then by the question,

EF =$25\sqrt{3}$m, $\angle$EGF =60°, FG =?

In the right angled triangle EFG,
We know that, $\frac{EF}{FG}$ = Tan 60°

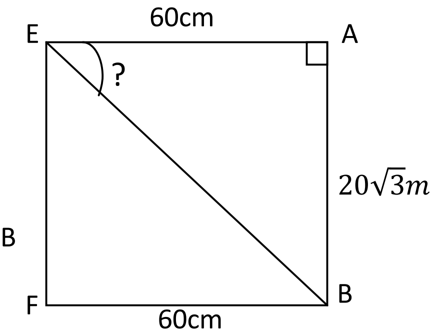
Or FG =$\frac{EF}{Tan\ 60°}$ =$\frac{25\sqrt{3}}{\sqrt{3}}$m =25m

So, the breadth of the river is 25m

Example 4:

The bottom of a house which is $20\sqrt{3}$m high, is observed from the roof of the opposite house 60m away from that house. Find the angle of depression if both of the houses have same height.

Solution:



Let AB be the house whose bottom point B is observed from the roof 'E' of the opposite house EF having equal height. Then by the question, FB = 60m and AB = $20\sqrt{3}$m.

To find ∠AEB. Join EA and EB.

Then in the right angled triangle EAB, $\frac{AB}{AE}$ =Tan ∠AEB

Or, $\frac{20\sqrt{3}}{BF}$ = Tan ∠AEB (AE =BF)

Or, Tan ∠AEB = $\frac{20\sqrt{3}}{60}$ = $\frac{1}{\sqrt{3}}$ = Tan 30°

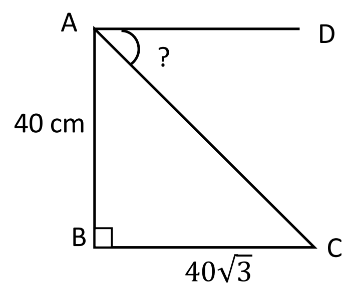
∴ ∠AEB = 30°

So, the angle of depression of the house is 30°

Example 5:

A pigeon on the ground is observed from the roof of a house, which is 40m high. If the pigeon is $40\sqrt{3}$m away from the bottom of the house on the ground, find the angle of depression of the pigeon from the observer.

Solution:



Let AB be the height of the house and 'C' be the position of the pigeon. Then by the question, AB = 40m and BC = $40\sqrt{3}$m. Let ∠DAC be the angle of depression of the pigeon. Then ∠DAC = ∠ACB

Now, in the right angled triangle ABC, $\frac{AB}{BC}$ = Tan ∠ACB

Or Tan ∠ACB = $\frac{40}{40\sqrt{3}}$ = $\frac{1}{\sqrt{3}}$ = Tan 30°

∴ ∠ACB = 30° = ∠DAC

So, the required angle of depression is 30°.

Example 6:

A 5ft tall person observed the top of a tower of 55 ft high from a point 50 ft away from the bottom of the tower on the horizontal level. Find the angle of elevation of that tower.

Solution:

Let PQ be the height of the tower and Rs be that of the man. Draw R T ll SQ.

Then by the question, SQ = RT = 50 ft

PQ = 55ft, RS = TQ = 5ft

$\therefore$ PT =(55-5)ft = 50ft

Join PR.

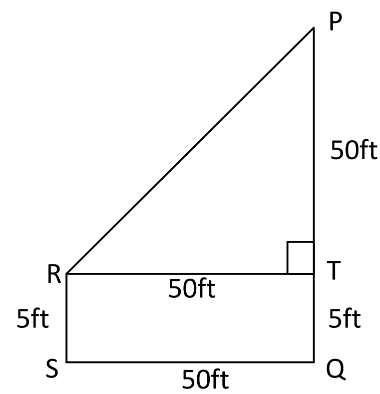
Now, in the right angled triangle PRT,

$\frac{PT}{RT}$ = Tan <PRT

Or Tan <PRT = $\frac{50}{50}$ =1=Tan 45°

$\therefore \angle$PRT = 45°

So, the required angle of elevation is 45°.



Example- 7:

An electric pole is erected at the centre of a circle of radius 10m. If the angle of elevation of the top of the pole is observed to be 60°from the circumference of the circle, what will be the height of the pole?

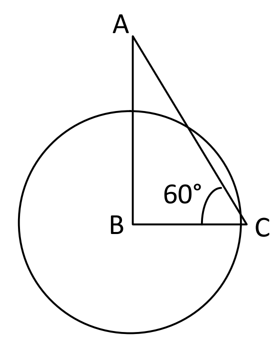
By the question, BC=10m, AB =? and <ACB =60°

In the right angled triangle ABC, we have, $\frac{AB}{BC}$ = Tan 60°

Or AB = BC Tan 60° = 10x$\sqrt{3}$

$\therefore$ AB = 10$\sqrt{3}$m

So the required height of the pole is 10$\sqrt{3}$.



Example- 8:

The shadow of a vertical pole of height 30$\sqrt{3}$m is found to be 90m at 4Pm. What will be the angle of inclination (elevation) of the sun at that time?

Solution:

Let PQ be the vertical pole and QR be its shadow at 4pm.

Join PR. Then in the right angled triangle PQR,

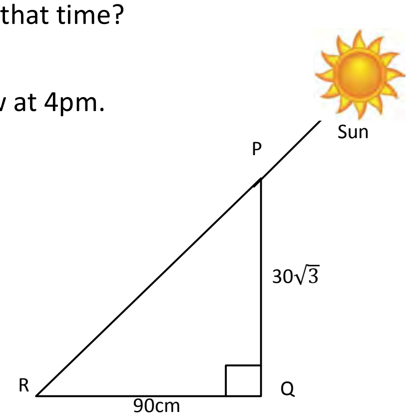
$\frac{PQ}{QR}$ = Tan $\angle$PRQ

Or $\frac{30\sqrt{3}}{90m}$ = Tan $\angle$R

Or Tan <R = $\frac{\sqrt{3}}{3}$ =$\frac{1}{\sqrt{3}}$ Tan 30°

$\therefore \angle$R = 30°

So the inclination of the sun at 4pm is 30°.



Example 9:

A pigeon on the roof of a house is observed from the top of a tower of height 120m at an angle of 30° to the horizontal line. Find the height of the house if the distance between the tower and the house is $60\sqrt{3}m$.

Solution:

Let EF be the height of the tower and GH be the height of the house.

Draw, EQ ll FH ll GP Then $\angle$QEG = $\angle$EGP = 30°

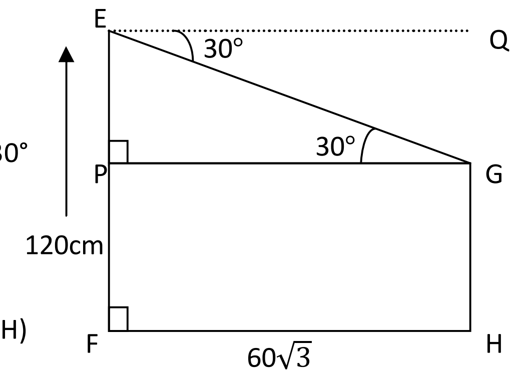
Now, in the right angled triangle EPG, $\frac{EP}{PG}$ = Tan 30°

Or $\frac{EP}{FH} = \frac{1}{\sqrt{3}}$ ($\because$ PG = FH)

Or EP = FH/$\sqrt{3}$ = $\frac{60\sqrt{3}}{\sqrt{3}}$ = 60m

$\therefore$ PF = EF – EP = 120m - 60m = 60m = GH (PF = GH)

So the height of the house is 60m.



Example- 10:

A tall tree breaks because of a strong wind. If 30m long broken part of the tree meets the ground making 30° angle with the horizontal level, how tall was the tree and how far does it meet the ground level from the bottom of the tree?

Solution:

Let PQ be the tree and MN be the broken part such that PM = MN = 30m. By the question, $\angle$MNQ =30°

Now in the right angled triangle MNQ, $\frac{NQ}{MN}$= Cos 30°

Or NQ = MN cos 30°

$= 30\frac{\sqrt{3}}{2} = 15\sqrt{3}$m

$\therefore$ NQ =$15\sqrt{3}$m

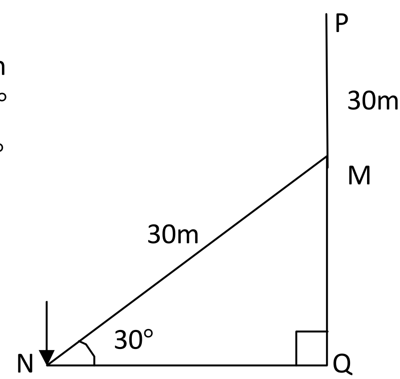
Again, $\frac{MQ}{MN}$ = sin 30° $= \frac{1}{2}$

Or, MQ = MN x $\frac{1}{2}$ = 30 x $\frac{1}{2}$ = 15m

So the total height of the tree = MQ + PM

= 15 + 30 = 45m

The broken part meets the ground $15\sqrt{3}$ m away from the bottom of the tree.



Exercise 16.2

1.(a) An observer sees the top of a tower at a distance of $28\sqrt{3}$m from the point of observation on the ground level. If the angle of elevation of the tower is found to be 30°, find the height of the tower.

(b) An observer is measuring the angle of elevation of a tower of height $50\sqrt{3}$m from the horizontal point 50m away from the foot of the tower. What angle will he measure ?

(c) A woman looks at the top of a tree which is 36m high from a point on the ground at on angle of 30° to the horizontal line. How far is she from the bottom of the tree?

(d) A man of height 1.5m observes the top of a telephone tower from a horizontal point 30m away from the bottom of the tower. If the angle of elevation of the tower is 60°, what will be the height of the tower ?

2.(a) A person looks at a pigeon on the ground level from his house of height 50m. If the pigeon be $50\sqrt{3}$m away from the bottom of the house, what will be the angle of depression of the pigeon?

(b) A shadow of a man of height $5\sqrt{3}$ft. is 15ft at 4 Pm. What will be the length of the shadow of a pole which is three times higher than the man at the same time, i.e. 4pm?

(c) A man, 1.4m tall, is inspecting the behavior of a bird sitting at the top of a pole of the height 43.4m. If the man is standing $14\sqrt{3}$m away from the bottom of the pole on the horizontal level, at what angle is he looking at the bird?

(d) A cat is focusing its eyes on a rat eating potatoes on the ground from a certain height. If the angle of depression is found to be 60° for the rat which is $12\sqrt{3}$m away from the height of the cat, how high is the cat sitting at?

3. (a) A poacher is targeting a dove sitting on the ground level from the roof of the house which is 45m high. If the dove be 45m away from the bottom of the house, at what angle should the poacher fix his catapult not to miss the target?

(b) A tree of height 40m is situated on the bank of a river. If the angle of elevation of the tree observed from the opposite bank of the same river be 45°, what will be the breadth of the river?

(c) A lamp post is erected at the centre of a circular pond of radius $12\sqrt{3}$m and depth 16m. If the angle of elevation of the lamp post observed from a point on the edge of the pond is found to be 30°, find the height of the lamp post from the bottom of the pond.

(d) An electric post is erected at the centre of a circular pond. The angle of elevation of the post of height $30\sqrt{3}$m from the level of water is 60°. What will be the shortest distance between the centre and the circumference of the circular pond, if that angle of elevation is measured from a point of the circumference of the pond?

4.(a) What will be the angle of elevation (inclination) of the sun if the length of the shadow of a vertical pole of height $10\sqrt{3}$m is 30m?

(b) What will be the length of the shadow of a vertical pole of height $18\sqrt{3}$m when the angle of elevation of the sun is 30°?

(c) What will be the height of a vertical pole when the length of its shadow is $36\sqrt{3}$m and the angle of elevation of the sun is 60°?

(d) A tall tree of height 51m is broken because of the strong wind. If the broken part of the tree touches the ground level and makes an angle of 30° with the horizontal level, what will be the length of the broken and remaining parts of the tree?

5. (a) A tall tree of height 60m is broken due to the strong wind. If the upper 40m broken part of the tree touches the level ground, calculate the angle made by the broken part with the horizontal line. Also calculate the horizontal distance between the bottom of the tree and the point at which the broken part of the tree touches the ground level.

(b) The angle of elevation of the top of a tree observed from the roof of a house which is 18m high is 30°. If the height of the tree be 38m, find the distance of the house from the bottom of the tree.

(c) A man of height 1.7m is observing a bird sitting at the top of a tree of height 53.7m at an angle of 30° with the horizontal direction. Find the distance between the man and the tree.

(d) The angle of depression of a house of height 15m observed from the top of a tower is 60°. Find the distance between the house and the tower if the height of the tower is 30m.

6 (a) The angle of elevation of a pole of height $150\sqrt{3}$m observed from the roof of a house is found to be 30°. If the distance between the house and the pole is 60m, find the height of the house.

(b) An observer of height 1.4m is observing the top of a telegraph tower of height 91.4m at a distance of $30\sqrt{3}$m from the tower. Find the angle of elevation of the top of the tower.

(c) The angle of depression of a boat sailing on the ocean is observed from the top of a cliff. If the height of the cliff is $20\sqrt{3}$m and the distance of the boat from the bottom of the cliff is 60m, what will be the angle of depression of the boat?

(d) A ladder rests on the vertical wall at a height of 18m. If the lower end of the ladder is $6\sqrt{3}$m away from the bottom of the wall, find the angle made by it with the ground level. Also find the length of the ladder.

7.(a) A lamp post is erected inside the water of a circular pond . If the angle of elevation of the top of the lamp post observed from two opposite points of the circumference of the pond are 30° and 60° and the height of the post above the level of water is 21m, what will be the diameter of the circular pond ? Also find the circumference of the pond ($\pi = \frac{22}{7}$).

(b) From the roof of a house of 90m height, the top of a pillar of height 30m is observed at an angle of 45° with the horizontal. Find the distance between the house and the pillar.

(c) 150m long cord of a kite has been unwound out by a boy who is flying it from a roof of a house of 20m height. If the angle of elevation of the kite be 30° from the roof of the house, find the height of the kite from the ground level.

(d) An electrical pole is erected at the centre of a circular pasture. If the height of the pole be $30\sqrt{3}$m and its angle of elevation from the observer sitting on the circumference of the pasture be 30°, find the diameter and the circumference of the pasture (Take $\pi = \frac{22}{7}$).

8. Make groups of students involving 3 students each. Then measure the height of the following objects by using the concepts of height and distance. Present them to the class and publish them in your school magazine if possible.

Objects:

i) Your own house

ii) Your own school building

iii) A temple

iv) A stupa

v) A church

vi) A tree or a pole

Unit: 17

Statistics

17.0 Review:

Discuss on groups and find the answer of the following questions.

i. Collect the marks obtained by each members of group in mathematics in mid-term examination.

ii. Find mean score (mark) and median mark of each group.

iii. Present the result to the class.

iv. Present the scores of all marks and list on the board. Can you calculate mean and median of the total data as before? Discuss in groups.

To calculate control values from large number of data we have to make frequency distribution from given data set. We are going to discuss frequency distribution.

17.1 Frequency distribution:

If there are small number of data repeated many times, then we can make the table of data with respective frequencies of the data series which is called discrete series.

If the data number is neither small nor repeated, what can we do? Discuss?

In this case we make the suitable class interval and make tally bar for the data that lie in that interval. At last column, we write the total number of tally in each interval called frequency.

Let the marks obtained by 40 students in a midterm exam be as follows:

25, 10, 31, 22, 37, 42, 45, 37, 32, 34, 45, 40, 29, 27, 28, 17, 19, 22, 25, 15, 14, 13, 28, 36, 38, 41, 42, 39, 25, 24, 31, 21, 22, 25, 26, 35, 36, 39, 49, 98.

Here, the minimum score is 10, so we make the following frequency distribution on table as the class interval 10-20 and so on.

Interval	Tally bar	No of student
10-20	𝍸 I	6
20-30	𝍸 𝍸 IIII	14
30-40	𝍸 𝍸 III	13
40-50	𝍸 II	7

The process of representation of data by using table is called frequency distribution. This table is called frequency table and the number of students in each class interval is called frequency of that class interval. In each range, the first data value (number) is called lower limit and the largest data value (number) is called upper limit of that interval (range).

Example 1:

Construct frequency table of class interval 10 of the following data:

8, 46, 32, 38, 15, 46, 22, 26, 13, 14, 12, 54, 9, 25, 27, 45, 53, 18, 32, 6, 34, 31, 38

Solution:

The minimum value of data is 6, so the first class interval is 0-10 and so on. Then frequency distribution is

Class interval	Tally Bar	frequency
0-10	ǀǀǀ	3
10-20	卌	5
20-30	ǀǀǀǀ	4
30-40	卌 ǀ	6
40-50	ǀǀǀ	3
50-60	ǀǀ	2

Here, 0 is lower limit and 10 is upper limit of 0-10.

The range of interval = length of interval = 10 – 0 = 10.

Exercise 17.1

1. Construct a frequency table of each of class interval 10 of the following data.

Age of family members (in years)
9, 20, 35, 42, 36, 2, 7, 15, 21, 25, 43, 53, 40, 38, 36, 22, 69, 65, 51, 47, 4, 14, 28, 60, 72, 77, 34, 21, 16, 75, 8, 15, 16, 29, 44

2. Construct a frequency table of class interval 4 of the following data.

weight of 30 students (in kg.)
31, 32, 31, 36, 45, 47, 50, 53, 60, 32, 35, 37, 45, 41, 55, 44, 48, 65, 63, 68, 40, 45, 49, 52, 35, 33, 39, 54, 32

3. The marks obtained by 50 students in a test is given below. Construct the frequency distribution table of class interval 30-40 with first class.

Marks obtained by 50 students
74, 62, 71, 63, 79, 73, 35, 43, 49, 48, 56, 59, 32, 35, 72, 58, 57, 62, 38, 49, 45, 42, 44, 43, 48, 52, 56, 72, 64, 39, 48, 62, 77, 44, 39, 75, 79, 83, 84, 81, 66, 69, 35, 44, 30, 83, 77, 44, 55, 48

4. The hourly wages of 36 workers of a factory are given below.

   74, 71, 79, 68, 74, 73, 63, 62, 84, 61, 75, 72, 79, 76, 67, 72, 61, 60, 69, 77, 81, 68, 67, 83, 72, 74, 78, 84, 80, 71, 66, 81, 64, 64, 73, 68

   Construct the frequency distribution of the above data with class interval of 5.

5. Divide the students in the suitable groups and tell them to collect the age of about 100 neighbors of family members and represent them in frequency distribution table.

7.2. Central tendency

The value of the given set of date that represent the characteristics of entire data is called central value. The calculation of such value is called measure of central tendency. The most common measure of central tendencies is mean, median, mode and quartiles.

7.2.1 Mean:

Mean: The mean is the sum of numerical values of each and every observation divided by the total number of observation. It is denoted by $\bar{x}$ (x bar) for variable x.

If $x_1, x_2,\ldots\ldots x_n$ be n discrete values of variable x then their arithmetic mean is given by $\bar{x} = \frac{x_{1+}x_{2+}\cdots x_n}{n} = \frac{\sum x}{n}$

If $x_1, x_2,\ldots\ldots x_n$ be n discrete values with respective frequencies $f_1, f_2,\ldots\ldots, f_n$, their arithmetic mean is given by

$\bar{x} = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \ldots\ldots + f_n} = \frac{\sum fx}{\sum f} = \frac{\sum fx}{N}$ Where N $=\sum f$ = Total frequency.

We have already discussed about those two formulae in our previous classes. Now we are going to discuss about the mean of grouped or continuous data.

Arithmetic mean for grouped data or continuous series

In grouped data (continuous series), the observations are classified with some suitable range values along with their class frequencies. To calculate arithmetic mean of grouped data, first we have to find the mid-value of each interval (range) as shown below

Mid-value (m) = $\frac{Lower\ limit + Upper\ limit}{2}$

Then we can use the formula of calculation of mean of discrete series with placing value of mid-value (m) instead of value of variable x.

Calculation of mean of grouped data

(a) If $m_1, m_2, \ldots\ldots. m_n$ be mid values of n intervals with frequencies $f_1, f_2, \ldots\ldots., f_n$ respectively then the mean is calculated by

$$\bar{x} = \frac{f_1 m_1 + f_2 m_2 + \cdots + f_n m_n}{f_1 + f_2 + \ldots\ldots + f_n} = \frac{\sum fm}{\sum f} = \frac{\sum fm}{N}$$

This method is called direct method of calculation of mean.

Example 1:

Calculate the mean (arithmetic mean) of the data given below.

weight of students (in Kg.)	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	12	18	27	20	17	6

Solution:

Calculation of mean,

Here,

weight (In kg.)	No of students (f)	mid value (m)	f.m
0-10	12	$\frac{0+10}{2} = 5$	60
10-20	18	$\frac{10+20}{2} = 15$	270
20-30	27	$\frac{20+30}{2} = 25$	675
30-40	20	$\frac{30+40}{2} = 35$	700
40-50	17	$\frac{40+50}{2} = 45$	765
50-60	6	$\frac{50+60}{2} = 55$	330
	$\sum f = \text{N} = 100$		$\sum fm = 2800$

Now,

We know, the mean $(\bar{X}) = \frac{\sum fm}{N} = \frac{2800}{100} = 28$ kg.

Therefore, the mean is 28kg.

(b) Deviation method (shortcut method)

Let us consider A = assumed mean of the data, then d = m-A

The mean is calculated by $\bar{x} = A + \frac{\sum fd}{N}$

Look at the above example, suppose A = 25

Weight (in Kg.)	**No. of students (*f*)**	**mid value (m)**	**d = m-A**	***f*.d**
0-10	12	5	-20	-240
10-20	18	15	-10	-180
20-30	27	25	0	0
30-40	20	35	10	200
40-50	17	45	20	340
50-60	6	55	30	180
	N = 100			$\sum fd = 300$

We know that, mean ($\bar{x}$) = $A + \frac{\sum fd}{N}$

=25 + $\frac{300}{100}$ = 25 + 3 = 28kg.

(c) Step deviation method: If h be the size of class and A be assumed the mean of given data set, we can calculate arithmetic mean $\bar{x}$ as below

$\bar{x} = A + \frac{\sum fd'}{N}$xh, where d'= $\frac{m-A}{h}$ and h = mid value of every interval

In the above example,

Suppose A = 25 and h = 10, then

weight (in Kg.)	mid value (m)	No. of students (*f*)	$d' = \frac{m-25}{10}$	f.d'
0-10	5	12	-2	-24
10-20	15	18	-1	-18
20-30	25	27	0	0
30-40	35	20	1	20
40-50	45	17	2	34
50-60	55	6	3	18
		N = 100		$\sum fd' = 30$

We have $\bar{X} = A + \frac{\sum fd'}{N}$xh

$= 25 + \frac{30}{100}$ x 10 = 25 + 3 = 28

Example 2:

If $\sum fm$ = 2700 and N = 50, find $\bar{x}$

Solution:

Here, $\sum fm$ = 2700, N = 50, $\bar{X}$ = ?

we know, $\bar{X} = \frac{\sum fm}{N} = \frac{2700}{50} = 54$

Example 3:

If assumed mean A = 40, $\sum fd = 20$ and mean ($\bar{x}$) = 42, find the value of N.

Solution:

Here, A=40, $\sum fd$ = 20, $\bar{X}$ =42, N =?

We know, $\bar{x}$ = A+$\frac{\sum fd}{N}$

Or, 42 = 40 +$\frac{20}{N}$

Or, $\frac{20}{N}$ = 42-40

Or, 2N = 20

$\therefore$ N = 10

Example 4:

If the mean height of the following data is 157.75 cm, find the value of K.

Height (cm)	140-145	145-150	150-155	155-160	160-165	165-170	170-175
No of Students	2	5	8	k	7	5	3

Solution:

Calculation of arithmetic mean

Height (in cm)	No of students (*f*)	mid value (m)	f.m
140-145	2	142.5	285
145-150	5	147.5	737.5
150-155	8	152.5	1220
155-160	k	157.5	157.5k

Height (in cm)	No of students (f)	mid value (m)	f.m
160-165	7	162.5	1137.5
165-170	5	167.5	837.5
170-175	3	172.5	517.5
	N = 30+k		$\sum fm$=4735+157.5k

We know that, $\bar{X} = \frac{\sum fm}{N}$

Or, 157.75 = $\frac{4735+157.5k}{30+k}$

Or, 4732.5+157.75k = 4735+157.5k

Or, 157.75k-157.50k = 4735-4732.5

Or, 0.25k = 2.5

∴ k = 10

Example 5:

Calculate the mean of the following data by constructing frequency table of class interval of length 10.

7, 22, 32, 47, 59, 16, 36, 17, 23, 39, 49, 31, 21, 24, 41, 12, 49, 21, 9, 8, 51, 36, 29, 18

Solution:

Construction of frequency table.

Class	Tally bar	Frequency (f)	Mid-value (m)	f×m
0-10	III	3	5	15
10-20	IIII	4	15	60
20-30	~~IIII~~ I	6	25	150
30-40	~~IIII~~	5	35	175
40-50	IIII	4	45	180
50-60	II	2	55	110
		N = 24		$\sum f\times m$=690

We have mean $\overline{X} = \frac{\sum f\times m}{N}$

$= \frac{690}{24} = 28.75$

∴ mean $\overline{X}$ = 28.75

Exercise 17.2

1. Find the mean of the following data.

   (a) 35, 36, 42, 45, 48, 52, 58, 59

   (b) 13.5, 14.2, 15.8, 15.2, 16.9, 16.5, 17.4, 19.3, 15.2

   (c)

x	5	8	10	12	14	16
f	4	5	8	10	2	2

   (d)

Age (in yrs)	12	13	14	15	16	17
No. of students	2	4	6	12	10	6

2. Calculate the mean of the following date by using direct method.

   (a)

Age (yrs)	0-10	10-20	20-30	30-40	40-50
No. of children	5	9	15	7	4

   (b)

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	1	4	10	8	7	5

   (c)

Daily wages (Rs)	200-400	400-600	600-800	800-1000	1000-1200
No. of workers	3	7	10	6	4

   (d)

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	7	5	6	12	8	2

3. Calculate the mean of the Q.(2) by

   i. Deviation method/short cut method

   ii. Step deviation method

4. Calculate the missing part of the following.

   (a) $\bar{x}$ = 49, $\sum fm$ = 980, N =?   (b) $\bar{x}$ = 102.25, N = 8, $\sum fm$ =?

(c) A = 100, $\bar{x}$ = 90, $\sum fd$ =?, N = 10 (d) $\bar{x}$ = 41.75, $\sum fd$ =270, N = 40, A =?

5.(a) If the mean of the given data is 32.5, find the value of k.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
Number of students	5	10	K	35	15	10

(b) If the mean of the following data is 14.2, find the value of P.

X	0-20	20-40	40-60	60-80	80-100
f	35	400	350	p	65

(c) If the mean age of the workers in a factory of the following data is 36.24, find the value of y.

Age (years)	16-24	24-32	32- 40	40-48	48-56	56-64
No. of workers	6	8	Y	8	4	2

(d) If the average expenditure per week of the students is Rs. 264.67, find the missing frequency.

Expenditure (Rs.)	0-100	100- 200	200-300	300-400	400-500	500-600
No. of students	20	30	?	20	18	12

6. Calculate mean of the following data by constructing frequency distribution table.

(a) 15, 51, 32, 12, 32, 33, 23, 43, 35, 46, 57, 19, 59, 25, 20, 38, 16, 45, 39, 40 (construct table of length 10)

(b) 25, 15, 24, 42, 22, 35, 34, 41, 33, 38, 54, 50, 36, 40, 27, 18, 35, 16, 51, 31, 23, 9, 16, 23, 31, 51, 7, 30, 17, 40, 60, 32, 50, 10, 23, 12, 21, 28, 37, 20, 58, 39, 10, 41, 13 (class of length 5)

7. (a) Find mean of the following data

X	0-9	10-19	20-29	30-39	40-49	50-59
F	8	10	14	10	8	10

(b)

Expenditure	0-400	500-900	1000-1400	1500-1900	2000-2400	2400-2800
No. of workers	1	2	3	4	1	2

8. Divide all students into the groups of 4. Collect the data about their age of at least 50 students of different classes from class 1 to 12, of your school. Construct the frequency distribution table. Calculate mean by using direct and deviation method. Prepare a report and present to the class.

17.2.2 Median:

The central value of a distribution that divides the entire data set into exactly two equal parts is called median. Median is also called mid-value of the distribution. The half of the data of the distribution lie below the median value and rest half of the data are above the median value. It is denoted by M_d.

For individual series to calculate the median;

i. Arrange all data in ascending or descending order.

ii. Use the formula $(\frac{n+1}{2})^{th}$ item, where n = total number of data in set.

Similarly, for discrete series the following three steps are used.

iii. Construct cumulative frequency distribution table.

iv. Find $\frac{N+1}{2}$, where N = $\sum f$, sum of frequencies = total numbers of data in data sets.

v. See cumulative frequency equal or just greater than $\frac{N+1}{2}$

vi. Locate the corresponding value of x in the table. which is median value.

Calculation of the median from grouped or continuous data

The following steps are used to calculate median of continuous data or grouped data:

i. Prepare less than cumulative frequency distribution table.

ii. Calculate $\frac{N}{2}$ to find the position of the median.

iii. See the cumulative frequency equal or just greater than $\frac{N}{2}$ and identify the median class (interval).

iv. Use following formula to find the median value.

M_d = L + $\frac{\frac{N}{2}-cf}{f}$ xh

Where, L = Lower limit of the median class

N = Total Frequency

c.f = Cumulative frequency of class preceding to median class

f = frequency of the median class

h = size of the median class

Example 1:

Calculate the median from the following data.

Mark obtained	15	18	22	2 6	27	29
No. of students	3	4	8	10	7	5

Solution:

Calculation of median

Mark obtained (x)	No of students (f)	cumulative frequency (cf)
15	3	3
18	4	7
22	8	15
26	10	25
27	7	32
29	5	37
	Σf = N =37	

We have,

The position of Median (M_d) $= \left(\frac{N+1}{2}\right)^{th}$ $item$

$= (\frac{37+1}{2})^{th} = 19^{th}$ item

∴ The cumulative frequency equal to or greater than 19 is 25. Therefore, the values of X corresponding to 25 is 26

Therefore, Median (M_d) = 26

Example 2:

Calculate the median of the following distribution.

Score of students	0-8	8-16	16-24	24-32	32-40	40-48	48-56
No. of students	6	10	16	18	12	10	8

Solution:

Calculation of median

Score of students (x)	No of students (f)	Cumulative frequency (c.*f*)
0-8	6	6
8-16	10	16
16-24	16	32
24-32	18	50
32-40	12	62
40-48	10	72
48-56	8	80
	Σf = N= 80	

Now, the position of median class $= (\frac{N}{2})^{th}$ item

$= (\frac{80}{2})^{th}$ item

$= 40^{th}$ item

The value of cumulative frequency equal or greater than 40 is 50.

So, median class is 24-32, where L = 24, c.f. = 32, f = 18, h = 8

By formula, Median (M_d) = L + $\frac{\frac{N}{2}-cf}{f}$ xh

$= 24 + \frac{40-32}{18}$ x8

$= 24 + \frac{64}{18}$

= 24 + 3.56 = 27.56

Example 3:

Find the missing frequency of the following distribution if the median value is 93.6

X	0-30	30-60	60-90	90-120	120-150	150-180
f	5	p	22	25	14	4

Solution:

Table for calculation of frequency

X	*f*	c.*f*.
0-30	5	5
30-60	p	5+p
60-90	22	27+p
90-120	25	52+p
120-150	14	66+p
150-180	4	70+p
	Σf = N = 70 + p	

Given that, M_d = 93.6

M_d lies in 90-120, where L = 90, f =25, c.*f*. = 27+p, h =30, N =70+p

Now,

M_d = L+ $\frac{\frac{N}{2}-cf}{f}$ xh

Or, 93.6 = 90+ $\frac{\frac{(70+P)}{2}-(27+P)}{25}$ x30

Or, 93.6-90 = $\frac{70+P-54-2P}{50}$ x30

Or, 3.6 = (16 - P)x$\frac{3}{5}$

Or, 3.6x5 = 48-3p

Or, 3P = 48 -18

Or, 3p = 30

$\therefore$ p = 10

∴ Missing frequency (P) = 10

Example 4:

Find the median height of the plants of the following data.

Height (in cm)	4-6	7-9	10-12	13-15	16-18	19- 21	22-24
No. of plants	2	3	10	7	4	3	2

Solution:

Here the classes are discontinuous. So we need to make them continuous by using adjustment/continuity/correction factor.

Correction factor = $\frac{Lower\ limit\ of\ second\ interval - upper\ limit\ of\ first\ interval}{2}$

= $\frac{7-6}{2}$ = 0.5

The class intervals are made continuous by adding 0.5 in upper limit and subtracting 0.5 in lower limit of each class interval.

Than the table for calculation of median is as follows

Height (cm)	**No. of plants (*f*)**	**Cumulative frequency**
3.5-6.5	2	2
6.5-9.5	3	5
9.5-12.5	10	15
12.5-15.5	7	22
15.5-18.5	4	26
18.5-21.5	3	29
21.5-24.5	2	31
	Σf = N = 31	

Now, the median class = $\left(\frac{N}{2}\right)^{th}$ item

= $\left(\frac{31}{2}\right)^{th}$ item = 15.5^{th} item

The value of c.f equal or greater than 15.5 is 22 in the column of cumulative frequency.
So median class is 12.5-15.5 where, L = 12.5, c.f = 15, f = 7 and h = 3

Median (M_d) = L+ $\frac{\frac{N}{2}-cf}{f}$ x h

= 12.5 + $\frac{15.5-15}{7}$ x 3

= 12.5 + $\frac{1.5}{7}$ = 12.71

∴ Median height of plant is 12.71 cm

Exercise 17.3

1. Calculate the median from the following data.

(a) 2.5, 4.5, 3.6, 4.9, 5.4, 2.9, 3.1, 4.2, 4.6, 2.2, 1.5

(b) 100, 105, 104, 197, 97, 108, 120, 148, 144, 190, 148, 22, 169, 171, 92, 100

(c)

Marks	18	25	28	29	34	40	44	46
No. of students	3	6	5	7	8	12	5	4

(d)

X	102	105	125	140	170	190	200
f	10	18	22	25	15	12	8

2. Calculate the median from the following frequency distribution table.

(a)

Wt (kg)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	5	7	11	10	3	1

(b)

Height (cm)	140-145	145-150	150-155	155-160	160-165	165-170	170-175
Frequency	2	5	8	10	7	5	3

(c)

Expenditure per day (Rs.)	0-100	100-200	200-300	300-400	400-500	500-600
Frequency	22	34	52	20	19	13

(d)

Marks obtained	less than 20	40	60	80	100
No. of students	21	44	66	79	90

3. Calculate the missing frequencies in the following table where;

(a) Median (M_d) = 35

Mark obtained	20-25	25-30	30-35	35-40	40-45	45-50
No. of students	2	5	8	k	4	5

(b) Median (M_d) = 132.5

Wages	100-110	110- 120	120-130	130-140	140-150	150-160
No. of workers	5	6	p	4	7	5

(c) Median (M_d) = 36

Age (yr)	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
No. of persons	50	70	100	300	k	220	70	60

4. Calculate the median of the following data:

(a)

Mark obtained	50-60	60-70	70-80	80-90	90-100
No. of students	2	5	11	16	20

(b)

Mark obtained	less than 20	40	60	80	100
No. of students	14	23	26	21	16

(c)

Income (Rs)	less than 600	700	800	900	1000
No. of workers	30	98	152	177	200

(d)

Temp (°c)	0-9	10-19	20-29	30-39	40-49
No. of days	8	10	20	15	7

5.(a) The mark obtained by 40 students of a class in a certain exam is as follow. Construct a frequency distribution table of class interval of 10 and calculate the median.

22, 56, 62, 37, 48, 30, 58, 42, 29, 39, 37, 50, 38, 41, 32, 20, 28, 16, 43, 18, 40, 52, 44, 27, 35, 45, 36, 49, 55, 40

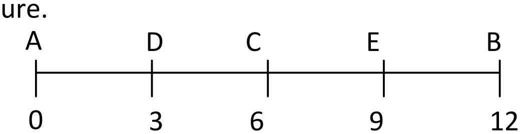
(b) The height (in cm) of 40 students of grade X is given below. Construct a frequency distribution table of class interval 5 and find the median.

142, 145, 151, 157, 159, 160, 165, 162, 156, 158, 155, 141, 147, 149, 148, 159, 154, 155, 166, 168, 169, 172, 174, 173, 176, 161, 164, 163, 149, 150, 154, 153, 152, 164, 158, 159, 162, 157, 156, 155.

6. Work in group of 5. Collect the age of 50 students of grade play group to grade 12 randomly and construct a frequency distribution table of suitable interval. Calculate the median of the age and present all process to the class.

17.3 Quartiles

Draw a line of length 12 cm or take a stick of length 12 inch. Mark a point on it such that it is at equal distance from each end points. Again mark points on the parts so that they are divided into two equal parts as shown in figure.



In this case, we can see that there are three points which divide the line/stick into four equal parts.
We call them quartile. They are denoted by Q_1, Q_2 and Q_3 respectively. Also note that Q_2 is median, since it divides the distribution into two equal parts.

Note: Q_1 is called lower quartiles and Q_3 is called upper quartile.

Calculation of quartiles

a) For individual series first arrange all data in ascending order and then use formula $(\frac{n+1}{4})^{th}$ item for Q_1 and $(\frac{3(n+1)}{4})^{th}$ item for Q_3 to locate the value of quartiles.

b) For discrete series we have to use the following steps.

i. Construct less than cumulative frequency distribution table.

ii. Use formula $(\frac{N+1}{4})^{th}$ for Q_1 and $(\frac{3N+1}{4})^{th}$ for Q_3 to locate the quartiles and find the value of quartiles in the column of X which is the corresponding value of c.*f*. just greater than $\frac{N+1}{4}$ and $\frac{3(N+1)}{4}$ respectively.

(c) To calculate quartiles Q_1 and Q_3 from continuous series, we have to follow the following steps.

i. Construct cumulative frequency distribution table.

ii. Find the values of $\frac{N}{4}$ and $\frac{3N}{4}$ for Q_1 and Q_3 respectively,

where N = $\sum f$ = total frequency

iii. The corresponding class interval of value of $\frac{N}{4}$ or greater than $\frac{N}{4}$ in the column of c.*f* for Q_1 and $\frac{3N}{4}$ or greater than $\frac{3N}{4}$ in c.*f* for Q_3 are called the classes of Q_1 and Q_3 respectively.

iv. Use the formula

$Q_1 = L + \frac{\frac{N}{4} - cf}{f} \times h$

where L = Lower limit of class containing Q_1,

c.*f* = Cumulative frequency of the class preceding the class containing Q_1,

f = frequency of class containing Q_1, and

h = length of class containing Q_1.

$Q_3 = L + \frac{\frac{3N}{4} - cf}{f} \times h$

Where, L = Lower limit of class containing Q_3

c.*f* = Cumulative frequency of the class preceding the class containing Q_3

f = frequency of class containing Q_3, and

h = length of class containing Q_3

Example 1:

Calculate the values of Q_1 and Q_3 from the following data.

Age of workers	20	25	28	30	32	35	42	46
No of workers	2	8	12	10	14	7	5	1

Solution:

Construction of cumulative frequency data

Age (X)	No of workers (*f*)	c.*f*.
20	2	2
25	8	10

28	12	22
30	10	32
32	14	46
35	7	53
42	5	58
46	1	59
	Σf = N = 59	

Here, the position of $Q_1 = (\frac{N+1}{4})^{th}$item

$= (\frac{60}{4})^{th}$item

$= 15^{th}$ item

∴ The value of c.f equal to or just greater than 15 is 22.

∴ The corresponding value of c.f. 22 in X is Q_1. i.e. $Q_1 = 28$

Again, the position of $Q_3 = (\frac{3(N+1)}{4})^{th}$ item

$= (\frac{3(60)}{4})^{th}$ item $= 45^{th}$ item

The value of c.f. equal to or just greater than 45 is 46.

∴ Q_3 is the value of X corresponding to c.f. 46

i.e. $Q_3 = 32$

Example 2:

Calculate the values of Q_1 and Q_3 from the following distribution table.

Mark obtained	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No of students	2	8	15	14	10	8	3

Solution:

Calculation of quartiles,

Mark obtained	frequency (f)	Cumulative Frequency (c.f.)
10-20	2	2
20-30	8	10
30-40	15	25
40-50	14	39
50-60	10	49

60-70	8	57
70-80	3	60
	Σf = N = 60	

Now, the position of $Q_1 = (\frac{N}{4})^{th}$ item

$= (\frac{60}{4})^{th}$ item

$= 15^{th}$ item

$\therefore$ Q_1 lies in the interval 30-40 since 25 is just greater than 15 in column c.f

$\therefore$ L = 30, c.f = 10, f = 15 and h =10

Now, $Q_1 = L + \frac{\frac{N}{4} - cf}{f} \times h$

$= 30 + \frac{15-10}{15} \times 10$

$= 30 + 3.34 = 33.34$

Again, the position of $Q_3 = \left(\frac{3N}{4}\right)^{th}$ item

$= \left(\frac{3x60}{4}\right)^{th}$ item

$= 45^{th}$ item

The value of c.f just greater than 45 is 49 in the column of c.f. So, Q_3 lies in the class 50-60.

Where, L = 50, c.f= 39, f = 10 and h = 10

We have, $Q_3 = L + \frac{\frac{3N}{4} - cf}{f} \times h$

$= 50 + \frac{45-39}{10} \times 10$

$= 50 + 6 = 56$

Exercise 17.4

1. Calculate the values of Q_1 and Q_3 from the following data.

(a) 10, 12, 14, 11, 22, 15, 27, 14, 16, 13, 25

(b) 250, 200, 150, 180, 190, 205, 208, 230, 155, 145, 149, 225, 202, 206, 257.

(c)

Mark	42	48	49	53	56	59	60	65	68	70
No of students	2	3	5	8	9	11	7	8	6	4

(d)

Wages	<200	210	215	220	225	230	>230
No. of workers	8	15	25	22	18	14	11

(e)

Marks	<35	<40	<50	<55	<60	<65	<75	<85
No. of students	3	10	22	40	70	95	110	123

2. Calculate the values of Q_1 and Q_3 from the following data.

(a)

Age of students	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
No. of students	5	12	25	26	24	28	20	15

(b)

Mark obtained	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	2	3	6	12	13	11	7

(c)

Height (cm)	100-110	110-120	120-130	130-140	140-150	150-160	160-170
No. of students	3	4	9	15	20	14	7

(d)

Wages (Rs)	100-150	150-200	200-250	250-300	300-350	350-400
No. of workers	6	11	21	34	25	22

(e)

Class	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency	8	12	15	14	12	9	10

3.(a) If $Q_1 = 8$, find value of k in the following table.

Age (yr)	0-6	6-12	12-18	18-24	24-30	30-36
No. of persons	9	6	5	k	7	9

(b) If $Q_1 = 31$, find value of missing frequency in the following table.

Class	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	5	?	8	7	6

(c) If $Q_3 = 51.75$, then find the value of k in the following table.

Weight (in kg)	40-44	44-48	48-52	52-56	56-60	60-64
Frequency	8	10	14	k	3	1

(d) What will be value of P if the upper quartile is Rs 460.

Income (Rs)	100-200	200-300	300-400	400-500	500-600
No. of person	15	18	P	20	17

4. Calculate the values of Q_1 and Q_3 from the following data

(a)

Height (cm)	<125	<130	<135	<140	<145	<150	<155
No. of students	0	5	11	24	45	60	72

(b)

Weight (kg)	110-119	120-129	130-139	140-149	150-159	160-169	170-179	180-189
No. of students	5	7	12	20	16	10	7	3

5.(a) The marks obtained by 30 students are as follows.

42, 65, 78, 70, 62, 50, 72, 34, 30, 40, 58, 53, 30, 34, 51, 54, 42, 59, 20, 40, 42, 60, 25, 35, 35, 28, 46, 60

Construct a frequency distribution table of each class length 10 and find the value of Q_1 and Q_3.

(b) Construct the class interval of length 20 and calculate lower and upper quartiles of the following data.

32, 87, 17, 51, P9, 79, 64, 39, 25, 95, 53, 49, 78, 32, 42, 48, 59, 86, 69, 57, 15, 27, 44, 66, 77, 92.

6. Work in groups of 3 students. Collect the data of 100 students of your school about the time required to reach the school from home. Present the data in the frequency distribution table. Find the value that divides the whole data into four equal classes and present your work to the class.

17.4 Use of cumulative frequency curves (Ogives)

Cumulative frequency is useful if detailed information about the data distribution is required. The curves of cumulative frequency are used to calculate the values of quartiles and median. Mainly there are two types of cumulative frequency curves. We call them as less than cumulative frequency curve and more than cumulative frequency curve. (In other words, more than ogives and less then ogives.) The point on X-axis corresponding to the point of intersection of more than cumulative frequency curve and less than cumulative frequency curve is called the median.

Example 1

Draw more than and less than cumulative curve of the following data

Height (cm)	90-100	100-110	110-120	120-130	130-140	140-150
frequency	5	22	30	31	18	6

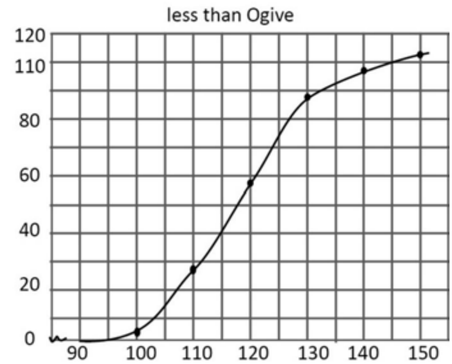
Solution:

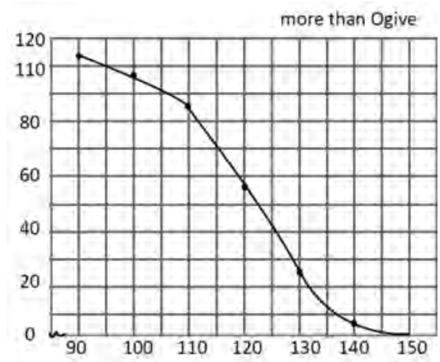
First construct the more than and less than frequency table as follow.

		less than cumulative frequency table		more than cumulative frequency table	
Height (cm)	*f*	Height (cm)	Less than c.*f.*	Height (cm)	More than *c.f.*
90-100	5	less than 100	5	more then 90	112
100-110	22	" " 110	27	" " 100	107
110-120	30	" " 120	57	" " 110	85
120-130	31	" " 130	88	" " 120	55
130-140	18	" " 140	106	" " 130	24
140-150	6	" " 150	112	" " 140	6

Now, for less than ogive, plot the points (100,5), (110,27), (120, 57) (130, 88), (140, 106) and (150, 112) in graph and join the points without using scale.

Similarly, for more than ogive plot the points (90, 112), (100,107), (110, 85), (120, 55), (130, 24), (140, 6) and join them without using scale. See the graph of these two ogives in the following figures.





Calculation of median, upper quartile and lower quartile by using cumulative frequency curve:

The following steps should be completed to find partition value (M_d, Q_1, Q_3) by frequency distribution curves.

i. Find the position of Q_1, M_d and Q_3 in Y-axis by using the formula $\frac{N}{4}$, $\frac{N}{2}$ and $\frac{3N}{4}$ respectively.

ii. Draw horizontal line from a point obtained in Y-axis such that the line meets the frequency curve.

iii. Draw vertical line from the point on the curve at which the horizontal line meet to X - axis.

iv. The point at X- axis is our required value.

Example 2:

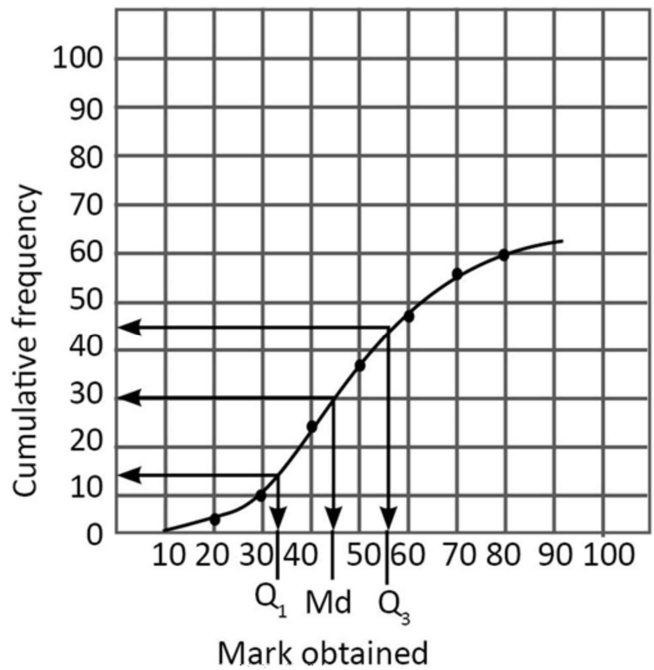
Compute Q_1, M_d, and Q_3 from the given data by using graphic method.

Mark obtained	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	2	8	15	14	10	8	3

Solution:

First construct less than frequency table.

marks less than	cumulative frequency
less than 20	2
30	10
40	25
50	39
60	49
70	57
80	60



Now, from less than ogive

i. Q_1 lies on $(\frac{N}{4})^{th}$ = $(\frac{60}{4})^{th}$ = 15^{th} item in Y-axis

So, the corresponding value of 15 in X-axis is 33.5. So first quartile is 33.5

ii. Median lies on $(\frac{N}{2})^{th}$ =$(\frac{60}{2})^{th}$ = 30^{th} item in Y-axis

So, the corresponding value of 30 in X-axis is 43.5 so median is approximately 44.

iii. Q_3 (upper quartile) lies on $\left(\frac{3N}{4}\right)^{th}$ = 3x15 = 45^{th} item in Y-axis Q_3 lies in the interval 50-60 and value in x axis corresponding to 45 is 56. Therefore, Q_3 = 56.

Exercise 17.5

1. **Draw less than ogive and find median class of the following data.**

(a)

Marks obtained	0-10	10-20	20-30	30-40	40-50	50- 60
No. of students	4	10	20	15	6	5

(b)

Wages (Rs)	100-150	150-200	200-250	250-300	300-350	350-400
No. of workers	5	8	15	12	7	3

(c)

Age of students	4-6	6-8	8-10	10-12	12-14	14-16	16-18
No. of students	7	12	21	15	14	11	10

(d)

Expenses (Rs.)	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No. of students	8	13	17	20	22	18	10	8	4

2. **Construct less than ogive and more than ogive of the following data**

(a)

Marks obtained	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of students	5	6	8	12	15	14	10

(b)

Class interval	5-15	15-25	25-35	35-45	45-55	55-65
Frequency	5	12	30	10	8	5

(c)

Wages (Rs)	20-40	40-60	60-80	80-100	100-120	120-140
No. of workers	4	5	10	8	7	6

(d)

Weight of students	40-44	40-48	40-52	40-56	40-60	40-64	40-68
No. of students	9	10	34	49	60	67	10

3. Calculate median class and value of median from the graph of question number 2.
4. Draw less than ogive of Q.N.1. and find the value of Q_1 and Q_3.
5. The daily expenses (in Rs) of 40 students of a class are given below.

   6, 12, 35, 23, 65, 40, 37, 39, 28, 44, 32, 25, 18, 12, 9, 32, 55, 62, 49, 52, 26, 40, 32, 55, 14, 16, 20, 26, 54, 49, 50, 66, 68, 35, 42, 45, 39, 50, 24, 29.

   Construct a frequency distribution of the class interval 10. After construct more than and less than ogive.

   Also find median, lower and upper quartiles by using graphical method.
6. Work in suitable group of students. Collect the data of 50 students about the numbers of days of their parents' visit in the school per year. Construct frequency distribution table with suitable length of interval. Construct less than and more than ogive. Calculate the class and value of Q_1, Q_2, Q_3 and than present to the class.

Unit: 18

Probability

18.0 Review:

Let's discuss about the following terms in groups.

i. Experiment and sample space with example.

ii. Outcomes and events of rolling two dice together.

iii. Favourable outcomes, equally likely outcomes.

iv. Probability of an event and total probability.

v. Empirical probability and probability scale.

After discussion in group, prepare and present the group report to the class. We have already studied about these concepts in grade 9. Now we are going to discuss about the principles of probability.

18.1. Principles of probability

Before starting this we must know about the following terminologies.

18.1.1. Mutually exclusive events

Write the following events while throwing two dice together.

i. The sum of the numbers displayed is 8.

ii. The sum of the numbers displayed is 9.

iii. Both the dice showing even number.

iv. Both the dice showing odd number.

In which two events have no common element (outcome)? Identify.

Here events i. and ii. have no common element (outcome). So the event i. and ii. are mutually exclusive events. Also iii. and iv. are mutually exclusive events.

Similarly, events i. and iii. have some common outcomes. So they are not mutually exclusive events.

When two or more events cannot occur at the same time then they are called mutually exclusive events. In other words, the occurrence of one event will prevent the occurrence of another event. In tossing a coin the occurrence of head prevents the occurrence of tail. So, getting head and getting tail are mutually exclusive events.

Example 1:

Judge/Identify whether the following pair of events are mutually exclusive or not.

(a) Getting odd number and getting even number while rolling a dice.

(b) Getting exactly two heads and at least one head when two coins tossed together.

(c) Drawing a king and an ace from a deck of cards.

Solution:

(a) Sample space S = {1, 2, 3, 4, 5, 6}

First event A = {getting odd number} = {1, 3, 5}

Second event B = {getting event number} = {2, 4, 6}

Since A and B do not have common outcome, they are mutually exclusive events.

(b) S = {HH, HT, TH, TT}

A = getting exactly two head = {HH}

B = getting at least one head = {HT, TH, HH}

Since outcome 'HH' is common to A and B, they are not mutually exclusive events.

(c) While drawing a card the sample space contains different 52 cards.

A = getting ace = {♣A, ♦A, ♥A, ♠A}

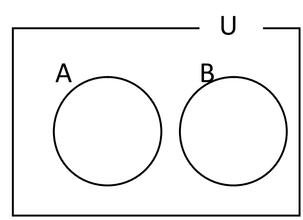
B = getting king = {{♣K, ♦K, ♥K, ♠K}}

Since event A and B have no common outcome, they are mutually exclusive events.

18.1.2. Additive law of probability "OR (SUM) rule of probability"

A coin is tossed. Then the events A = Turning of H = {H}

B = Turing of T = {T}



Also $A \cap B = \phi$ then we can write P(A) = 1/2 and P(B) = 1/2

P(A) + P(B) = 1 ....i.

Also, $A \cup B$ = {H,T}

$P(A \cup B)$ = 2/2 = 1 ……………ii.

from i. and ii.

$P(A \cup B) = P(A) + P(B)$

Where A and B are mutually exclusive events.

In another way, if A and B are two mutually exclusive events then they are disjoint subsets of sample space S.

Then $n(A \cup B) = n(A) + n(B)$

Dividing both side by n(S)

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$$

Therefore,

$P(A \cup B) = P(A) + P(B)$ is called additive law for mutually exclusive events of a sample space S.

Similarly, if A, B and C are three mutually exclusive events of sample space S, then we can write P(A∪B∪C) = P(A) + P(B) + P(C) and so on.

Example 2:

What will be the probability of getting both head or both tail in tossing a coin twice?

Solution:

The sample space S = {HH, HT, TH, TT}

$\therefore$ n(S) = 4

let event A = getting of both heads = {HH}

$\therefore$ n(S) = 1

event B = getting of both tails = {TT}

$\therefore$ n(N) = 1

Now, P(A) = $\frac{1}{4}$ and P(B) = $\frac{1}{4}$

P(getting of both heads or both tails} = P(A∪B) = ?

We have P(A∪B) = P(A) + P(B)

$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Example 3:

Find the probability of getting either diamond or a black card from a well shuffled pack of 52 cards.

Solution:

If a deck of cards have 52 cards $\therefore$ n(S) = 52

let A = getting a diamond

$\therefore$ n(A) = 13

P(A) = $\frac{13}{52}$

B = getting black card

$\therefore$ n(B) = 26 [all cards of club and spade]

P(B) = $\frac{26}{52}$

Since diamond is red, A and B are mutually exclusive events.

$\therefore$ P(A∪B) = P(A) + P(B) = $\frac{13}{52} + \frac{26}{52} = \frac{3}{4}$

Example 4:

A bag contains 5 red, 8 green and 7 blue identical balls. What will be the probability of getting a red ball or a green ball when a ball is drawn randomly?

Solution:

Here total balls = 5 + 8 + 7 = 20

Let, R = getting red balls. So, P(R) = $\frac{5}{20}$

G = getting green ball. So P(G) = $\frac{8}{20}$ and

B = getting blue ball So, P(B) = $\frac{7}{20}$

Since all balls are of distinct colours, events R, G and B are mutually exclusive.

So P(R or G) = P (R∪G) = P(R) + P(G)

$$= \frac{5}{20} + \frac{8}{20} = \frac{13}{20}$$

Example 5:

Find the probability of getting a letter M or T from the word "MATHEMATICS" when a letter is selected randomly.

Solution:

We have S = {M, A, T, H, E, M, A, T, I, C, S}

n(S) = 11

Let, A = getting 'M'

∴ n(A) = 2

B = getting 'T'

∴ n(B) = 2,

P(A∪B) = ?

Here A and B are mutually exclusive events

∴ P(P or B) = (A∪B) = P(A) + P(B)

$$= \frac{2}{11} + \frac{2}{11} = \frac{4}{11}$$

Example 6:

In a community survey of some women, the following data is found.

Job	**No of women**
Teacher	25
Farmer	35
Administrator	15
Doctor	5

A woman is selected randomly. What will be the probability that she is either a farmer or a doctor?

Solution:

Here, total number of women is 80.

$\therefore$ n(S) = 80

Let, A be the set of the women farmers

$\therefore$ n(A) = 35

B be the set of women doctors

$\therefore$ n(B) = 5

P(A or B) = P(A$\cup$B) = ?

Since here the women farmer is not a doctor, the events are mutually exclusive.

$\therefore$ P(A$\cup$B) = P(A) + P(B)

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} = \frac{35}{80} + \frac{5}{80} = \frac{40}{80} = \frac{1}{2}$$

* In case of not mutually inclusive events, event A and event B have some common outcomes, then we can write by using set theory.

n(A$\cup$B) = n(A) +n(B) -n(A$\cap$B)

Dividing both side by n(S)

$$\frac{\mathrm{n}(A\cup B)}{\mathrm{n}(S)} = \frac{\mathrm{n}(A)}{\mathrm{n}(S)} + \frac{\mathrm{n}(B)}{\mathrm{n}(S)} - \frac{\mathrm{n}(A\cap B)}{\mathrm{n}(S)}$$

P(A$\cup$B) = P(A) + P(B) - P(A$\cap$B)

Example 7:

A card is drawn from a well shuffled deck of 52 cards. What will be the probability of getting a diamond or a face card?

Solution:

Here, n(S) = 52

Let A = getting diamond

$\therefore$ n(A) = 13

B = getting face card

$\therefore$n(B) = 12

P(A) = $\frac{13}{52}$ and P(B) = $\frac{12}{52}$

Since there are 3 face cards in diamond n(A$\cap$B) = 3

$\therefore$ (A$\cap$B) = $\frac{3}{52}$

$\therefore$ P(A or B) = (A $\cup$ B) = P(A) + P(B) - P(A$\cap$B)

$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}$$

Exercise 18.1

1. Judge which of the following events are mutually exclusive and which are not?
   (a) A: a black card and B: a queen, in card drawn from a pack of 52 cards.
   (b) A: at least one head and B: 2 tails, in a simultaneous toss of two coins.
   (c) A: a total of 9 and B: Both dice have odd number; in a simultaneous throw of two dice together.
   (d) A: 2 heads and B: at least 2 heads in three successive tosses of the coins
   (e) A: a multiple of 3 and B: a multiple of 7, in a single draw of a card from a pack numbered from 1 to 20.
2. Find the probability of;
   (a) getting at least one head while tossing two fair coins together.
   (b) getting prime number when a fair dice is rolled once
   (c) getting (i) an ace (ii) a diamond (iii) a face card in a draw of card from well shuffled deck of 52 cards
   (d) getting (i) total of 11 (ii) total of 9 in simultaneous throw of two fair dice together.
   (e) Obtaining a yellow ball from a bag of 5 red, 10 yellow and 7 pink identical balls in a single pick.

3.(a) What will be the probability of getting at least one tail or no tail in a single toss of two coins? Find it.

(b) Find probability of getting three heads or three tails when three fair coins are tossed simultaneously.

(c) Find the probability of getting a prime number or getting 6 in a single throw of a die.

(d) What will be the probability of getting total of 8 or 11 in a single throw of two dice? Find it.

4. A card is drawn randomly from a well shuffled deck of card. Find the probability of;
   (a) getting an ace and a jack.
   (b) getting a spade or a red card.
   (c) getting a 5 or a 6.
   (d) getting an ace or a Jack or a king.
   (e) getting a face card or a 7.

5.(a) Find the probability of getting a multiple of 6 or a multiple of 7 when a number card is drawn from a pack of number cards from 1 to 40.

(b) What will be the probability of getting a letter 'S' or a letter 'U' from the word 'SUCCESSFULNESS' when a letter is drawn randomly? Find it.

(c) A number is drawn from the bag of identical ball numbered from 1 to 50. Find the probability of obtaining a multiple of 2 or a multiple of 11.

(d) An urn contains 7 blue, 8 green, 10 black and 5 yellow identical marbles. If a marble is drawn randomly what is the probability that the marble is a black or a green or a yellow? Find it.

6. In a survey of students of a college about the use of communication the following information is found.

Communication	Landline	Internet	Smart phone	Cell phone
No. of students	12	20	25	8

If a student is chosen at random, what is the probability that he/she is;

(a) either using land line or smart phone?

(b) either using smart phone or cell phone?

(c) using land line or internet or cell phone?

7. In SLC exam of 2072 a certain school obtained the following result in compulsory mathematics.

Grade	A^+	A	B^+	B	C^+	C	D^+
Number of students	8	10	12	12	6	5	2

Find the probability of;

(a) having grade A^+ or B^+ or C^+

(b) having grade A^+ or A but not other.

(c) having grade A or B or C but not A^+, B^+, C^+ or D^+

8. A card is drawn from a well shuffled deck of 52 cards. Find the probability of;

(a) getting an ace or a black card.

(b) getting a diamond or a king.

(c) getting a face card or a queen.

(d) getting a red card or 5 or 6.

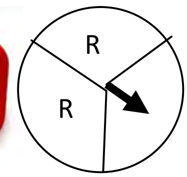
(e) getting a spade or a face card.

18.2. Multiplication rule of probability [AND rule]

Before discussing multiplication rules we need to know about independent and dependent events. Let's consider the following example.

A die is rolled and a spinner with red, yellow and green colour is spun together. The sample space is given as;

	Die	**1**	**2**	**3**	**4**	**5**	**6**
Spinner	R	(R,1)	(R,2)	(R,3)	(R,4)	(R,5)	(R,6)
	Y	(Y,1)	(Y,2)	(Y,3)	(Y,4)	(Y,5)	(Y,6)
	G	(G,1)	(G,2)	(G,3)	(G,4)	(G,5)	(G,6)



Here, the way in which the die lands does not affect the possible ways in which spinner can land and conversely. So getting 5 on die does not affect in getting 'Red' in spinner. So these two events are called independent events.

> If A and B are two events of sample space S. Then events A and B are said to be independent events if the occurrence (or non-occurrence) of one event has no effect on the occurrence (or non occurrence) of the other event. For example, A is an event of getting a head on coin and B is the event of getting 1 on a dice when they are tossed simultaneously, then A and B are called independent events.

18.81. The multiplication law of probability

On the above discussion, (the dice rolled and spinner spun), let A as getting 'red' in spinner and B as getting 5 in dice, then A and B are independent events.

The probability of red in spinner and 5 on a dice is probability of red and 5.

i.e. P(Red and 5) = P(A and B)

The total sample space S is:

S = {(R, 1), (R,2), (R,3), (2,4), (R,5), (R,6), (Y,1), (Y,2), (Y,3), (Y,4), (Y,5), (Y,6), (G,1), (G,2), (G,3), (G,4), (G,5), (G,6)

n(S) = 18

Then, P(Red and 5) = P (A and B) = $\frac{1}{18}$

Also P(Red) = $\frac{1}{3}$ and P(5) = $\frac{1}{6}$

$\therefore$ P(Red) x P(5) = P(A) x P(B) = $\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

$\therefore$ P(A and B) = P(A) x P(B)

> If two events A and B are independent events of sample space S, then the probability of the occurrence of A and B is equal to the product of the probability of occurrence of A and the probability of occurrence of B. This law is called multiplication law of probability. i.e. P(A and B) = P(A) x P(B) or P(A $\cap$ B) = P(A) x P(B)

Note: If A, B and C are three independent events of a sample space S, then

P(A and B and C) = $P(A \cap B \cap C)$ = P(A) x P(B) x P(C)

Example 1

Identify whether the following events are independent or not.

(a) getting tail on the first coin and head on the second coin after tossing two coins simultaneously

(b) getting 5 on the first dice and odd number on the second dice while rolling two dice.

(c) getting head on a coin and even number on a dice when a coin is tossed and a die is rolled simultaneously.

(d) picking an ace in the first draw and queen in the second pick form a deck of 52 card without replacement.

Solution:

(a) Independent: since the occurrence of tail does not affect the occurrence of head in the second dice.

(b) The occurrence of 6 does not affect the occurrence of odd number in the second. So independent.

(c) Independent: (Why?)

(d) Not independent (Why?)

Example 2:

A coin is tossed and a die is rolled simultaneously. Find the probability of getting head on the coin and even number on the die.

Solution:

For coin: S = {H, T}

Let A: an event of a head = {H}

$\therefore$ n(A) = 1

P(A) = 1/2

for die S = {1, 2, 3, 4, 5, 6}

B = an event of even number = {2, 4, 6}

$\therefore$ n(B) = 3

P(B) = $\frac{3}{6}$

P(head and even number) = P(A and B) = $P(A \cap B)$ = ?

$P(A \cap B)$ = P(A). P(B)

$= \frac{1}{2} \times \frac{3}{6} = \frac{3}{12} = \frac{1}{4}$

Alternatively,

S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

= {H, 1), (H, 2), (H, 3), ...}

$\therefore$ n(S) = 12

Let, A: getting head B: getting even number

P(A and B) = ?

$A \cap B = \{(H, 2), (H, 4), (H, 6)\}$

$\therefore n(A \cap B) = 3$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{12} = \frac{1}{4}$

Example 3:

Two cards are drawn from a well shuffled deck of 52 cards one after another with replacement before the second draw. Find the probability that both of them are black cards.

Solution:

Here, n(S) = total no. of cards = 52

Let A = no. of black cards

n(A) = 26

Let A_1: The first draw in black card. $P(A_1) = \frac{n(A_1)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

A_2: The second draw in black card.

Since second draw is after the replacement of the first card;

$\therefore n(A_2) = 26, P(A_2) = \frac{n(A_2)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

$\therefore$ The probability of both black cards $= P(A_1 \cap A_2)$

$= P(A_1) \times P(A_2) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Example 4:

An urn contains 5 black, 8 green and 12 pink identical balls. If two balls are drawn one after another with replacement, find the probability of;

i. the first ball is black and the second is pink.

ii. the first ball is green and the second is pink.

iii. both are green balls

Solution:

Total no. of balls = 5 + 8 + 12 = 25

P(B) = P(black ball) = $\frac{5}{25} = \frac{1}{5}$

P(P) = P(pink ball) = $\frac{12}{25}$

P(G) = P(green ball) = $\frac{8}{25}$

i. P(black and pink ball) = P(B ∩ P) = P(B).P(P) = $\frac{1}{5} \times \frac{12}{25} = \frac{12}{125}$

ii. P(green and pink ball) = P(G ∩ P) = P(G).P(P) = $\frac{8}{25} \times \frac{12}{25} = \frac{96}{625}$

iii. P(both green) = P(G ∩ G) = P(G) x P(G) = $\frac{8}{25} \times \frac{8}{25} = \frac{64}{625}$

Examples 5:

The events X and Y are such that P(X) = $\frac{1}{7}$ and P(Y) = $\frac{3}{7}$. If X and Y are independent, find P(X ∩ Y) and P(X ∪ Y).

Solution:

Here, P(X) = $\frac{1}{7}$ and P(Y) = $\frac{3}{7}$

Since X and Y are independent, P(X∩Y) = P(X).P(Y) = $\frac{1}{7} \times \frac{3}{7} = \frac{3}{49}$

Again, we have P(X ∪ Y) = P(X) + P(Y) - P(X ∩ Y)

$$= \frac{1}{7} + \frac{3}{7} - \frac{3}{49}$$

$$= \frac{4}{7} - \frac{3}{49} = \frac{28-3}{49} = \frac{25}{49}$$

Exercise 18.2

1. Define independent events of an experiment.
2. What do you mean by multiplicative rule of probability? Give an example.
3. Identify whether the following events are independent or not.
   (a) getting head on the first coin and tail on the second coin in a simultaneous toss of two coins.
   (b) getting the first card an ace and the second card a king without replacement.
   (c) landing in red colour of spinner and getting tail on coin.
   (d) getting both even numbers in a simultaneous rolling of two dice.

4.(a) Two coins are tossed together. What will be the probability of getting tail on both coins? Find it.

(b) Three coins are tossed at a time, what is the probability of getting the head on the first coin, tail on the second and third coins? Find it.

(c) A coin and a dice are thrown together. What is the probability of the coin landing on tail and the die landing on odd number? Find it.

(d) Two dice are rolled together. Find the probability of the first die land on even and second on 1.

5.(a) Two cards are drawn from a well shuffled deck of 52 cards one after another with replacement. What will be the probability of;

i. the first card is 10 and the second is a face card?

ii. the first card is a spade and the second is a red card?

(b) Two identical marbles are drawn one after another with replacement from an urn containing marbles numbering from 1 to 50. Find the probability that one is the multiple of 7 and the other is the multiple of 8.

(c) What will be the probability of getting a red card and a green card, if two cards are drawn from a well shuffled pack of coloured cards containing 12 red cards, 15 green cards and 10 blue cards with replacement before the second draw? Find it.

(d) Two letters are drawn one after another with replacement from the word 'UNFORTUNATELY'. Find the probability of getting the first letter 'U' and the second N or A.

6(a) An urn contains 12 red, 13 green and 15 yellow identical balls. Two balls are drawn one after another with replacement of the first before the second drawn. Find the probability of;

i. both are yellow.

ii. the first is yellow and the second is red.

iii. the first is red and the second is green.

iv. the first is yellow and the second is red or green.

(b) The probability of two events A and B are 0.85 and 0.75 respectively. If A and B are independent, find the probability of getting event A and event B.

(c) If two children are born in a family. Calculate the probability that;

i. both are boys. ii. the first is a girl and the second is a boy.

(d) Three children are born in a family. Calculate the probability that:

i. all three are sons. ii. two are sons and the third is a daughter.

(e) The probability of that Enjal can solve a problem is $\frac{3}{4}$ and the probability that Susant can solve is $\frac{1}{4}$. If both of them try, what will be the probability that Enjal and Susant both can solve the problems? Also find the probability that Enjal or Susant will solve.

7. Project work

Work in groups of students. Distribute the following objects into each group.

(i) A deck of playing card (ii) A dice and a coin

(iii) A bag of 3 types of different colour balls of the same size.

Pick one card or ball and replace by one after another and find the probability of the combined events.

18.3 Tree diagram

A tree diagram is a schematise representation of all the events and their outcomes of an experiment. The events are denoted by branches of the tree diagram. In a tree diagram, the events are clearly shown and that makes easy to find the probabilities of required events.

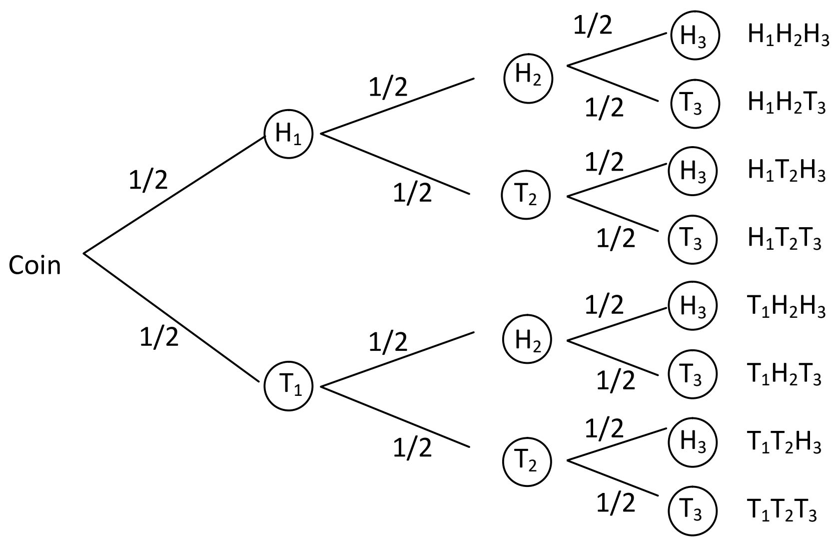
Example 1:

A fair coin is tossed three times. Make a tree diagram and find the probability of obtaining

(i) three heads (ii) at least two heads.

Solution:

The tree diagram of tossing a fair coin for three times is as follows:



∴ The sample space (S) = $\{H_1H_2H_3, H_1H_2T_3, H_1T_2H_3, H_1T_2T_3, T_1H_2H_3, T_1H_2T_3, T_1T_2H_3, T_1T_2T_3\}$

(i) P(all three heads) = $P(H_1H_2H_3) = P(H_1).P(H_2).P(H_3) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

(ii) Again, P(two heads) = $P(H_1 \cap H_2 \cap T_3) + P(H_1 \cap T_2 \cap H_3) + P(T_1 \cap H_2 \cap H_3)$

$$= P(H_1).P(H_2).P(T_1) + P(H_1).\ P(T_2).P(H_1) + P(T_1).P(H_2).P(H_3)$$

$$= \frac{1}{2}.\frac{1}{2}.\frac{1}{2} + \frac{1}{2}.\frac{1}{2}.\frac{1}{2} + \frac{1}{2}.\frac{1}{2}.\frac{1}{2} = \frac{3}{8}$$

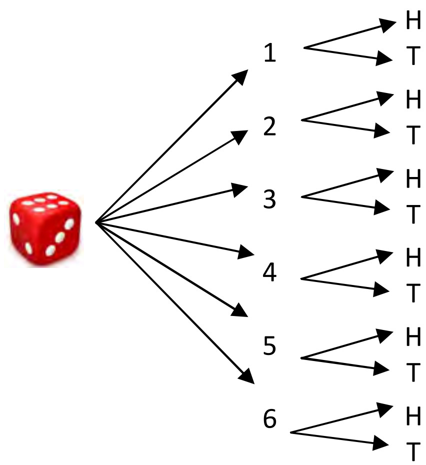
P(at least two heads) = P(exactly three heads) + P(two heads)

$$= \frac{1}{8} + \frac{3}{8} - \frac{4}{8} = \frac{1}{2}$$

Example 2:

Draw a tree diagram of rolling a die and tossing a coin and find the probabilities of each outcomes.

Solution:



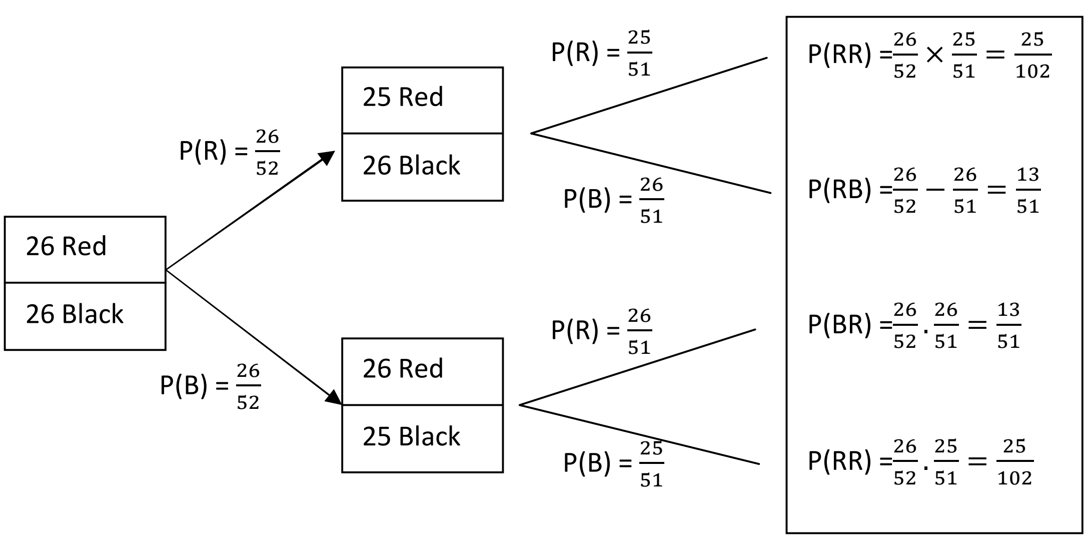
Outcomes	Probabilities
1H	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
1T	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
2H	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
2T	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
3H	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
3T	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
4H	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
4T	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
5H	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
5T	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
6H	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
6T	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$

Example 3:

Two cards are drawn from a well shuffled deck of 52 cards without replacing the first draw. Make a tree diagram and find the probability that both the cards are Red cards.

Solution:

The tree diagram is a follows



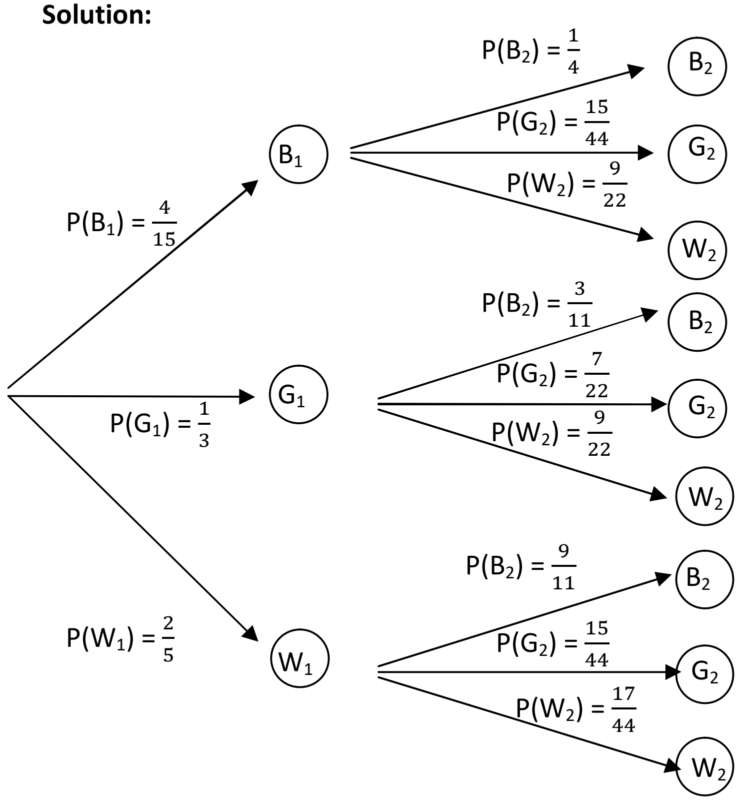
∴ The probability of both cards boing red = P(RR) = $\frac{25}{102}$

In this case, the probability of the second (event) occurrence depends on the probability of the first occurrence. So they are called dependent events.

Example 4:

A bag contains 12 blue, 15 green and 18 white identical balls. Two balls are drawn one after another without replacement. Make a tree diagram of i. both are blue balls ii. one is blue and other is white iii.the first is white and the second is green.

Solution:



$P(B_1B_2) = \frac{4}{15} \times \frac{1}{4} = \frac{1}{15}$

$P(B_1G_2) = \frac{4}{15} \times \frac{15}{44} = \frac{1}{11}$

$P(B_1W_2) = \frac{4}{15} \times \frac{9}{22} = \frac{6}{55}$

$P(G_1B_2) = \frac{1}{3} \times \frac{3}{11} = \frac{1}{11}$

$P(G_1G_2) = \frac{1}{3} \times \frac{7}{22} = \frac{7}{66}$

$P(G_1W_2) = \frac{1}{3} \times \frac{9}{22} = \frac{3}{22}$

$P(W_1B_2) = \frac{2}{5} \times \frac{3}{11} = \frac{6}{55}$

$P(W_1G_2) = \frac{2}{5} \times \frac{15}{44} = \frac{3}{22}$

$P(W_1W_2) = \frac{2}{5} \times \frac{17}{44} = \frac{17}{110}$

By tree diagram,

i. P(both are blue balls) = $P(B_1B_2) = \frac{1}{15}$

ii. P(one blue other white) = $P(B_1W_2) + P(W_1B_2) = \frac{6}{55} + \frac{6}{55} = \frac{12}{55}$

iii. P(first white ball and second green ball) = $P(W_1G_2) = \frac{3}{22}$

Exercise 18.3

1.(a) A fair coin is tossed three times. Make a tree diagram and find the probability of;

i. all three tails. ii. at least two heads. iii. exactly two tails.

(b) A coin is tossed and a spinner with three colours red, blue and green is spun. Make a tree diagram and find;

i. the coin landing on head and red in spinner.

ii. the coin land on head and spinner spun at any colour.

(c) A dice is rolled and a coin is tossed. Draw a tree diagram and find the probability that the dice lands on odd number and the coin on head.

(d) The cards are drawn from a deck of cards with replacement of the first before the second draw. Find the probability that both the cards are club by drawing a tree diagram.

2. The cards are drawn from well shuffled deck of 52 cards one after another without replacement. Draw a tree diagram and find the probability of;

(a) both are diamond.

(b) the first is a diamond and the second is other card.

(c) the first is a face card and the second is not.

(d) both are face cards.

(e) both are same colour cards.

3. Three balls are drawn from a bag containing 7 black and 3 white identical balls. Draw a tree diagram to show possible outcome and find the probability of;

(a) all three white balls.

(b) two white and a black.

(c) two black and a white.

(d) the first black, the second white and the third black.

4.(a) Three children are born in a family. Draw a tree diagram to show the possible outcomes and find the probability that;

i. all the children are boys.

ii. two are boys and a girl.

iii. at least one is a girl.

(b) The probability of winning a game by Amrit is 2/3 and that of Ashish is 1/3. If the play three matches, draw a tree diagram played by them and find the probability that;

(i) all three matches will be won by Amrit.

(ii) first two games by Amrit and third by Ashish.

(iii) Ashish will win at least one game.

5. Divide the class into suitable groups. Then each group has to take one of the following experiment and write the possible outcomes by drawing a tree diagram.

   i. Tossing three coins simultaneously.

   ii. Tossing a coin and rolling a die.

   iii. Rolling two fair dice simultaneously.

   iv. Spinning a 4 coloured spinner and rolling a die.

   v. Drawing two cards one after another without replacement and the cards are different suits.

   Also find the probability of each outcome and present to your teacher.

Answer sets

Exercise 1.1

1. (a) 6 (b) 7 (c) 3 (d) 10 (e) 3 (f) 4 (g)3

2. Show your teacher.

3.(a) [i.140 ii.440] (b) 70 (c) 11

4.(a) (i) 35% (ii) 10% (b) (i) 46% (ii) 282

5.(a) 23, 13 (b) (i) 15 (ii) 45 (iii) 70

Exercise 1.2

1. (a) 8 (b) 2 (c) 1 (d) 2 (e) 4 (f) 9

2. (a) 9 (b) 57 (c) 2 (d) 9 (e) 44 (f) 8

3. Show your teacher. 4.(a) 10 (b)11, 59, 25 (c) 40, 135

5.(b) (i) 21 (ii) 59 (iii) 15 (c) (i) 30 (ii) 125

5. (a)(i) 5 (ii) 52 (b) (i) 45% (ii) 5%

6. (c) (i) 300 (ii) 120

7. (a)(i) 10 (ii) 30 (b) (i) 75% (ii) 35% (iii) 35% (iv) 25%

Exercise 2.1

1. Show your teacher

2.(a) Rs. 4746 (b) Rs. 14949.9 (c) Rs. 75,145 (d) Rs. 1,19,328 (e) 4,29,400

3.a() Rs. 6000 (b) Rs. 5000 (c) Rs. 7,80,000 (d) Rs. 1,60,000 (e) Rs. 1,70,000

4.(a) Rs. 6441 (b) Rs. 5582.20 (c) Rs. 1,88,145 (d) Rs. 15368

5.(a) Rs. 7500, Rs. 825.75 (b) 12870, Rs. 11000 (c) Rs. 2223, Rs. 18,000
(d) Rs. 43200, Rs. 41284

6. (a) Rs. 113 (b) Rs. 7777.77; Rs. 7910 (c) Rs. 250,000; Rs. 3,02,275 (d) 13%

7. (a) Rs. 8,3660 (b) Rs. 6619.43

Exercise 2.2

1. (a) Rs. 2120 (b) Rs. 14,8116.5 (c) Rs. 91,455 (d) Rs. 69322.5
(e) Rs. 622717.2 (f) Rs. 720154.8 (g) Rs. 128239.5 (h) Rs. 34896

(i) Rs. 2085373 (j) Rs. 149568

2. (a) Rs. 40370.4 (b) Rs. 133640; Rs. 427648 (c) Rs. 49422 (d) Rs. 69,992.4

3. (a) $ 321 (b) 85 (c) 4700 (d)(i) 80468.085 yen

(ii) 9231.14 (iii) 7082.39 (iv) 472750 (v) 8063965.88 (vi) 6656.10

4. (a) 76.72 (b) 5.88 (c) 881.25 (d) 359.55 (e) 2646.95

(f) 451764

5. (a) Rs. 49,34,250 (b) Rs. 11,84,718.6 (c) 66,103.57 J.Y. (d) Rs. 15,58,000

6. (a) Rs. 85555.20 (b) Rs. 28333.35 (c) 3.94%

Exercise 3

2.(a) Rs. 1764 (b) 2.5years (c) Rs. 17,500 (d) 10.5%

3.(a)(i) Rs. 5796, Rs. 45796 (ii) Rs. 13555.75, Rs. 99555.75

(iii) Rs. 11,4695, Rs. 1129695 (iv) Rs. 1785.68, Rs. 11795.68 (b) same as (a)

4. (a) Rs. 23152.5, Rs. 3152.5 (b) Rs. 66550, Rs. 16550

(c) Rs. 173522.55; Rs. 23522.55 (d) Rs. 506250; Rs. 106250

5.(a) Rs. 1693.40 (b) 3906.25 (c) Rs. 5508 (d) Rs. 1171.88

6.(a) Rs. 862.02 (b) Rs. 54080, Rs. 4080 (c) Rs. 3397.78 (d) Rosani Rs. 3241.62

7.(a) 1600 (b) 1066.11 (c) Rs. 80,400 (d) Rs. 7500 (e) Rs. 16,000

8.(a) Rs. 55,000 (b) 10%, Rs. 12,000 (c) 15%, Rs. 8000 (d) Rs. 20,000

9.(a)(i) Rs. 23170 (ii) Rs. 23806.70 (b) Rs. 21,000, Rs. 20,000

(c) (d) 2yr (e) 2 yr

Exercise 4.1

1. (a) 2144415 (b) 27214685 (c) 6655 (d) 152756

2. (a) 1151 (b) 122982 (c) 4.33 (d) Rs. 3016.65

3.(a) 4000 (b) 188×10^n (c) Rs. 495867.80

4.(a) 5% (b) 4% (c) 2 years (d) 3 years

5.(a) 167,076 (b) 10000 (c) 62492 (d) 7009

Exercise 4.2

1.(a) Rs 2187 (b) Rs 1049760 (c) Rs 196520 (d) 2.07×10^7

2.(a) 20% (b) 25% (c) 3yrs (d) 2 yrs

3.(a) Rs 1,00,000 (b) Gain Rs 11736 (c) i.Rs.90,000, ii.Rs.59049 (d) Rs26500

4.(a) 30 (b) Rs 8,77,500 , 8775

Exercise 5.1

1. a) 24 cm^2 b) 30cm^2 c) 84cm^2 d) 12cm^2 e) $16\sqrt{3}$ cm^2

2. a) 179.9 cm^2 b) 90.51 cm^2 c) 253.24cm^2 d) 24cm^2

3. a) 32cm^2 b) 45cm^2 c) 48cm^2 d) 27cm^2

4. a) $6\sqrt{6}cm^2$ b) $36\sqrt{3}cm^2$ c) $25\sqrt{3}cm^2$ d) 6cm

5. a) 11.2cm b) 26cm c) 16cm, 12 cm^2 d) 24cm, 64cm

6. a) 9000cm^2 b) 336cm^2 c) 3cm, 4cm, 5cm d) 336cm^2

7. Show your teacher.

Exercise 6.1

1. a) 440cm^2 , 597.14cm^2 b) 176cm^2, 253cm^2 c) 290.4cm^2, 401.28cm^2.

2. a) 198cm^2, 225.72cm^2 b) 1628cm^2 c) 3080cm^2.

3) 385cm^2 4) 2002cm^2 5) 17.5cm 6) 1848cm^2 7) 17248cm^3

8) 1558.85cm^3 9) 3234cm^3 10) 3.5cm, 80cm 11) 5cm

12) 14cm, 4cm 13) 1cm, 3960cm^3 14) 2.156 kg 15) 44.88cm

Exercise 6.2

1. a) 154cm^2 b) 616cm^2 c) 1386cm^2 d) 5544cm^2

2. a) 38.8cm^3 b) 11498.66cm^3 c) 195.51cm^3 d) 310.46cm^3

3. a) 314.28cm^2, 523.8cm^3. b) 559.02cm^2, 956.54cm^3

4. a) 7cm b) 3.5cm 5.a) 1437.33cm^3 b) 11498.66cm^2.

6. a) 10.5cm b) 3cm 7.a) 110.88cm^2, 166.32cm^2

b) 81.46 cm^2, 122.19 cm^2

8. a) 942.86cm^2 b) 9355.5cm^2.

9. a) Surface is 3 times more than the original surface and the volume is 7 times more than the original volume.

b) 3 times the original surface area

10. 1191.31cm^2

11. 18cm 12) 8.32cm 13) 4cm 14) 360cm 15) 2.52cm

Exercise 7.1

1. a) $30cm^2$, $600cm^2$,$660cm^2$ b) $49\sqrt{3}cm^2$, $504cm^2$, $602\sqrt{3}cm^2$

   c) $168cm^2$, $2560cm^2$, $2896cm^2$,

2. a) $270cm^2$ b) $644cm^2$ c) $357cm^2$ d) 16cm

3. a) $1080cm^3$ b) $4500cm^3$ c) $180cm^3$

4. a) 6cm b) 10cm, 10cm, $10\sqrt{2}cm$

5. a) $2100cm^3$ b) 12cm, 72 cm^3

6. $267.76cm^2$, $203.65cm^3$

Exercise 7.2

1. a) $220cm^2$, $374cm^2$ b) $550cm^2$, $704cm^2$ c) 100.57 cm^2, $150.85cm^2$

2. a) $57.75cm^3$ b) $1005.71cm^3$ c) $2514.28cm^3$

3. a) $1386cm^3$, b) $1232cm^3$

4. a) $4004cm^2$ b) $301.71cm^2$ c) $2310cm^2$

5. a) 24cm, $1232cm^3$ b) $50.28cm^3$

6. a) 24cm b) 14cm c) 44cm

7. a) 14cm b) 48cm c) 96cm

8. a) 8.36cm b) 16cm

Exercise 7.3

1. a) $270cm^2$, $351cm^2$ b) $960cm^2$, $1536cm^2$

   c) $240cm^2$, $340cm^2$ d) $672cm^2$, $868cm^2$

2. a) $297cm^3$ b) $12544cm^3$ c) $1568cm^3$

3. a) $360cm^2$, $400cm^3$ b) $340cm^2$, $363.33cm^3$

4. a) 6cm, b) $1280cm^3$ c) $384cm^2$

5. a) $240cm^2$, $384cm^3$ b) $179.37cm^2$

6. a) 10cm b) $1920cm^2$ c) 8cm

Exercise 7.4

1. a) $473cm^2$, $511.5cm^2$, $782.83cm^3$ b) $968cm^2$, $112.2cm^2$, $3028.67cm^3$

   c) $429cm^2$, $438.35cm^2$,$372.16cm^3$

2. a) $2266cm^2$, $2420cm^2$, $7238cm^3$ b) $39.6cm^2$, $42.74cm^2$, $18.23cm^3$,

c) $273.43cm^2$, $301.71cm^2$, $377.14cm^3$

3. a) $203.28cm^2$, $258.72cm^3$ b) $858cm^2$, $1950.66cm^3$

c) $308.88cm^2$, $421.34cm^3$

4. a) $200.2cm^2$, $215.6cm^3$ b) $134.64cm^2$, $115.80cm^3$

c) $7260cm^2$, $46200cm^3$

5. a) 7cm b) 14cm 6.a) $163.42\ m^2$ b) $528\ m^2$

7. a) 8cm b) $1251.58cm^3$ 8.a) $414.86\ cm^2$ b) $11628.57\ cm^3$

9. a) $581.43cm^2$ b) $1257.14cm^2$

10. a) $383.68cm^2$ b) $216.83cm^3$

Exercise 7.5

1.a) 18000 litres b) Rs. 25000

2. a) Rs. 2933.33 b) Rs. 33000

3. Rs. 2016 4. Rs. 16354.8 litres

5. Rs. 38400, 173250 litres. 6. Rs. 2729

Exercise 8.1

1.(a) $4xy$ b) $6x^2y^2$ c) a^2bc d) $5x^2y^3$

2.a) $a + b$ b) $x - 3$ c) $2x + 1$ d) $x + y$

3.a) $x^2 + xy + y^2$ b) $a^2 - a + 1$ c) $4m^2 - 2mn + n^2$ d) $x^2 + 1 + \frac{1}{x^2}$

4.a) $a - 2$ b) $x - 2$ c) $m + 2$ d) $y(2y + 1)$

5.a) $x^2 + xy + y^2$ b) $x^2 + x + 1$ c) $a^2 - ab + b^2$ d) $x^2 - x + 1$

6.a) $a + b + c$ b) $x^2 + x + 1$ c) $2x + 1$ d) 1 e) $4x^2 - 6xy + 9y^2$

Exercise 8.2

1. a) $6x^2y$ b) $30x^2y^2z$ c) $2a^2b^4x$ d) $120a^4b^5c$

2. a) $2(x+y)(x^2-4y^2)$ b) $3(m-2)(m^2-9)$

c) $xy(x+y)^2(x-y)$ (d) $b(a^2-b^2)(a^2+ab+b^2)$

3. a) $(x-y)(x^4+x^2y^2+y^4)$ b) $(a-1)(a^4+a^2+1)$

c) $\left(m-\frac{1}{m}\right)\left(m+1+\frac{1}{m^4}\right)$ d) $a^6 - b^6$

4. a) $2a^2(a^2-4)(a-1)(a^2+2a+4)$ b) $2x(x^2-4)(x+1)(x^2+2x+4)$

c) $y(4y^2-1)(y-1)(4y^2-2y+1)$ d) $x^2(x+1)(x^4+x^2+1)$

5. a) $a^6 - 1$ b) $x(x^6-1)$ c) $m^6 - \frac{1}{n^6}$

6.a) $(a^2-1)(a^2-4)$ b) $x^2(x^6-1)$ c) $(x+y+z)(x+y-z)(y+z-x)(z+x-y)$

d) $2(a+1)(a^2-4)(a^2+2a+4)$

Exercise 9.1

1.a) 3 b) 7 c) P

2.a) $2\sqrt{3}$ b) $6\sqrt{2}$ c) d) $\sqrt[3]{4x^2}$ e) $\sqrt[3]{9x^2y^2}$

3.a) $3\sqrt{2}$ b) $5\sqrt[3]{2}$ c) $2x\sqrt[4]{2x}$

4.a) $\sqrt{20}$ b) $\sqrt[3]{54a^3}$ c) $-\sqrt[3]{320x}$ d) $\sqrt{x^2 - y^2}$

5.a) $\sqrt[6]{9}\ and\ \sqrt[6]{8}$ b) $2\sqrt[12]{16}$, $\sqrt[12]{27}$ and $\sqrt[12]{16}$ c) $\sqrt[6]{9}$, $\sqrt[6]{6}$ and $\sqrt[6]{126}$,

6.a) $\sqrt{3} > \sqrt[3]{2}$ b) $\sqrt[6]{162} > \sqrt{5}$ c) $\sqrt[4]{12} > \sqrt[6]{8}$

7.a) $\sqrt[3]{4}, \sqrt[6]{27}, \sqrt[2]{5}$ b) $\sqrt[4]{8}, 3\sqrt[3]{4}, 2\sqrt[3]{4}$ c) $\sqrt[6]{6}, \sqrt[4]{8}, \sqrt[3]{7}$

8.a) $10x$ b) $30\sqrt{3}$ c) $7a\sqrt[3]{2}$

9.a) $\sqrt{2}$ b) $x\sqrt[3]{3}$ c) $2x\sqrt[4]{y}$

10.a) $3\sqrt{5}$ b) $\frac{3}{\sqrt{2}}$ c) $3\sqrt{2} - 2\sqrt{5}$

11.a) $6\sqrt{30}$ b) $8x\sqrt[6]{243}$ c) $\frac{1}{\sqrt[3]{(a-b)^4}}$

12. a) $5\sqrt{3}$ b) $20\sqrt{2}$ c) $\frac{1}{3\sqrt[6]{12}}$

13.a) $3a^3b^3$ b) $\frac{z^4}{xy^5}$ c) x

14. a) $4a + 4\sqrt{ab} - 3b$ b) $25x - 9y$ c) 1

15.a) $\sqrt{\frac{a+b}{a-b}}$ b) $\frac{1}{(x-y)^2\sqrt[3]{(x-y)^2}}$ c) $\sqrt{\frac{x-y}{x+y}}$

Exercise 9.2

1.a) $\sqrt{5}$ b) $\sqrt{x+1}$ c) $2 + \sqrt{3}$ d) $\sqrt{x} - \sqrt{a}$

2.a) $\frac{5\sqrt{3}}{3}$ b) $3x\sqrt{2}$ c) $\frac{5x\sqrt{6}}{2}$ d) $\frac{5ab\sqrt{2}}{3}$

3.a) $4(\sqrt{3} - \sqrt{2})$ b) $\frac{3}{2}(5 - \sqrt{15})$ c) $\frac{8-\sqrt{55}}{3}$ d) $\frac{144-41\sqrt{6}}{30}$

4.a) $\frac{x-\sqrt{x^2-y^2}}{y}$ b) $-\left(\frac{11+6\sqrt{2}}{7}\right)$ c) $\frac{2x^2+2y}{x^2-y}$ (d) $\frac{2(x+a)}{x-a}$ (e) $\frac{12x\sqrt{y}}{x^2-9y}$

5.a) 5 b) $4a\sqrt{a^2-1}$ c) 0 d) $\frac{1}{5}\left(2\sqrt{30}-7\sqrt{15}+20\right)$

6.a) $a=\frac{27}{23}$ and $b=\frac{10}{23}$ b) 64 c) 10, 98

Exercise 9.3

1.a) 11 b) 16 c) 6

2.a) 3 b) 15 c) 4

3.a) 9 b) 9 c) 4

4.a) – c) No solution

5.a) 9 b) 5 c) 4

6.a) $\frac{64}{29}$ b) 2 c) $\frac{4a}{5}$ d) 1

7.a) 12 b) 36 c) $\frac{1}{16}$ d) 28

8.a) 3a b) ±5 c) $\frac{1}{3}$ d) 5

Exercise 10.1

1.a) $\frac{9}{4}$ b) 72 c) $\frac{3}{2}$ d) 26 e) 21

2.a) $\frac{x(1-x^2y^2)}{y(x-y)}$ b) $\frac{1}{20}$ c) $\frac{6}{11}$ d) $\frac{7}{2}$ e) 1

3.a) 1 b) 1 c) 1 d) 1

4.a) $\left(\frac{x}{y}\right)^{2a}$ b) $\left(\frac{x}{y}\right)^{\frac{a+b}{a-b}}$ c) $\left(\frac{a}{b}\right)^{x+y}$ d) $\frac{y^2}{(p-y)^p}$

5.a) 1 b) 1 c) 1

Exercise 10.2

1.a) 4 b) $\frac{1}{2}$ c) 2 d) ±2 e) 2

2.a) 0 b) 1 c) - 1 d) 3 e) 6

3.a) 0,2 b) 0,1 c) 1 d) 1,2 e) 1,2 f) 0, -3

4.a) ±1 b) ±1 c) 1, 2 d) ±2 e) ± 2 f) -1, 2

Exercise 11.1

1.a) $\frac{-(x+13)}{(x+3)(x-2)}$ b) $\frac{(y-3(y+2)}{(y-6)(y-2)}$ c) $\frac{2(x^2+y^2)}{x^2-y^2}$ d) $\frac{16}{(a+3)(a+5)}$ e) $\frac{3(x^2+y^2)}{x^2-y^2}$ f) $\frac{a+b}{ab}$

2.a) $\frac{1-5a}{(a+1)(a-1)^2}$ b) $\frac{2m^3}{m^2-n^2}$ c) $\frac{4a^2}{x(x+2a)}$ d) $\frac{2y^3}{1-y^2}$ e) $\frac{x-3y}{2x-3y}$ f) $\frac{4xy}{4x^2-y^2}$

3.a) $\frac{3x}{x^2-y^2}$ b) 0 c) $\frac{2}{2a+5}$ d) $\frac{2(x-y)}{x+y}$ e) $\frac{2n}{m^2-n^2}$ f) $\frac{4x^2}{x^2-4a^2}$

4.a) $\frac{4-x}{(x-1)(x-2)(x-3)}$ b) 0 c) $\frac{9x-19}{(x-1)(x-2)(x-3)}$ d) $\frac{3x-7}{(x-2)(x-3)}$

5.a) o b) $\frac{2(y-1).y^2}{1-y+y^2}$ c) $\frac{-4p}{1+p^2+p^4}$ d) 2

6.a) $\frac{2(a-b)}{a^2-ab+b^2}$ b) $\frac{2(x-2)}{x^2-2x+4}$ c) $\frac{2(a+x)}{a^2+ax+x^2}$

7.a) 1 b) 1 c) 0

8.a) 0 b) 0 c) $\frac{2b}{4x^2-1}$ d)) $\frac{\sqrt{x}}{2(1-x)}$

9. a) $\frac{8}{1-x^8}$ b) $\frac{8a^7}{a^8-b^8}$ c) $\frac{1}{(1-x)(x^3+1)}$

10. a) $\frac{1}{1-x}$ b) 1 c) $\frac{1}{a-1}$

Exercise 12.1

1.a) x = 10, y = 7 b) x = 3, y = 1 c) x = 4, y = 5

d) x = 6, y = 10 e) x = 1, y = 1

2.a) 17, 12 b) 60°, 45° c) 9, 27

3.a) 1400m^2 b) 6300m^2 c) length = 40m, breadth = 30m

4.a) Rs. 1500, Rs. 700 b) Rs. 210, Rs. 90 c) Rs. 700, Rs. 400

5.a) 47 years, 7 years b) 19 years, 15 years c) 26 years, 6 years

6.a) 63 b) 38 c) 75

7.a) $\frac{11}{13}$ b) $\frac{2}{3}$ c) $\frac{4}{7}$

8.a) 12:00 noon b) 15km/hr, 11km/hr

Exercise 12.2

1.a) ±6 b) 1,8 c) $-\frac{1}{2},\frac{2}{3}$ d) $1 \pm \sqrt{3}$ e) -1, $\frac{3}{5}$

2. a) ±5 b) 8 c) +5, 24

3.a) 7, 8 b) 11, 13 c) 4, 6

4.a) 5 years b) 3 years c) 10 years, 16 years

5.a) 45 b) 45 c) 53

6.a) 34 ft b) 30ft, 25 ft c) 28 m

7. a) 5cm, 12cm b) $6\sqrt{10}cm$, $2\sqrt{10}cm$ c) 6cm, 8cm, 10cm

Exercise 13

1) $10cm^2$ 2) 5cm 3) 10cm 4) 15 cm 5) 10cm 6) $36cm^2$

Exercise 15.1

1. (a) 100^o, 40^o,40^o (b) 40^o, 40^o, 100^o (c) $\angle$ADB = 25^o and $\angle$DOC = 50^o
2. (a) $\angle$BEC = 55^o (b) $x^o = 40^o$, $\angle A = 80^o$, $\angle C = 100^o$, $\angle B = 100^o$

   (c) $\widehat{AD}^{\cdot} = 90^o$ $\widehat{BC}^{\cdot} = 100^o$
3. (a) $x = 40^o$ (b) $\angle$QPS = 30^o (c) $x^o = 40^o$, $z = 120^o$, $y = 80^o$

Exercise 15.2

1. (a) QR = 12cm (b) ABF = 40^o (c) $\angle$QAP = 70^o
2. (a) $\angle$POR = 80^o (b) $\angle$PMR = 60^o (c) $x = 44^o$, $\angle$DCB = 22^o

Exercise 16.1

1. (a) 22.5 cm^2 (b) 42 cm^2 (c) $18\sqrt{3}\ cm^2$ (d) 9 cm^2 (e) $36\sqrt{3}\ cm^2$
2. (a) 6 cm^2 (b) $15\sqrt{3}\ cm^2$ (c) $11\sqrt{3}\ cm^2$ (d) $40\sqrt{3}\ cm^2$

   (e) 18 cm^2 (f) 30°
3. (a) $32\sqrt{3}\ cm^2$ (b) 30 cm^2 (c) 75 cm^2
4. (a) 10.5 cm^2 (b) AD=4cm, BC=32cm (c) DC=20/3 cm
5. (a) $75\sqrt{3}\ cm^2$ (b) 144 cm^2 (c) $60\sqrt{2}\ sq\ cm$
6. (a) $91\sqrt{3}\ cm^2$ (b) 42 sq. cm and 84 sq. cm (c) $18(\sqrt{3}+1) sq\ cm$

   (d) $48\sqrt{3}\ sq\ cm$ (e) 40sq. cm

Exercise 16.2

1 (a) 28 m (b) 60° (c) $36\sqrt{3}\ m$ (d) 53.46m

2 (a) 30° (b) 45m (c) 60° (d) 36m

3 (a) 45° (b) 40m (c) 28m (d) 30m

4 (a) 30° (b) 54m (c) 108m (d) 34m and 17 respectively

5 (a) 30°, $20\sqrt{3}\ m$ (b) $20\sqrt{3}\ m$ (c) $52\sqrt{3}\ m$ (d) $5\sqrt{3}\ m$

6 (a) $130\sqrt{3}\ m$ (b) 60° (c) 30° (d) 60°, $12\sqrt{3}\ m$

7 (a) $28\sqrt{3}\ m$, $88\sqrt{3}\ m$ (b) 60m (c) 95 m (d) 180m, 565.71m

Exercise 17.2

1.(a) 46.875 (b) 16 (c) 10.32 (d)12.95

2. (a) 24 (b) 43.86 (c) 706.67 (d) 28.75

3. (i) (a) 24 (b) 43.86 (c) 706.67 (d) 28.75

   (ii) (a) 24 (b) 43.86 (c) 706.67 (d) 28.75

4.(a) 10 (b) 818 (c) -100 (d) 35

5.(a) 25 (b) 150 (c) 12 (d) 50

6.(a) show your teacher.

7.(a) 29.5 (b) 1507

Exercise 17.3

1. (a) 3.6 (b) 121 (c) 34 (d) 140

2. (a) 64.54kg (b) 157.5 cm (c) Rs. 246.15 (d) 40.90

3. (a) 6 (b) 3 (c) 150

4. (a) *78.33* (b) *80* (c) *703.70* (d) *25.5*

5.(a) 40 (b) 157.5

Exercise 17.4

1. (a) 12, 22 (b) 155, 225 (c) 53, 65 (d) 215, 225 (e) 55, 65

2. (a) 7.54, 13.91 (b) 32.08, 64.09 (c) 121.33, 152.14
   (d) 230.35, 334.5 (e) 40,98.33

3.(a) *8* (b) *10* (c) *16* (d) *30*

4. (a) 137.69, 148 (b) 136.10, 169.5

5.(a) (b) Show your teacher.

Exercise 17.5

1 -5. Show your teacher.

Exercise 18.1

1. (a) not	(b) yes	(c) yes	(d) not	(e) yes
2. (a) 3/4	(b)1/2	(c)1/13,1/4,3/13	(d)1/18,1/9	(e)5/11
3. (a) 1	(b) ¼	(c) 2/3	(d) 7/36	
4. (a) 2/3	(b)3/4	(c) 4/13	(d) 3/13	(e)4/13
5. (a) 11/40	(b)5/4	(c) 1/5	(d) 23/30	
6. (a) 37/65	(b) 33/65	(c) 8/13		
7. (a) 26/55	(b) 18/55	(c) 27/55		
8.(a) 7/13	(b) 4/13	(c) 15/52	(d) 15/26	(e) 11/26

Exercise 18.2

3. (a) Independent	(b) Dependent	(c) Independent	(d) Independent	
4. (a) ¼	(b)1/8	(c) 1/4	(d) 1/12	
5. (a)3/169,1/8	(b)21/1250	(c) 180/1369	(d) 6/169	
6. (a) i. 9/64	ii. 9/80	iii. 39/400	iv. 15/64	(b)0.6375
(c)1/4,1/4	(d) 1/8,1/8	(e) 3/16;13/16		

Exercise 18.3

1.(a)1/8 , 1/2, 3/8	(b) 1/6, 1/2	(c) 1/4	(d) 1/16	
2.(a)3/17	(b) 13/68	(c) 40/24	(d) 11/221	(e) 1/2
3.(a) 27/1000	(b) 189/1000	(c) 441/1000	(d) 63/1000	
4.(a) 1/8 ; 3/8 ; 7/8	(b) 8/23; 4/27; 19/27			